Optimal Penalty Level, Manipulation, and Investment Efficiency

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Abstract

In this study we examine whether it is efficient to impose a penalty based on an earlier optimistic signal and a bad outcome, and what should be the optimal penalty level to maximize the overall efficiency. We find that imposing a penalty helps firms with good projects to stand out from bad firms, thus helps to improve the investment efficiency, but it also brings a "signalling" cost for good firms to deter bad firms' mimicking. We show that when a good firm has a much larger chance to achieve a good outcome than a bad firm or when a large proportion of the penalty can be reimbursed to the investor, imposing the penalty to achieve the least-cost separating equilibrium is optimal, because the benefit from eliminating the investment inefficiency outweighs the expected signalling cost. On the other hand when even a good firm has a high chance to get bad outcome and there is not a big difference between good and bad projects, and when the reimbursement proportion is very small, imposing no penalty and allowing a pooling equilibrium become optimal, because the benefit of distinguishing firm-types is small while the cost of distinguishing firms is high.

We also consider introducing an ex-ante direct cost of manipulation besides the ex-post penalty. We show that with the introduction of the direct cost, the initial information quality before manipulation becomes relevant and plays a role in determining the optimal penalty level. In addition, imposing a sufficiently-high penalty has one more benefit of reducing the expected direct manipulation cost because the penalty dampens the firm's manipulation.

1 Introduction

It has been widely acknowledged that firms' financial accounting reports, although regarded to be the best source of financial information, are limited in revealing information fully and truthfully to financial statements users. Although in many cases firms try to improve the informativeness of their financial reports, in some other cases firms may have incentives to exploit the flexibility of GAAP and provide misleading financial information. To fight misleading manipulations of financial information, the SEC and the FASB have taken numerous measures to counter-balance opportunistic financial reporting practices to protect investors, including imposing stricter scrutiny and higher penalties. In reality, a firm's poor performance following optimistic accounting information usually triggers class action lawsuits and investigations by the SEC, and results in both monetary and non-monetary costs to the firm, which we refer to as the "penalty" in our paper.[1]

Our study examines whether it is efficient to impose a penalty based on an earlier optimistic signal and a bad outcome, and what should be the optimal penalty level to maximize the overall efficiency. We study a setting in which both firms with good projects (which we call "good firms") and firms with bad projects (which we call "bad firms") are able to manipulate the realization of a public signal regarding their types. The public signal is used by a representative investor who decides whether to fund the project. We assume that a penalty may be charged to a firm if the outcome of its project is bad while the previous signal was good.

We start with a setting in which a firm can freely choose between a high signal and a low signal, but faces a penalty if it releases a high signal but the later outcome is bad. We find that imposing a penalty helps good firms to stand out from bad firms by releasing high signals, thus helps to improve the investment efficiency, but it also brings a "signalling" cost for the good firms to deter the bad firms' mimicking. We show that when the good firm has a much larger chance to achieve a good outcome than the bad firm or when a large proportion of the penalty can be reimbursed to the investor, imposing the penalty to achieve the least-cost separating equilibrium is optimal to maximize the overall efficiency. This is because in this case by imposing the least-cost separating penalty the benefit from eliminating the investment inefficiency outweighs the expected signalling

[1] Karpoff, Lee, and Martin (2008) estimate that the reputational penalty imposed by the market (which is defined as the expected loss in the present value of future cash flows due to lower sales and higher contracting and financing costs) is over 7.5 times the sum of monetary legal penalties per firm. Therefore, in our study the penalty includes but is not limited to monetary costs such as litigation costs and penalties paid to the SEC.

cost for the good firm to deter the bad firm's mimicking. On the other hand when even the good firm has a high chance to get bad outcome and there is not a big difference between good and bad projects, and when the reimbursement proportion is very small, imposing no penalty and allowing a pooling equilibrium become optimal, because the benefit of distinguishing firm-types is small while the cost of distinguishing firms is high.

We further consider introducing an ex-ante direct cost of manipulating signals besides the ex-post penalty. Firms' manipulation not only bears potential ex-post penalties such as litigation costs and potential reputation losses, but may also bear ex-ante direct costs or restrictions of manipulation such as the restrictions by GAAP, managers' psychic suffering, costs of forging documents and misleading the board of directors, etc. (Gao and Zhang 2016, Laux and Stocken 2012). With the direct cost, a firm is unable to release its desired signal with certainty, although it can influence the chance of a signal's realization. In this setting the expected cost of manipulation comes from both the potential penalty and the direct cost of manipulation. Our analysis shows that in general the results in the main setting still hold. However, with the introduction of the direct cost, the initial information quality before manipulation becomes relevant and plays a role in determining the optimal penalty level. In particular, when the initial information quality is high, imposing a penalty does not affect a good firm's decision much but is more effective in dampening a bad firm's upward manipulation. In addition, a high initial information quality also facilitates the possibility of a separating equilibrium in which investment inefficiency is eliminated. Moreover, with the direct manipulation cost there is one more benefit of imposing a penalty. That is, imposing a sufficiently-high penalty helps to reduce the expected direct manipulation cost as the penalty dampens firms' manipulation.

The remainder of the paper proceeds as follows. Section 2 discusses related studies. Section 3 outlines the model's main setup and analyzes a firm's strategy and the optimal penalty level. Section 4 introduces a direct cost of manipulation and analyzes the effects of the direct cost on the firm's equilibrium strategies as well as the optimal penalty. Section 5 concludes the paper. All the proofs are in the appendix.

2 Related Literature

Our paper is related to the literature examining the relationship between firms' accounting reporting and litigation. Evans and Sridhar (2002) examine firms' disclosures to deter rivals' entrance in a product market competition and to obtain financing from the capital market. They show that the role of shareholders litigation sometimes is redundant and cannot complement other mechanisms to discipline misreporting. Dye (2011) evaluates firms' voluntary disclosure decisions when they have a duty to disclose material information and withholding material information results in litigation for damages. He shows that when the material threshold is high, a value-maximizing firm is more likely to disclose its private information when the threshold is further raised. Caskey (2014) examines a setting in which investors can file class action lawsuits against a firm if the firm releases news that contradicts its previous report. His study shows that investors' anticipation of litigation costs leads to an amplified negative reaction to bad news. Our paper deviates from these studies by focusing on whether imposing a penalty is optimal to maximize the overall efficiency, and we find that sometimes it is efficient to impose a sufficiently large penalty to distinguish firm-types in order to improve the investment efficiency, while sometimes it is optimal not to impose any penalty and allow firms to pool together.

Laux and Stocken (2012) also examine the effects of penalties and they find sometimes a higher penalty level results in more manipulation. Specifically, when the entrepreneur is sufficiently more optimistic than investors about the chance of his project success, an increase in the expected penalty may lead to more managerial manipulation. The driving force of their result is the managerial over-optimism: the entrepreneur is over-optimistic and thus does not believe he will be penalized, while the investor requests for a lower equity stake as the larger penalty (which is reimbursable) increases the investor's expected payoff. In contrast, in our model we do not have different believes about the project outcome, and managerial over-optimism does not exist. The driving force to decide whether imposing a penalty is efficient comes from the trade-off between the benefit of distinguishing firm-types to improve the investment efficiency and the dead-weight loss of the signalling cost due to the penalty. In addition, in Laux and Stocken's paper there is no information asymmetry given an unmanaged report, and manipulation is always detrimental, while in our study there is information asymmetry as the firm observes its type but the investor only observes a signal, and manipulation is beneficial when a good-type firm manipulates to enhance the chance of a good signal. Furthermore, the focuses of these two studies are different: while Laux and Stocken (2012) focus on illustrating a case in which higher penalties may lead to more manipulation due to managerial over-optimism, our study concentrates on under what conditions imposing a penalty is efficient and under what conditions imposing no penalty is efficient, and what is the optimal penalty level to maximize the overall efficiency.

There is a huge literature on earnings management (Dye, 1988; Arya, Glover, and Sunder, 1998; Demski, 1998, etc.). Some studies show that tolerating firms' manipulation can be optimal. For example, Arya et al. (1992) show that sometimes the manager's report can be useful only if he is permitted to misstate his performance. Hofmann (2008) shows that with excessive earnings management, it may be efficient not to use an indicator of the earnings management in contracting. Beyer (2009) finds that earnings manipulation may help to reduce the market's perceived variance in a setting where both the mean and the variance of the cash flow distribution are unknown. Laux (2014) shows that more convex executive pay plans increase the magnitude of manipulation, which leads to less efficient investment. Meanwhile, highly convex pay plans are more effective to induce executive efforts. Due to this trade-off, the information manipulation still exists even with the optimal contract. Caskey and Laux (2016) find that more conservatism induces more earnings manipulation but this helps to improve efficiency. In our paper the firm's manipulation can also be beneficial when a good firm manipulates the signal to increase the chance that it obtains an accurate signal. Some other studies in this literature show that less stringent controls can be efficient. For instance, Ewert and Wagenhofer (2005) find that tightened accounting standards to restrain accrual manipulation not only induce more real manipulation, but may also result in more accrual manipulation. The intuition is that tightened standards improve the earnings quality, and the increased value relevance provides firms with more incentive to engage in accrual manipulation. Drymiotes (2011) shows that oversight may result in more manipulation because of the managers' potential ability to undermine oversight through manipulation choices. Mittendorf (2010) focuses on the effectiveness of audits and shows that more relaxed audit thresholds may actually mitigate inefficiencies brought by misreporting. In a similar spirit, we find that sometimes it is optimal not to impose any penalty upon an inconsistency between an optimistic signal and a later bad outcome.

Our paper is also related to studies on endogenous information precision. Nan and Wen (2014) examine accounting biases in a setting in which firms are able to improve the informativeness of a signal about their profitability through efforts, and they find that firms with high profit prospects are more motivated to improve the information quality in a more downward-biased system. Bertomeu and Magee (2011) examine the interaction between politically based information regulations and the economic cycle. They show that a shift of accounting information quality driven by a downturn in economy may result in more bad loans and higher interest rates. Liang and Nan (2014) study a model in which managers exert both productive effort and performance-reporting effort to improve the information quality of the performance measure, and analyze the interaction between these two efforts in various settings. Our paper is similar to these studies to the extent that we also examine a setting with endogenous information quality, but we differentiate from these papers by focusing on how imposing penalties affects firms' manipulation decisions, and what is the optimal penalty level to maximize the overall efficiency.

Another line of related literature is the literature on investment inefficiency and imperfect information. Stiglitz and Weiss (1981) analyze the investment inefficiency related to credit rationing. They show that while a bank may use interest rates as screening devices to identify borrowers who are more likely to repay, the interest rate it offers will also affect firms' risk-taking behavior: a higher interest rate induces firms to take more risky projects. As a consequence, the bank will not raise the interest rate when it has an excess demand for credit and will instead deny loans. Myers and Majluf (1984) study a setting in which a firm seeks financing through equity issuance to undertake a valuable project and the firm has private information about the firm value. They find that sometimes the firm will refuse to issue shares even if it has to give up a good project. This is because when the firm issue shares the investors will interpret the issuance as an indicator of bad news, and that will affects the price the investors are willing to pay, which in turn affects the firm's decision to issue shares. In our study, we also examine a setting with investment inefficiency and information asymmetry. We show that investment inefficiency may exist in the equilibrium when the cost for good firms to stand out from bad firms is too high.

3 The Main Setting

3.1 The Setup

We consider a representative firm that seeks financing for its project. The project will generate a cash flow of X_H in case of success, and will bring a cash flow of X_L if the project fails; $X_H > X_L \geq 0$. We use z to denote the outcome of the project, $z \in \{X_H, X_L\}$. The project can be either of a good type, g, or of a bad type, b.[2] We denote the type of the project by T, $T \in \{g, b\}$. We assume that a good project has a higher probability of success ($P_g = \Pr(z = X_H|T = g)$) than a bad project ($P_b = \Pr(z = X_H|T = b)$), $0 < P_b < P_g < 1$, where P_g and P_b are the probabilities of success for the good project and the bad project, respectively. For convenience, we often refer to a firm with a good project as "a good firm" and a firm with a bad project as "a bad firm," and refer to a firm's project type as "a firm's type."

The type of the project is privately observed by the firm. For outsiders, the prior belief of the probability that the firm has a good project is θ; i.e., $\Pr(T = g) = \theta$. The outsiders are unable to observe the firm's type, but a signal S is generated and released to the public. The signal is binary and can be either a high signal, S_H, or a low signal, S_L.

The firm can freely choose whether to release a high signal or a low signal. Although the firm is "free" to choose either signal, the firm faces a penalty F if it chooses to release a high signal but later the outcome turns out to be bad. This assumption about F is to capture the fact that, in reality, class action lawsuits and investigations by the SEC are usually triggered by poor performance following optimistic signals. For example, Groupon, the Chicago-based online coupon retailer, was recently sued in a class action lawsuit upon a reduction of its revenue in the fourth quarter of 2011 by $14.3 million for failing to disclose negative trends in its business earlier. This also triggered a probe by the SEC into Groupon's book-keeping procedures. Although we call F the "penalty" for our convenience, F can include not only monetary penalties such as penalties paid to the SEC

[2] Our main results are robust to a continuous setting in which the project outcome X is continuous (a good firm yields a higher expected X than a bad firm) and the penalty is applied when X is sufficiently lower than the reported signal S. The trade-off to determine the optimal penalty level remains the same. The main benefit of imposing a penalty is to help the good firm to stand out from the bad firm by upward manipulation so that the investment efficiency is improved. The main cost of penalty is the dead-weight loss from the penalty. If the probability of a sufficiently low outcome is much lower for the good firm than the bad firm, the benefit of investment-efficiency improvement dominates the dead-weight loss and it is optimal to impose a positive penalty. Otherwise the optimal penalty is zero.

and vicarious liabilities, but can also include non-monetary costs such as a significant distraction of management, an effort to cooperate with a SEC investigation, a loss of reputation due to the scrutiny, etc.. Notice that even though we assume the same F for both good and bad firms, ex ante the expected penalty is higher for the bad firm than for the good firm because the bad firm is more likely to obtain a bad outcome.[3,4] With the firm's ability to choose its desired signal, it is a signalling game in which the good firm decides whether to signal by releasing S_H which may result in a penalty once the outcome unfortunately turns to be bad, and the bad firm decides whether it is worthy to mimic the good firm by releasing S_H.

Observing the signal S, a representative investor decides whether to fund the project. We denote the investing decision by v, $v \in \{I, 0\}$, where $v = I$ represents the decision that the project is funded and undertaken while $v = 0$ means the project is forgone. We assume the investor is risk-neutral and participates in a competitive capital market. If the investor decides to fund the project, she provides the required investment I in exchange for a portion of the firm's shares, denoted by $\alpha \in [0, 1]$, to break even. The project is undertaken once the firm gets financing from the investor, and has to be forgone if the firm does not get financing.[5,6]

In our model, a high signal followed by a project failure does not necessarily indicate that the

[3] If the penalty varies with the ex-post likelihood that the firm issued a misleading report, say $F' = F \cdot P(b|S_H, X_L)$, the main results in our setting qualitatively remain. With $F' = F \cdot P(b|S_H, X_L)$, a good firm's expected payoff by releasing a high signal becomes $\Pi_g (S = S_H) = [P_g X_H + (1 - P_g) X_L] [1 - \alpha (S_H)] - (1 - P_g)F'$, and a bad firm's expected payoff by releasing a high signal becomes $\Pi_b (S = S_H) = [P_b X_H + (1 - P_b) X_L] [1 - \alpha (S_H)] - (1 - P_b)F'$. Since $P(b|S_H, X_L)$ is correctly conjectured by the investor in equilibrium, our analysis remains the same except that F is replaced by F'. When F' approaches $\hat{F}$, the likelihood that a bad firm releases a high signal (denoted by $\Pr (S_H|b)$) will be very low and the ex-post endogenous likelihood $P(b|S_H, X_L) \equiv \frac{(1-\theta)(1-P_b) \Pr(S_H|b)}{(1-\theta)(1-P_b) \Pr(S_H|b)+\theta\left(1-P_g\right) \Pr(S_H|g)}$ will also be very low. By choosing a sufficiently-high F such that $F = \frac{F'}{P(b|S_H,X_L)}$, we can achieve a mix-strategy equilibrium where F' approaches $\hat{F}$ and $\Pr (S_H|b)$ approaches zero (but can never reach zero). By continuity, the firm's ex-ante expected payoff in such a mix-strategy equilibrium approaches the firm's ex-ante expected payoff in the least-cost separating equilibrium in our main setting ($\Pi(F = \hat{F})$). Therefore, our results in the main setting qualitatively remain the same by using this alternative assumption.

[4] In our model, the penalty includes but is not limited to monetary fines, therefore we do not impose a restriction that the penalty should be less than X_L. Nevertheless, in our model we show that the optimal level of penalty can never be larger than the least-cost separating equilibrium penalty level $\hat{F} \equiv \frac{P_g X_H + \left(1-P_g\right) X_L - I}{\frac{(1-P_b)[P_g X_H + (1-P_g) X_L]}{P_b(X_H - X_L) + X_L} - \beta\left(1-P_g\right)}$, and it is possible that $\hat{F}$ is not greater than X_L. Therefore our main results remain qualitatively if we impose the restriction $F < X_L$.

[5] Although in this study we assume that the firm gets financing from equity investors, our main results still hold if the firm seeks financing from debtholders. This is because regardless of the firm's capital structure choices, the firm has the incentive to influence the signal to lower its cost of financing, and the penalty threat plays the exact same role in both the equity-financing and the debt-financing settings.

[6] If we instead assume that the contract between the investor and the firm occurs before the signal is observed, the investor could offer a menu of contracts ex ante (screening contracts), which specifies the requested portion as a function of the signal (i.e., $\alpha (S_H)$ and $\alpha (S_L)$). With such a contract menu, the results in our main setting remain the same.

signal was inaccurate, because even a good firm who releases an accurate high signal may end up in failure. Our model stresses the fact that, in practice, although some misleading behavior and incentives can be detected correctly ex post, in many cases it is hard to tell whether a previous signal of a firm's performance that turns out to be inconsistent with the later outcome is legitimate or not. For example, Facebook faced class action lawsuits alleging that the company had misled investors about revenue projections for the social network right after its IPO in May 2012. A Facebook spokesman said the company "believes the lawsuit is without merit and will defend ourselves vigorously." In addition, the incurrence of penalties may not necessarily indicate any wrongdoing. Many companies choose to settle class action lawsuits simply to avoid the costs of ongoing litigation, but deny any wrongdoing. For example, in September 2012, Bank of America settled for $2.43 billon the class action lawsuit alleging the bank misled investors about the acquisition of Merrill Lynch, but denied the allegations. The bank's chief executive, Brian T. Moynihan, said in a statement that "resolving this litigation removes uncertainty and risk and is in the best interests of our shareholders." Even after class action lawsuits are settled, in many cases it remains unclear whether the companies had manipulated information to mislead investors or the information was legitimate before a bad outcome is realized. In fact, some empirical studies provide evidences that there is no significant correlation between the incidence of securities settlements and fraudulent behavior (Alexander, 1991; Choi, 2004).

Some monetary penalties, such as penalties paid to the SEC and some of the vicarious liabilities under Rule 10b-5, can be reimbursed to investors. To capture this reimbursement, we assume that a portion of the penalty, βF, is reimbursed to the investor when the penalty is incurred, $\beta \in [0, 1)$. That is, upon a project failure with a high signal, the investor will receive βF. We further assume that $\beta F < I - X_L$. The assumption $\beta F < I - X_L$ implies that the reimbursement upon a project failure is not enough to cover the loss of the failed project. This assumption is to ensure that the investor is more willing to fund a good project than a bad project. More specifically, $\beta F < I - X_L$ is the sufficient condition that the investor requests a lower proportion of shares when her posterior belief that the firm has a good project is higher.

Figure 1 shows the time line of this setting.

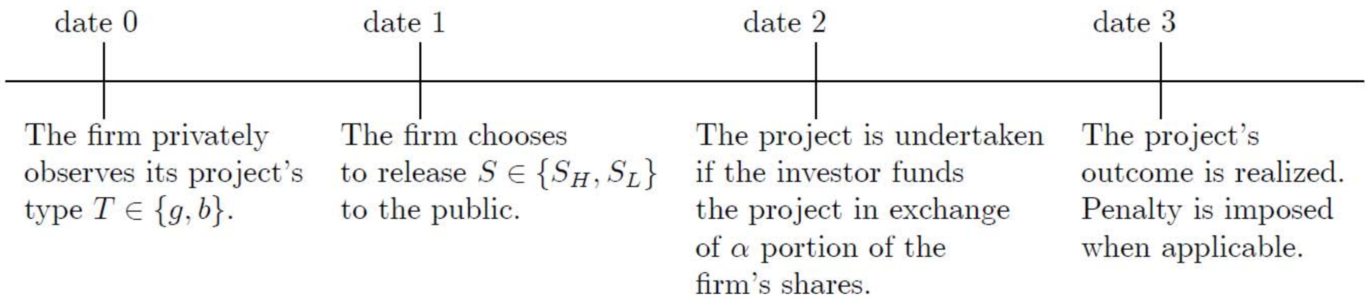

Figure 1: Time line.

3.2 The Analysis

By backward induction, at date 2 upon the signal S, the representative investor decides whether to fund the project. Based on the observed signal $S \in \{S_H, S_L\}$, the investor updates her belief about the probability that the firm is a good type. The updated beliefs upon S_H and S_L are denoted as θ_H and θ_L, respectively. The investor's valuation of the firm's value based on the updated belief is

$$V(S_H) \equiv [\theta_H P_g + (1-\theta_H)P_b](X_H - X_L) + X_L$$

upon S_H, and is

$$V(S_L) \equiv [\theta_L P_g + (1-\theta_L)P_b](X_H - X_L) + X_L$$

upon S_L.

The investor requests a proportion of shares $\alpha(\cdot)$ to break even if she determines to invest in the project:

$$\begin{cases} \alpha \cdot V(S_H) + \Pr(X_L|S_H)\beta F = I, & \text{if } S = S_H \text{ and } v = I; \\ \alpha \cdot V(S_L) = I, & \text{if } S = S_L \text{ and } v = I. \end{cases} \tag{1}$$

The requested proportion of shares upon a signal therefore is

$$\begin{aligned} \alpha(S_H) &= \frac{I - \Pr(X_L|S_H)\beta F}{V(S_H)}, \quad \text{if } S = S_H \text{ and } v = I; && (2) \\ \alpha(S_L) &= \frac{I}{V(S_L)}, \quad \text{if } S = S_L \text{ and } v = I; \\ \alpha(S_H) &= \alpha(S_L) = 0, \quad \text{if } v = 0. \end{aligned}$$

We consider three cases. In the first case, only good firms have positive NPV projects (i.e., $P_g X_H + (1 - P_g) X_L > I, P_b X_H + (1 - P_b) X_L < I$). In this case the investor's investing decision may be contingent on the signal. We call it the *signal-contingent case*. In the second case, both good and bad firms have positive NPV projects (i.e., $P_g X_H + (1 - P_g) X_L > P_b X_H + (1 - P_b) X_L > I$). In this case, regardless of the signal, the investor is always willing to fund the project. We call this case the *always-fund case*. In the third case, neither good nor bad firms have positive NPV projects (i.e., $P_b X_H + (1 - P_b) X_L < P_g X_H + (1 - P_g) X_L < I$), and therefore the investor never funds the project regardless of the signal. We call this case the *never-fund case*.

The never-fund case is trivial, as the firm's signal plays no role and thus the penalty plays no role. We therefore exclude the never-fund case in our analysis and only concentrate on the other two cases.

3.2.1 The Signal-Contingent Case

We start with the signal-contingent case in which only good firms have positive NPV projects ($P_g(X_H - X_L) + X_L > I, P_b(X_H - X_L) + X_L < I$). In this case, the investment efficiency is maximized when we achieve a separating equilibrium in which good firms are distinguished and get funds, while bad firms do not get funded. However, the separating equilibrium may not be optimal. Sometimes it may be too costly for a good firm to signal its type and deter bad firms, and a pooling equilibrium may be optimal. Nevertheless, for our convenience, although the investment decision is contingent on the signal only in a separating equilibrium, we still call this case "signal-contingent" case as this is the only case in which the investment decision may be contingent on the signal.

The Firm's Strategy Given F We first analyze the firm's strategy of releasing the signal given the penalty F. For a good firm, if it releases a high signal, its expected payoff (denoted by $\Pi_g (S = S_H)$) is

$$\Pi_g (S = S_H) = [P_g X_H + (1 - P_g) X_L] [1 - \alpha (S_H)] - (1 - P_g)F. \tag{3}$$

For a bad firm, if it releases a high signal, its expected payoff (denoted by $\Pi_b (S = S_H)$) is

$$\Pi_b (S = S_H) = [P_b X_H + (1 - P_b) X_L] [1 - \alpha (S_H)] - (1 - P_b)F. \tag{4}$$

It is easy to see $\Pi_g(S = S_H) > \Pi_b(S = S_H)$, which indicates that the good firm has a stronger incentive to release a high signal than the bad firm. This is because the good firm has a higher expected cash flow from the project and a lower expected penalty (due to the lower likelihood of failure).

Given different levels of the penalty F, we have different cases of good and bad firms' reporting strategies, which are summarized in the following proposition:

Proposition 1 *In the signal-contingent case, we have the following equilibria depending on the level of the penalty F:*

(i) ***pooling equilibrium****: when $F \leq \underline{F}$ or $F \geq \bar{F}$, both firm-types report the same signals and the investor makes the investment decision only based on her prior belief;*

(ii) ***mixed-strategy equilibrium****: when $\underline{F} < F < \hat{F}$, the good firm reports S_H and the bad firm reports S_H or S_L with some probability, and the investor only funds upon S_H;*

(iii) ***separating equilibrium****: when $\hat{F} \leq F < \bar{F}$, the good firm reports S_H and the bad firm reports S_L, and the investor only funds upon S_H.*

$$
\begin{aligned}
0 &\leq \underline{F} < \hat{F} < \bar{F}. \\
\underline{F} &\equiv \max\left[\frac{[\theta P_g + (1-\theta) P_b](X_H - X_L) + X_L - I}{\frac{(1-P_b)[[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L]}{P_b(X_H-X_L)+X_L} - \beta[1 - \theta P_g - (1-\theta) P_b]}, 0\right], \\
\hat{F} &\equiv \frac{P_g X_H + (1-P_g) X_L - I}{\frac{(1-P_b)[P_g X_H+(1-P_g)X_L]}{P_b(X_H-X_L)+X_L} - \beta(1-P_g)}, \\
\text{and } \bar{F} &\equiv \min\left[\frac{P_g X_H + (1-P_g) X_L - I}{(1-P_g)(1-\beta)}, \frac{\left(\frac{P_g X_H+(1-P_g)X_L}{[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L} - 1\right) I}{(1-P_g)(1-\beta)}\right].
\end{aligned}
$$

Proposition 1 shows that there are three types of equilibria, depending on the penalty level. When the penalty is very small (i.e., $F \leq \underline{F}$), the threat from a potential penalty is negligible, and therefore both good and bad firms report S_H. That is, we achieve a pooling equilibrium. In this case, as all firms report S_H, the signal is no longer informative and will be ignored by the investor. The investor thus makes her investment decision only based on the prior belief about the project's expected NPV. In this case we have either overinvestment or underinvestment inefficiency: when the prior belief about the project's NPV is positive, the investor always funds the project and even bad firms get funded; when the prior belief is negative, the investor does not fund any project and even good projects are forgone.

As the penalty F gets higher but is still low (i.e., $\underline{F} < F < \hat{F}$), the good firm still reports a high signal, but the bad firm becomes indifferent between reporting S_H and S_L as its expected benefit from implementing the project equals its expected penalty. The bad firm reports S_H or S_L with some probability and we achieve a mixed-strategy equilibrium. In this case we have overinvestment inefficiency as the bad firm may also get funded.

When the penalty F is high but not prohibitively high (i.e., $\hat{F} \leq F < \bar{F}$), a separating equilibrium is achieved in which the good firm reports S_H and the bad firm reports S_L. With a high penalty level, the good firm still finds it beneficial to release S_H, but the bad firm cannot afford S_H any more. In this case there is no investment inefficiency. Notice that when $F = \hat{F}$ we achieve the least-cost separating equilibrium. That is, $\hat{F}$ is the minimum penalty that induces a separating equilibrium.

When the penalty level F becomes prohibitively high (i.e., $F \geq \bar{F}$), to avoid the huge expected penalty upon failure, neither good nor bad firms release S_H. Again, we achieve a pooling equilibrium. As all firms report S_L, the signal is no longer informative, and the investor makes the investment decision only based on her prior belief. This case, therefore, reduces to be the same as the case of $F \leq \underline{F}$. Since all the analysis of the case of $F \geq \bar{F}$ can be duplicated by the case of $F \leq \underline{F}$, in the following analysis we will exclude the $F \geq \bar{F}$ case to avoid tedious repetition.

Optimal Penalty Analysis We now analyze the optimal penalty that maximizes the social welfare or the overall efficiency, which is denoted by F^*. In our setting, the social welfare or the overall efficiency can be represented by the firm's ex-ante expected payoff before it observes its type because the investor always breaks even. We denote the firm's ex-ante expected payoff to be Π.

We find that imposing a penalty brings both a benefit of improving the investment efficiency and a dead-weight "signalling cost" for the good firm to deter bad firms. The benefit of the penalty comes from improving the investment efficiency by helping the good firm to stand out from bad firms by releasing S_H, and this benefit is a result of two effects, one through the endogenous ownership stake requested by the investor (in other words the endogenous implicit financing cost), and one through the expected penalty. First, the penalty reduces the implicit financing cost for both good and bad firms as the investor anticipates a reimbursement βF upon a failure and S_H. More

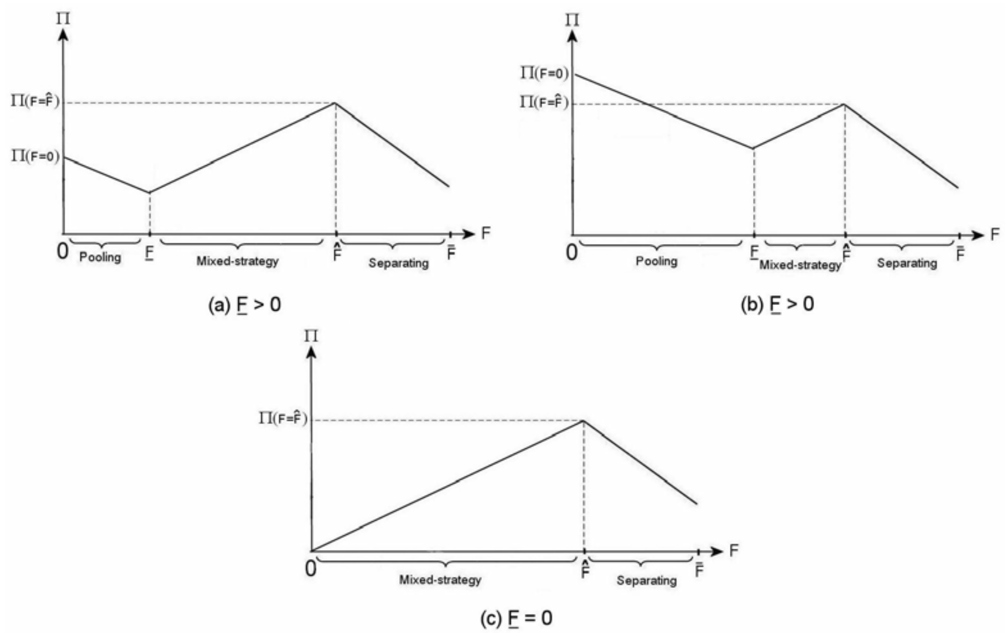

Figure 2: The effects of F on Π.

importantly, although both good and bad firms enjoy a lower $\alpha(S_H)$, the good firm benefits more from the lower implicit financing cost because the good firm's expected cash flow is higher and thus the net value for its remaining stake is higher. Second, imposing a penalty has different impacts on the good firm's and the bad firm's expected penalties. The expected penalty for the good firm is small as the good firm is less likely to have a failure, while the expected penalty for the bad firm is large as the bad firm is more likely to fail. Therefore a higher penalty level discourages the bad firm from releasing a high signal more than discouraging the good firm's high-signal decision. These two effects, together, make it easier for the good firm to be distinguished from bad firms with a higher penalty level, as the good firm may still choose to obtain a high signal while the bad firm will not. As a result, the investment efficiency is improved. However, this improvement of investment efficiency is at the cost of the good firm's dead-weight signalling cost: by choosing the high signal to stand out from the bad firms, the good firm must bear the expected penalty. This trade-off between the investment efficiency benefit and the dead-weight loss determines the optimal penalty level.

Figure 2 shows the three types of equilibria depending on the level of penalty F and how the firm's ex-ante expected payoff Π changes with F. In the pooling equilibrium in which both firm-types report S_H, there is no benefit of investment efficiency improvement from increasing the penalty because the investor cannot identify good firms from bad firms. A higher penalty only brings more dead-weight loss. Therefore, the firm's ex-ante expected payoff decreases in F. In the mixed-strategy equilibrium in which the good firm reports S_H and the bad firm plays a mixed strategy by reporting S_H or S_L with some probability, the firm's ex-ante expected payoff Π increases in F, because as the penalty level increases the bad firm is less likely to report S_H and thus the investment efficiency is improved. The investment efficiency reaches its maximum when we increase F to $\hat{F}$ and achieve the least-cost separating equilibrium. Once we achieve the least-cost separating equilibrium, if we keep increasing F, Π starts decreasing in F because the investment efficiency can no longer be improved but the higher penalty level brings more dead-weight loss.

It is easy to see that the firm's ex-ante expected payoff maximizes at either the least-cost separating equilibrium or the zero-penalty pooling equilibrium. That is, the optimal penalty level is either $F^* = \hat{F}$ or $F^* = 0$. Our further analysis shows that the least-cost separating equilibrium is optimal when P_g is high and P_b is low, or when β is high, while the zero-penalty pooling equilibrium is optimal otherwise. We summarize this result in the following proposition.

Proposition 2 *In the signal-contingent case,*

- *if* $\underline{F} = 0$, $F^* = \hat{F}$;
- *if* $\underline{F} > 0$, $F^* = \hat{F}$ *when* $\beta > \dfrac{\frac{\theta[P_g X_H + (1-P_g) X_L - I]}{(1-\theta)[I - P_b X_H - (1-P_b) X_L]} - \frac{\frac{P_g}{1-P_g} X_H + X_L}{\frac{P_b}{1-P_b} X_H + X_L}}{\frac{\theta[P_g X_H + (1-P_g) X_L - I]}{(1-\theta)[I - P_b X_H - (1-P_b) X_L]} - 1}$ *is satisfied (that is, when* P_g *is sufficiently high and* P_b *is sufficiently low, or when* β *is sufficiently high); otherwise,* $F^* = 0$.

When $\underline{F} = 0$, the zero penalty level cannot be optimal. Because the investor has a prior belief that the project's expected NPV is negative,[7] with zero penalty, the signal is useless and the investor will not fund any project. In other words, with $\underline{F} = 0$ the optimal penalty level must be the least-cost separating level, $F^* = \hat{F}$.

With $\underline{F} > 0$, the optimal penalty level is the least-cost separating level when P_g is sufficiently high and P_b is sufficiently low. That is, when the good firm has a much larger chance to achieve

[7] Recall that $\underline{F} \equiv \max[\frac{[\theta P_g + (1-\theta) P_b](X_H - X_L) + X_L - I}{\frac{(1-P_b)[[\theta P_g + (1-\theta) P_b](X_H - X_L) + X_L]}{P_b (X_H - X_L) + X_L} - \beta[1 - \theta P_g - (1-\theta) P_b]}, 0]$, and thus $\underline{F} = 0$ implies the prior expected NPV, $[\theta P_g + (1-\theta) P_b](X_H - X_L) + X_L - I$, is negative.

the good outcome than the bad firm, imposing a sufficiently-high penalty is optimal to help the good firm distinguish itself from bad firms. With P_g much higher than P_b, the expected penalty for the good firm is quite low as the penalty only applies upon an unlikely failure, but the bad firm cannot afford the expected penalty as its chance of failure is much higher. In this case the least-cost separating equilibrium is optimal because the investment inefficiency is completely eliminated (i.e., no expected loss from a bad project) and the expected signalling cost for the good firm to deter bad firms is low. On the other hand when P_g is close to P_b, even the good firm has a high chance to get a bad outcome and there is no big difference between good and bad projects. The benefit of distinguishing firm-types is small and is dominated by the dead-weight loss from the good firm's signalling cost, and thus the zero-penalty pooling equilibrium becomes optimal.

Proposition 2 also indicates that the least-cost separating equilibrium is optimal when the reimbursement proportion β is high. This is because a higher β reduces the good firm's expected signalling cost to deter bad firms. To understand this result, notice that the firm's ex-ante expected payoff at the least-cost separating equilibrium is

$$\begin{aligned}\Pi\left(F=\hat{F}\right) &= \theta\Pi_g\left(F=\hat{F}\right)+(1-\theta)\underbrace{\Pi_b\left(F=\hat{F}\right)}_{=0}\\ &= \theta\{[P_gX_H+(1-P_g)X_L][1-\alpha(S_H)]-(1-P_g)\hat{F}\}.\end{aligned}$$

Once we substitute $\alpha(S_H)$ we can rewrite $\Pi\left(F=\hat{F}\right)$ to be

$$\Pi\left(F=\hat{F}\right)=\underbrace{\theta\left[P_gX_H+(1-P_g)X_L-I\right]}_{\text{The expected return from a good project}}-\underbrace{(1-\beta)\,\theta(1-P_g)\hat{F},}_{\text{The expected signalling cost to deter bad firms}}$$

where the second term, $(1-\beta)\,\theta(1-P_g)\hat{F}$, represents the good firm's expected signalling cost to deter bad firms, and it is easy to see that this signalling cost decreases in β.

With a higher β, the implicit financing cost becomes lower for both good and bad firms. The lower implicit financing cost, on the one hand, encourages both firm-types to report high signals and calls for a higher penalty level to achieve the separating equilibrium (recall that $\hat{F}\equiv\frac{P_gX_H+(1-P_g)X_L-I}{\frac{(1-P_b)[P_gX_H+(1-P_g)X_L]}{P_b(X_H-X_L)+X_L}-\beta(1-P_g)}$, which increases in β); on the other hand, as we explained earlier, because the good firm benefits more from the lower implicit financing cost than the bad firm, a higher β in fact makes it easier for the good firm to stand out, despite the higher equilibrium penalty level. We highlight this interesting result in the following corollary.

Corollary 1 *As β increases, the minimum penalty level to achieve the separating equilibrium $\hat{F}$ increases, but the good firm's expected signalling cost to deter bad firms decreases.*

3.2.2 The Always-Fund Case

In the always-fund case in which both good and bad firms have positive NPV projects, the investor always chooses to finance the project regardless of the signal and there is no investment inefficiency. Therefore, imposing a positive penalty no longer helps to improve the investment efficiency by distinguishing firm-types. As the only benefit of the penalty disappears but the dead-weight loss of the penalty still exists, it is obvious that the optimal penalty must be zero.

Proposition 3 *In the always-fund case, $F^* = 0$.*

4 The Introduction of a Direct Manipulation Cost

In the main setting we assume that the firm is able to release whichever signal it desires to release. In other words, the firm can manipulate upward to release S_H or manipulate downward to release S_L without any direct cost ex ante. However, in the real world firms' manipulation not only bears potential ex-post penalties such as litigation costs, penalties paid to the SEC, and potential reputation losses, but there may also be some ex-ante direct costs of manipulation, or some ex-ante restrictions on firms' manipulation. For example, a firm's financial reports must be prepared in accordance with GAAP, which restricts the firm from reporting whatever earnings number that it desires to report. In addition, to release an optimistic financial report, a firm must convince its auditor, which is another kind of restriction. The direct cost of manipulation may also come from managers' psychic suffering, costs of forging documents and misleading the board of directors, etc. (Gao and Zhang 2016, Laux and Stocken 2012).

We now examine the effects of a direct cost of manipulation. In contrast to the main setting in which the firm can release whichever signal it wants, here we assume there is a direct cost of manipulation that prevent the firm from obtaining its desired signal with certainty. The firm

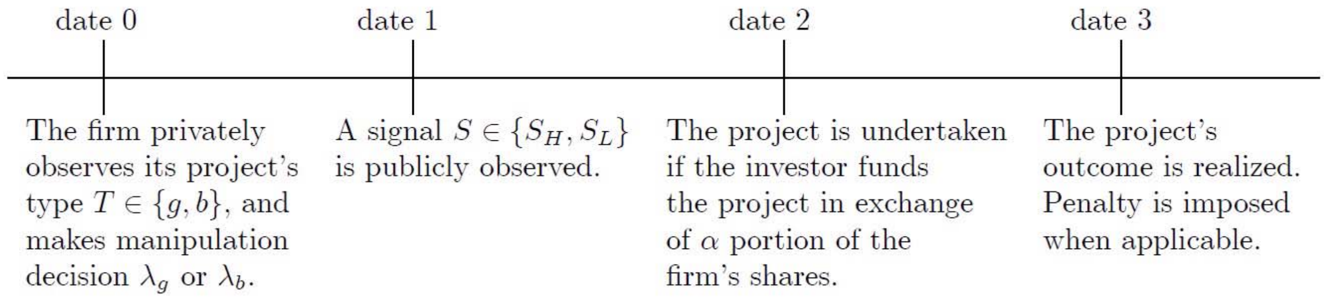

Figure 3: Time line with the direct cost of manipulation.

can make an unobservable manipulation λ ex ante to influence the chance of the realization of a specific signal, but it is not guaranteed that it will obtain its desired signal for sure even with the manipulation. We assume that the ex-ante direct cost of manipulation is $C(\lambda)$, $C(\lambda) = \frac{K}{2}\lambda^2$, where K is a publicly-known cost parameter.[8] We denote the good firm's manipulation and the bad firm's manipulation to be λ_g and λ_b, respectively. We allow both upward manipulation to increase the chance of a high signal and downward manipulation to decrease the chance of a high signal. That is, λ_g/λ_b can be positive or negative. With λ_g and λ_b, the conditional probabilities of signals for different firm-types are:

$$P(S_H \mid g) = q + \lambda_g, \; P(S_L \mid g) = 1 - q - \lambda_g, \tag{5}$$
$$P(S_H \mid b) = 1 - q + \lambda_b, \text{ and } P(S_L \mid b) = q - \lambda_b.$$

The parameter q represents the initial information quality of the signal before the firm's manipulation. We also assume the cost parameter K is sufficiently high so that the bad firm is unable to achieve a high signal for certain by manipulation (i.e., $1 - q + \lambda_b < 1$). Otherwise, if even the bad firm can manipulate the signal to S_H for sure, then it reduces to the previous main setting which we already analyzed. In addition, we exclude the case when K is extremely high, because that case is trivial and not interesting as no firm will manipulate. Figure 3 shows the time line with the direct cost of manipulation.

[8] Our analysis qualitatively remains the same if we assume that the direct manipulation cost depends on the firm's type (e.g., K_g for a good firm and K_b for a bad firm, $K_g \neq K_b$).

Notice that although both the direct manipulation cost and the penalty F discourage the firm from manipulation, the direct manipulation cost incurs ex ante when the firm manipulates, while the penalty incurs ex post when the project fails after a high signal. Also notice that in our previous main setting the initial information quality is irrelevant; regardless of the initial information quality, the firm is able to release whichever signal it desires. However, once we introduce the direct cost of manipulation, the initial information quality q becomes relevant and plays an active role, as the distribution of signal realization depends on both the firm's manipulation and the initial information quality q. Moreover, in the previous main setting, we have a signalling game and the equilibrium could be a pure-strategy equilibrium because the firm can report a specific signal with certainty. With the direct manipulation cost, however, because the firm can no longer release with certainty whatever signal it desires, it is no longer a "pure" signalling game.

To analyze the setting with the direct manipulation cost, we also consider the signal-contingent case and the always-fund case. We again exclude the never-fund case in which both good and bad firms have negative-NPV projects, as in that case no firm has incentive to manipulate. In addition, in the signal-contingent case, although good firms have positive NPV projects and bad firms have negative NPV projects, if the initial information quality q is very low and the prior belief of the project's NPV is negative (i.e., $[\theta P_g + (1-\theta) P_b] (X_H - X_L) + X_L < I$), the case is reduced to be the same as the never-fund case.[9] We also exclude this particular sub-case in our analysis.

Since with the introduction of the direct cost the model is no longer a pure signalling game, we examine the firm's manipulation decision instead of analyzing its signalling strategy. To study the optimal penalty, we will first analyze the firm's best-respond manipulation decision given F (λ_g^* or λ_b^*), and then we will be able to characterize the optimal penalty level F^*.

4.1 The Firm's Manipulation Decision

Given a penalty F, the firm's manipulation decision depends on the trade-off between the benefit and the cost of obtaining a high signal. A higher chance of a high signal means a higher chance to get the project funded and yields a higher expected cash flow from the project, but at the same

[9] This is because when the initial information quality is very bad, the signal will be very noisy even if good firms try their bests to distinguish themselves by manipulation. As a result, the investor ignores the signal and simply does not fund any project. In return, no firm has incentive to manipulate and it is obvious that the optimal penalty is zero.

time also implies a higher expected penalty. For the good firm, the benefit from the project cash flow is larger than for the bad firm, while the cost from expected penalty is smaller than for the bad firm. As a consequence, the good firm is more motivated to manipulate upward than the bad firm. Notice that this result is also reflected in the main setting. In the main setting, the good firm almost always releases a high signal except in the extreme case when the penalty level is prohibitively high; in contrast, the bad firm only releases a high signal when F is extremely low or releases S_H with some probability in the mixed-strategy equilibrium.

Similarly, when the reimbursement portion of penalty β increases, the good firm benefits more from the lower implicit financing cost than the bad firm, and thus the good firm's upward manipulation increases more in β than the bad firm's. This echoes the intuition of Corollary 1 in the main setting: as β increases, it becomes easier for the good firm to release S_H to be distinguished because the good firm benefits more from the lower financing cost than the bad firm.

We summarize these results in the following lemma.

Lemma 1 *With the direct manipulation cost,*

(i) the good firm is more motivated to manipulate upward than the bad firm;

(ii) both λ_g^* and λ_b^* increase in β, but λ_g^* increases more in β than λ_b^*.

To analyze the optimal penalty level in this scenario, we also examine how F affects λ_g^* and λ_b^*. In the previous main setting, the firm chooses whichever signal to release at no direct cost and the equilibrium cannot reflect clearly how the penalty level marginally affects the firm's reporting decision, while in the scenario with the direct cost, we are able to show the marginal effect of the penalty on the firm's manipulation decision. We list our findings of F's effects on the firm's manipulation below.

Lemma 2 *With the direct manipulation cost,*

(i) $\frac{d\lambda_b^*}{dF} < \frac{d\lambda_g^*}{dF}$*;*

(ii) $\frac{d\lambda_b^*}{dF} < 0$*;*

(iii) $\frac{d\lambda_g^*}{dF} \geq 0$ *when P_g is sufficiently high and P_b is sufficiently low, or when β is sufficiently high;* $\frac{d\lambda_g^*}{dF} \leq 0$ *otherwise.*

First, we find that an increase in penalty discourages the bad firm's upward manipulation more than the good firm's upward manipulation ($\frac{d\lambda_b^*}{dF} < \frac{d\lambda_g^*}{dF}$), because a bad project is more likely to fail and the expected penalty is larger for the bad firm. This is consistent with the result in the main setting: Proposition 1 shows that as F increases, while the good firm still releases S_H, the bad firm switches from releasing S_H to releasing S_L.

Lemma 2 also shows that a higher penalty level always discourages the bad firm's upward manipulation ($\frac{d\lambda_b^*}{dF} < 0$), but a higher penalty level may encourage the good firm to manipulate upward more ($\frac{d\lambda_g^*}{dF} \geq 0$), even though the higher penalty level directly increases the good firm's expected penalty. To understand the intuition of these results, notice that when the good firm has a much larger chance of getting a good outcome than the bad firm, the expected penalty for the good firm is quite low as the penalty only applies upon an unlikely failure, but the bad firm cannot afford the expected penalty as its chance of failure is much higher. Therefore a larger penalty helps the good firm to stand out from bad firms more easily and thus it motivates the good firm to manipulate upward more. In addition, when the reimbursement proportion β is high, as the penalty increases, the good firm benefits more from a lower implicit financing cost upon a high signal. As the benefit from a lower implicit financing cost outweighs the direct increase in the expected penalty, the good firm will manipulate upward more upon a higher penalty level to enhance the chance of a high signal. On the other hand for the bad firm, a higher penalty level always deters its upward manipulation because there is not much benefit from getting a high signal but with a high signal the bad firm suffers from a larger expected penalty. In fact, even if the reimbursement proportion β approaches 1, the benefit of a lower implicit financing cost $\alpha(S_H)$ still cannot outweigh the higher expected penalty cost for the bad firm, and therefore λ_b^* always decreases in the penalty level F.

4.2 Optimal Penalty with the Direct Cost of Manipulation

We now analyze the optimal penalty with the direct manipulation cost, first in the signal-contingent case, and then in the always-fund case.

4.2.1 Signal-contingent Case with the Direct Cost of Manipulation

Recall that in the signal-contingent case in the main setting, the optimal penalty is either zero or the least-cost-separating penalty. In addition, in the main setting, by imposing different levels of penalty, we are able to achieve a pooling equilibrium in which both firm-types release the same signal, a separating equilibrium in which good firms are distinguished perfectly from bad firms, or a mixed-strategy equilibrium in which good firms release S_H while bad firms play a mixed strategy of releasing either S_H or S_L. Here with the direct cost of manipulation, however, we may not be able to achieve a separating equilibrium and thus unable to have the least-cost-separating penalty. Moreover, we cannot achieve a pooling equilibrium in which both firm-types have the same signal, as the direct cost of manipulation restricts firms (at least bad firms) from obtaining their desired signal with certainty. Nevertheless, we are able to tackle the characteristics of the optimal penalty to maximize the overall efficiency by analyzing how the changes in penalty level affect the equilibrium strategies as well as the firm's ex-ante expected payoff.

As the penalty level increases, we find three different sets of equilibrium strategies:

(i) **Upward-manipulation equilibrium:** When F is small, both good and bad firms manipulate upward. Although both types manipulate upwards, as F increases, the bad firm's upward manipulation λ_b^* is discouraged more than the good firm's manipulation λ_g^* (recall that $\frac{d\lambda_b^*}{dF} < \frac{d\lambda_g^*}{dF}$ by Lemma 2).

(ii) **Mixed-strategy without manipulation equilibrium:** When the penalty increases beyond a certain level, the good firm still manipulates upward because the expected penalty is lower for the good firm than for the bad firm. The bad firm, however, no longer manipulates upward. Instead, the bad firm plays a mixed strategy in equilibrium by quitting the game with some probability and staying in the game without manipulation with some probability. The reason is that the bad firm's payoff upon S_H is reduced to zero due to the higher penalty level and thus the bad firm no longer manipulates ($\lambda_b^* = 0$). As F increases, the bad firm increases the probability of quitting. Anticipating this, the investor's posterior belief upon S_H increases in F, which results in lower $\alpha(S_H)$. With a lower $\alpha(S_H)$ and a higher F , the bad firm's payoff upon S_H still remains at zero. Therefore, both strategies (i.e., quitting the game and staying in the game without manipulation) result in zero payoff to the bad firm. Notice that the bad firm will not manipulate downward. This is because a downward manipulation brings a direct manipulation cost to the bad firm, while there is no benefit of obtaining S_L as its payoff upon S_L is zero. By a downward manipulation, the bad firm's expected payoff would be negative.

(iii) **Separating equilibrium or mixed-strategy with downward manipulation equilib-**

rium: When F becomes sufficiently large, we have two cases, depending on whether it is possible to achieve a separating equilibrium. In contrast to the main setting, here it is not guaranteed that a separating equilibrium can be achieved.

With a very high level of penalty, the condition to achieve a separating equilibrium is that the bad firm always quits the game even given the highest posterior belief about a good type upon S_H, and the good firm is able to obtain S_H with certainty. Otherwise, we cannot achieve a separating equilibrium. If even the good firm obtains S_L with some chance, the bad firm will never completely quit the game. This is because if the bad firm would always quit, the investor would believe any firm staying in the game must be a good firm and would fund the firm's project regardless of its signal. Anticipating this, the bad firm would not always quit, and instead would play a mixed strategy by quitting the game with some probability and staying in the game with a downward manipulation with some probability. To understand why the bad firm has an incentive to manipulate downward, notice that the bad firm's probability of quitting is quite high when F is sufficiently large. Considering the high probability that the bad firm quits, the investor's posterior belief upon S_L can be high enough such that the investor is willing to fund even a firm with S_L. Once the investor funds upon both S_H and S_L, the bad firm will have an incentive to manipulate downward to avoid the large penalty upon S_H. That is, the bad firm yields a negative payoff upon S_H but a positive payoff upon S_L, and the bad firm's expected payoff of staying in the game is still zero in this mixed-strategy equilibrium. We summarize the condition of a separating equilibrium in the following lemma.

Lemma 3 *In the signal-contingent case with the direct manipulation cost, if* $P_g X_H + (1 - P_g) X_L - I - (1 - \beta)(1 - P_g)\hat{F} \geq (1 - q)K$ *is satisfied, a separating equilibrium is able to be achieved, and the least-cost separating equilibrium is achieved at* $\hat{F}$ *.*

Notice that if we are able to achieve a separating equilibrium, the least-cost separating penalty level $\hat{F}$ is actually the same as in the main setting.

In addition, Lemma 3 shows that a high initial information quality q makes it easier for a separating equilibrium to be achieved, as the good firm is able to obtain S_H even without much manipulation. In addition, the larger the direct manipulation cost K, the less likely that we are able to achieve a separating equilibrium.

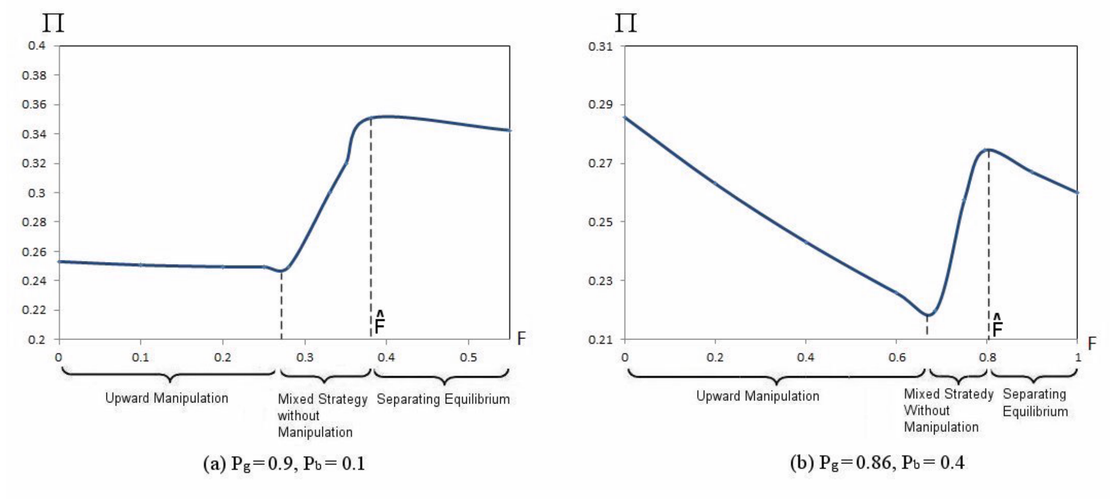

Figure 4: The optimal penalty level in the signal-contingent case with the direct manipulation cost when a separating equilibrium can be achieved. $\theta = 0.5$, $\beta = 0$, $q = 0.8$, $X_H = 3, X_L = 1$, $I = 2$, and $K = 3$.

The following numerical examples illustrate how the firm's equilibrium strategy and the firm's ex-ante expected payoff Π change with the penalty F. Figure 4 illustrates cases when a separating equilibrium can be achieved (that is, when the condition in Lemma 3 is satisfied). Our numerical analysis shows that, when a separating equilibrium can be achieved, the optimal penalty level is either the least-cost separating penalty $\hat{F}$ or zero penalty, which echoes our result in the main setting. Although it is analytically intractable to completely prove this result with the direct manipulation cost, our numerical analysis verifies this conclusion without finding any counter example.[10] For example, as shown in Figure 4, in Panel (a) when P_g is sufficiently higher than P_b ($P_g = 0.9, P_b = 0.1$), the optimal penalty is the least-cost separating penalty ($F^* = \hat{F}$), while in Panel (b) when P_g is not much higher than P_b ($P_g = 0.86, P_b = 0.4$), the optimal penalty level is zero ($F^* = 0$).

Figure 5 illustrates cases when the condition in Lemma 3 is not satisfied; that is, when a separating equilibrium cannot be achieved. In Panel (a), the optimal penalty is positive when P_g is sufficiently higher than P_b ($P_g = 0.8, P_b = 0.1$). In Panel (b), the optimal penalty level is zero

[10] To show that the firm's ex-ante expected payoff reaches the maximum either at $F = 0$ or at $F = \hat{F}$, we need to show that (1) in the region of the separating equilibrium, the ex-ante payoff Π decreases in the penalty F, and (2) in the region of the mixed-strategy without manipulation equilibrium, the ex-ante payoff Π increases in the penalty F. It is easy to prove that in the region of the separating equilibrium, Π does decrease in F, because once we reach the least-cost separating equilibrium, any increase in F only brings more cost to the firm without any benefit. However, it is analytically intractable to prove that Π increases in F in the mixed-strategy without manipulation region due to the complexity of the equilibrium. Nevertheless, our numerical analysis verifies that this is true.

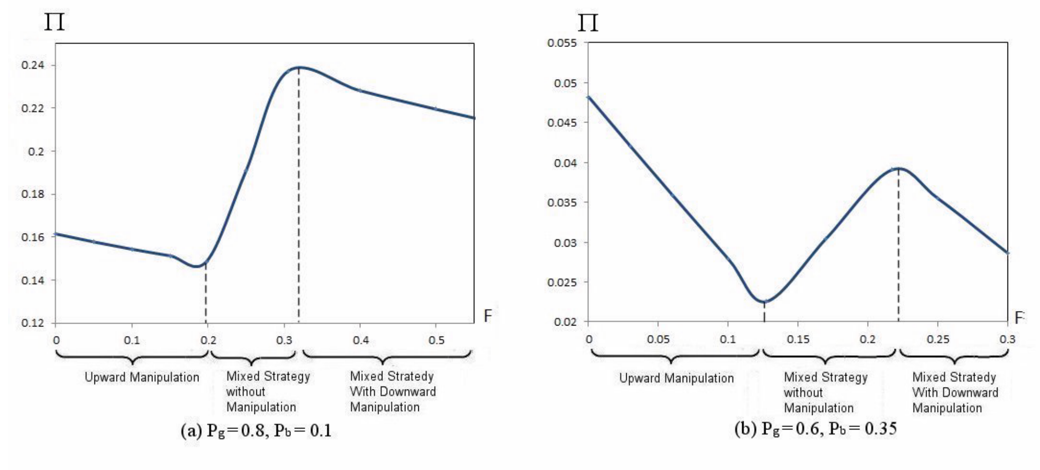

Figure 5: The optimal penalty level in the signal-contingent case with the direct manipulation cost when a separating equilibrium cannot be achieved. $\theta = 0.5$, $\beta = 0$, $q = 0.8$, $X_H = 3, X_L = 1$, $I = 2$, and $K = 3$.

when P_g is not sufficiently higher than P_b ($P_g = 0.6, P_b = 0.35$).

Analytically, we prove that the optimal penalty level to maximize the firm's ex-ante expected payoff is positive when P_g is sufficiently high and P_b is sufficiently low, or when β is sufficiently high, and the optimal penalty is zero otherwise. This echoes our results in the main setting and the intuition is similar.

Proposition 4 *In the signal-contingent case with the direct manipulation cost, $F^* > 0$ when P_g is sufficiently high and P_b is sufficiently low, or when β is sufficiently high; otherwise, $F^* = 0$.*

Although the results are in general similar to the results in the main setting, there does exist one difference: recall that in the main setting when the investor has a prior belief that the project's expected NPV is negative, zero penalty cannot be optimal, because with zero penalty both types will report S_H but the investor will not fund the project as the signals are useless. However, here with the direct cost of manipulation, this is no longer the case and it is possible that zero penalty is optimal. For example, Panel (b) in Figure 5 actually shows a case with negative prior about the expected NPV, and it shows that the optimal penalty level is zero. This is because the direct manipulation cost restricts the firm from reporting its desired signal with certainty, and thus even with zero penalty the signal is still informative about the firm's type.[11]

[11] From the analysis in this section, one can see that our study differs from Laux and Stocken (2012) in several

4.2.2 Always-fund Case with the Direct Cost of Manipulation

Recall that in the always-fund case in the main setting, the optimal penalty is always zero because a penalty only brings a dead-weight loss without any benefit. In contrast, with the direct manipulation cost, we find that imposing a positive penalty can be optimal because it helps to save on the firm's expected manipulation cost. By imposing a penalty, the firm's upward manipulation incentive is dampened and its expected manipulation cost is lowered, which is an additional benefit of imposing the penalty.

In particular, both the reimbursement proportion β and the initial information quality q play roles in a positive optimal penalty. When β is high, the dead-weight loss from the penalty is small. Although the good firm's manipulation may be encouraged by the penalty when β is high, which results in more manipulation cost (Lemma 2), when q is high, this encouragement of manipulation for the good firm is limited while the penalty is more effective in dampening the bad firm's manipulation. Overall, we have a lower expected manipulation cost upon a higher penalty level. When the saving of manipulation cost outweighs the dead-weight loss from the penalty, it can be optimal to have a positive penalty in the always-fund case.

Proposition 5 *In the always-fund case with the direct manipulation cost, $F^* > 0$ when both β and q are sufficiently high; otherwise $F^* = 0$.*

In the always-fund case, as the penalty level changes from low to high, the firm's equilibrium

important results.

First, in Laux and Stocken's paper a higher penalty level induces more managerial upward-manipulation, because the investor requests for a lower equity stake by anticipating more reimbursement while the manager does not believe he will be penalized that much. In their model, once the reimbursement proportion becomes zero, a higher penalty level can no longer induce more managerial manipulation (Proposition 5 in Laux and Stocken 2012). In contrast, in our setting even if the reimbursement proportion is zero, a higher penalty level may still induce the good firm to manipulate upward more (Lemma 2). The reason of this difference is that the result in Laux and Stocken's model is driven by the managerial over-optimism, while in our study the driving force is that the higher penalty level benefits the good firm not merely through a lower stake requested by the investor, but more importantly through helping the good firm to be distinguished from bad firms.

Second, in Laux and Stocken's model the managerial manipulation always destroys value because it leads to overinvestment. In our setting, however, manipulation may be beneficial and improve the investment efficiency when a higher penalty level induces more upward manipulation from the good firm.

Third, In Laux and Stocken's study when the direct manipulation cost is low, the firm will always report a high signal regardless of the penalty level, because a larger penalty is more likely to increase manipulation due to the managerial optimism (Proposition 6). With a low cost of manipulation, the investor believes that the report is more likely to have been manipulated, and therefore expects a higher reimbursement upon a project failure, while the manager does not believe he will be penalized due to over-optimism and thus still reports a high signal. In contrast, in our setting, with a low direct manipulation cost, when the penalty level is high enough, we achieve the separating equilbrium in which only the good firm reports a high signal and the bad firm quits because a high penalty threat drives bad firms away.

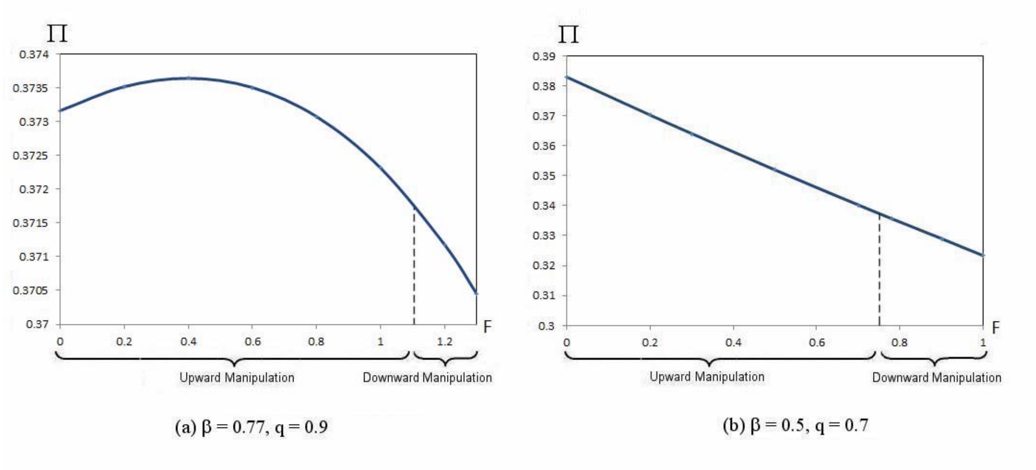

Figure 6: The optimal penalty level in the always-fund case with the direct manipulation cost. $\theta = 0.5$, $P_g = 0.9$, $P_b = 0.5$, $X_H = 3, X_L = 1$, $I = 2$, and $K = 3$.

strategy changes from an upward manipulation equilibrium in which both firm-types manipulate upward, to an equilibrium in which the bad firm manipulates downward while the good firm still manipulates upward, as illustrated by Figure 6. When the penalty level is low, both firm-types manipulate upward, because the firm's financing cost is lower upon S_H. As the penalty level becomes higher, both types' upward-manipulation incentives are dampened by the larger expected penalty. When the penalty level is high enough, the bad firm chooses to manipulate downward since the expected penalty outweighs the benefit from the lower financing cost upon S_H. Under this situation, the bad firm does not quit the game because it still yields positive expected payoff by the downward manipulation. Figure 6 Panel (a) also shows that the optimal penalty level is positive when both β and q are sufficiently high ($\beta = 0.77, q = 0.9$), and Panel (b) shows that the optimal penalty is zero when β and q are low ($\beta = 0.5, q = 0.7$).

4.3 The Interaction between the Penalty and the Direct Manipulation Cost

So far we have thoroughly analyzed the optimal penalty level given K. In reality, the direct manipulation cost may also be influenced by regulators. For example, if the SEC and the FASB convert US GAAP to IFRS (arguably, this may increase the flexibility of financial reporting and provide more discretionary options), it will become less costly to manipulate (K becomes lower). Therefore, an interesting question remaining is what will be the optimal manipulation cost to maximize the overall efficiency if K can also be chosen.

When P_g is sufficiently high and P_b is sufficiently low, or when β is sufficiently high, our analysis has shown that the benefit of improving the investment efficiency by distinguishing firm-types outweighs the dead-weight loss from imposing the penalty. The investment efficiency reaches the maximum when we are able to fully distinguish good firms from bad firms and the overall efficiency is maximized at the least-cost separating equilibrium. If both F and K are choices by regulators, then it is optimal to impose the least-cost separating penalty $F^* = \hat{F}$ and choose the zero manipulation cost ($K^* = 0$). The reason is that (1) a low K helps us to achieve a separating equilibrium (i.e., helps the condition in Lemma 3 to be satisfied) so that the investment efficiency reaches the maximum, and (2) given we have achieved a separating equilibrium, increasing the direct manipulation cost only brings more loss but no more benefit to the firm.

On the other hand when P_g does not differ much from P_b and β is low, our analysis has shown that the dead-weight loss by the penalty dominates the benefit from the improved investment efficiency, and therefore it is optimal not to impose any penalty ($F^* = 0$). In this case, if K is a choice variable, the optimal manipulation cost must be positive ($K^* > 0$). This is because given no penalty, with $K = 0$ both firm-types report S_H, and thus the signals are useless. It is then better to increase the direct manipulation cost so that the firm's manipulation is restricted and thus the signal becomes informative about the firm-type. The more-informative signal can help the good firm distinguish itself and thus the overall efficiency can be improved.

The above discussion suggests that the ex-post penalty F and the ex-ante direct manipulation cost have a relationship that resembles substitutes, although it is not a straight-forward substitutional relationship. This analysis may provide some regulatory implications. It indicates that to maximize the overall efficiency, regulators may choose either a combination of high litigation costs and plentiful discretionary options in financial reporting, or a combination of no litigation costs and tightened financial reporting rules, depending on the economy conditions.

5 Conclusions

In this paper we study whether it is efficient to impose a penalty based on the inconsistency between an optimistic signal and a bad outcome. We find that when a good project has a much larger chance to achieve a good outcome than a bad project or when a large proportion of the penalty can be reimbursed to the investor, improving the investment efficiency is more important, and imposing a penalty so that firms with good projects can be distinguished from firms with bad projects is optimal. On the other hand when a good project is not much better than a bad project on generating a good outcome and when the reimbursement proportion is very small, the dead-weight loss by the penalty dominates the benefit from the investment efficiency improvement, and therefore imposing no penalty and allowing a pooling equilibrium become optimal.

We also analyze how the introduction of a direct manipulation cost influences the equilibrium. We find that in general the results are similar to the main setting, but the direct cost does play some role and changes the firm's and the investor's equilibrium strategies. Our analysis may provide some implications on the regulatory measures to fight firms' opportunistic financial reporting.

Appendix

Proof. (of Proposition 1) From (3), one can see that the good firm would always report a high signal if

$$F \leq \frac{\left[P_g(X_H - X_L) + X_L\right](1 - \alpha(S_H))}{(1 - P_g)}. \tag{6}$$

Similarly from (4), the bad firm would always report a high signal if

$$F \leq \frac{\left[P_b(X_H - X_L) + X_L\right](1 - \alpha(S_H))}{(1 - P_b)}. \tag{7}$$

We first analyze the case when the penalty is very small. When both (6) and (7) are satisfied, a pooling equilibrium is achieved where both good and bad firms report S_H. When the prior belief about the project's NPV is positive ($[\theta P_g + (1-\theta) P_b](X_H - X_L) + X_L > 0$), the investor will fund all projects and request $\alpha(S_H) = \frac{I-(1-\theta P_g-(1-\theta)P_b)\beta F}{[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L}$. Substituting $\alpha(S_H)$ into (7), after some simplification, we have

$$F \leq \frac{\left[\theta P_g + (1-\theta) P_b\right](X_H - X_L) + X_L - I}{\frac{(1-P_b)[[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L]}{P_b(X_H-X_L)+X_L} - \beta\left[1 - \theta P_g - (1-\theta) P_b\right]}. \tag{8}$$

Note that the above threshold is positive iff the prior expected NPV is positive. If the prior expected NPV is negative, (8) can never be satisfied and a pooling equilibrium is achieved only when $F = 0$, where the investor ignores the signal and does not fund any project. Therefore, we have $\underline{F} \equiv \max[\frac{[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L-I}{\frac{(1-P_b)[[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L]}{P_b(X_H-X_L)+X_L}-\beta[1-\theta P_g-(1-\theta)P_b]},0]$, and a pooling equilibrium is achieved when $F \leq \underline{F}$.

As the penalty level gets higher, (8) does not hold and a mixed-strategy equilibrium is achieved where the bad firm is indifferent between S_H and S_L. That is,

$$\Pi_b\left(S=S_H\right)=\left[P_b X_H+\left(1-P_b\right) X_L\right]\left[1-\alpha\left(S_H\right)\right]-(1-P_b)F=0, \tag{9}$$

where $\alpha\left(S_H\right)=\frac{I-(1-\theta_H P_g-(1-\theta_H)P_b)\beta F}{[\theta_H P_g+(1-\theta_H)P_b](X_H-X_L)+X_L}$, $\theta_H=\frac{\theta}{\theta+(1-\theta)q'}$, and q' is the probability that the bad firm reports a high signal in equilibrium. One can see that since $\alpha\left(S_H\right)$ increases in q', if F is higher, to hold equation (9), q' must be lower in equilibrium. Therefore, in the mixed-strategy equilibrium, q' decreases in F. When F is high enough, q' would be reduced to zero and a separating equilibrium is achieved. The lowest F to achieve the separating equilibrium satisfies (9) when $q'=0$ ($\theta_H=1$). We denote the threshold to be $\hat{F}$,

$$\hat{F} \equiv \frac{P_g X_H+\left(1-P_g\right) X_L-I}{\frac{(1-P_b)[P_g X_H+(1-P_g)X_L]}{P_b(X_H-X_L)+X_L}-\beta\left(1-P_g\right)}.$$

When the penalty level F becomes prohibitively high, neither good nor bad firms release S_H to avoid the huge expected penalty upon failure. When the prior belief about the project's NPV is positive, good firms will still get funded upon S_L by pooling with bad firms. The good firm will always report S_L when its payoff upon S_H is less than its payoff upon S_L,

$$\left[P_g X_H+\left(1-P_g\right) X_L\right]\left[1-\alpha\left(S_H,\theta_H=1\right)\right]-(1-P_g)F \leq \left[P_g X_H+\left(1-P_g\right) X_L\right]\left[1-\alpha\left(S_L,\theta_L=\theta\right)\right],$$

where $\alpha\left(S_H,\theta_H=1\right)=\frac{I-(1-P_g)\beta F}{P_g(X_H-X_L)+X_L}$, and $\alpha\left(S_L,\theta_L=\theta\right)=\frac{I}{[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L}$. That is,

$$F \geq \frac{\left(\frac{P_g X_H+(1-P_g)X_L}{[\theta P_g+(1-\theta)P_b](X_H-X_L)+X_L}-1\right) I}{\left(1-P_g\right)\left(1-\beta\right)}.$$

When the prior belief about the project's NPV is negative, good firms won't get funded upon S_L.

The good firm will report S_L when its payoff upon S_H is negative,

$$[P_g X_H + (1 - P_g) X_L] [1 - \alpha (S_H, \theta_H = 1)] - (1 - P_g) F \leq 0.$$

That is,

$$F \geq \frac{P_g X_H + (1 - P_g) X_L - I}{(1 - P_g)(1 - \beta)}.$$

Combining the two cases, we have that $\bar{F} \equiv \min[\frac{P_g X_H + (1-P_g) X_L - I}{(1-P_g)(1-\beta)}, \frac{\left(\frac{P_g X_H + (1-P_g) X_L}{[\theta P_g + (1-\theta) P_b](X_H - X_L) + X_L} - 1\right) I}{(1-P_g)(1-\beta)}]$ and the good firm will report S_L when $F \geq \bar{F}$. ■

Proof. (of Proposition 2 and Corollary 1) We start from the case in which the investor only funds the project upon S_H. Since the investor breaks even, the overall efficiency can actually be represented by a firm's ex-ante expected payoff before the firm observes its type, which is denoted by Π, given by

$$\Pi = \underbrace{\theta \Pr (S_H|g) [(P_g(X_H - X_L) + X_L)(1 - \alpha (S_H)) - (1 - P_g) F]}_{\text{The expected return from a good project upon a high signal}} + \underbrace{(1 - \theta) \Pr (S_H|b) [(P_b(X_H - X_L) + X_L)(1 - \alpha (S_H)) - (1 - P_b) F]}_{\text{The expected return from a bad project upon a high signal}}, \tag{10}$$

where $\Pr (S_H|g)$ ($\Pr (S_H|b)$) represents the likelihood that the good (bad) firm chooses to report S_H. Substituting $\alpha(S_H)$ from (2) into the above equation, we have

$$\Pi = \underbrace{\theta \Pr (S_H|g) [P_g(X_H - X_L) + X_L - I] + (1 - \theta) \Pr (S_H|b) [P_b(X_H - X_L) + X_L - I]}_{\text{The project's expected return}} - \underbrace{(1 - \beta) F \cdot [\theta \Pr (S_H|g)(1 - P_g) + (1 - \theta) \Pr (S_H|b)(1 - P_b)]}_{\text{Expected penalty net of reimbursement}}. \tag{11}$$

When $F \leq \underline{F}$, F has no effect on the project's expected return and only brings more loss from penalty, thus Π decreases in F. For the same reason, when $\hat{F} \leq F < \bar{F}$, Π also decreases in F. In the case of the mixed-strategy equilibrium (that is, when $\underline{F} < F < \hat{F}$), equation (9) must hold, so we have

$$F = \frac{[P_b X_H + (1 - P_b) X_L] [1 - \alpha (S_H)]}{(1 - P_b)}. \tag{12}$$

The ex-ante payoff Π in the mixed-strategy equilibrium is

$$\begin{aligned}\Pi &= \theta\Pi_g\left(S=S_H\right) + \underbrace{(1-\theta)\Pi_b\left(S=S_H\right)}_{=0} \\ &= \theta\left[\left[P_gX_H + \left(1-P_g\right)X_L\right]\left[1-\alpha\left(S_H\right)\right] - (1-P_g)F\right].\end{aligned}$$

Substituting (12) into the above Π, we have

$$\Pi = \theta\left[1-\alpha\left(S_H\right)\right]\frac{\left(P_g-P_b\right)X_H}{\left(1-P_b\right)}.$$

Since $\alpha\left(S_H\right)$ decreases in F, we have Π increases in F in the mixed-strategy equilibrium. Given the above analysis on the effect of penalty in each case, it is obvious that the firm's ex-ante payoff reaches the maximum either at $F=0$ or at $F=\hat{F}$. When the prior expected NPV is negative (i.e., $\underline{F}=0$), it is obvious that $F^*=\hat{F}$.

When the prior expected NPV is positive (i.e., $\underline{F}>0$), we need to compare the firm's ex-ante expected payoffs at $F=0$ and at $F=\hat{F}$. At the least-cost separating penalty level $F=\hat{F}$, the firm's ex-ante expected payoff is

$$\begin{aligned}\Pi\left(F=\hat{F}\right) &= \theta\Pi_g\left(F=\hat{F}\right) + (1-\theta)\underbrace{\Pi_b\left(F=\hat{F}\right)}_{=0} \\ &= \theta\{\left[P_gX_H + \left(1-P_g\right)X_L\right]\left[1-\alpha\left(S_H\right)\right] - (1-P_g)\hat{F}\}.\end{aligned}$$

Substituting $\alpha(S_H)$ from (2) into the above equation, we can rewrite $\Pi\left(F=\hat{F}\right)$ to be

$$\Pi\left(F=\hat{F}\right) = \underbrace{\theta\left[P_gX_H + \left(1-P_g\right)X_L - I\right]}_{\text{The expected return from a good project}} - \underbrace{\left(1-\beta\right)\theta(1-P_g)\hat{F}.}_{\text{The expected signalling cost to deter bad firms}} \tag{13}$$

As shown in (13), the ex-ante payoff is the expected return from implementing a good project minus the expected signalling cost for a good firm to deter bad firms from mimicking. In addition, we are able to show that $\Pi\left(F=\hat{F}\right)>0$.

With $\underline{F}>0$, at the pooling equilibrium at $F=0$, both good and bad firms release S_H and the investor funds all projects because she has a prior belief that the project is a positive-NPV project (recall that $\underline{F}>0$ implies that the prior belief of a project's NPV is positive). The firm's ex-ante expected payoff therefore is

$$\Pi(F=0) = \theta\Pi_g(F=0) + (1-\theta)\Pi_b(F=0) \tag{14}$$
$$= \underbrace{\theta\left[P_g X_H + (1-P_g) X_L - I\right]}_{\text{The expected return from a good project}} - \underbrace{(1-\theta)\left[I - P_b X_H - (1-P_b) X_L\right]}_{\text{The expected loss from a bad project}}.$$

As shown in (14), $\Pi(F=0)$ is the expected return from a good project minus the expected loss from a bad project.

The optimal penalty level when $\underline{F} > 0$, therefore, can be solved by comparing $\Pi\left(F=\hat{F}\right)$ in (13) and $\Pi(F=0)$ in (14). Then we have $\Pi\left(F=\hat{F}\right) > \Pi(F=0)$ which implies

$$\underbrace{(1-\beta)\,\theta(1-P_g)\frac{P_g X_H + (1-P_g) X_L - I}{\frac{(1-P_b)[P_g X_H + (1-P_g) X_L]}{P_b X_H + (1-P_b) X_L} - \beta(1-P_g)}}_{\text{The expected signalling cost to deter bad firms}} < \underbrace{(1-\theta)\left[I - P_b X_H - (1-P_b) X_L\right]}_{\text{The expected loss from a bad project}}.$$

After some simplification, we get the condition of $F^* = \hat{F}$ which is

$$\beta > \frac{\frac{\theta[P_g X_H + (1-P_g) X_L - I]}{(1-\theta)[I - P_b X_H - (1-P_b) X_L]} - \frac{\frac{P_g}{1-P_g} X_H + X_L}{\frac{P_b}{1-P_b} X_H + X_L}}{\frac{\theta[P_g X_H + (1-P_g) X_L - I]}{(1-\theta)[I - P_b X_H - (1-P_b) X_L]} - 1}.$$

From (13), $\Pi\left(F=\hat{F}\right)$ decreases in P_b, and from (14), $\Pi(F=0)$ increases in P_b. Thus, if P_b is lower, it is more likely that $\Pi\left(F=\hat{F}\right)$ dominates $\Pi(F=0)$ and the optimal level of penalty is $\hat{F}$. In particular, when P_g approaches one, the signalling cost is reduced to zero and $\Pi\left(F=\hat{F}\right)$ must be higher than $\Pi(F=0)$. From continuity, we have $F^* = \hat{F}$ when P_g is sufficiently high and P_b is sufficiently low.

From (13), it is obvious that the expected signalling cost in the least-cost separating equilibrium (i.e., $(1-\beta)\,\theta(1-P_g)\hat{F}$) decreases in β. Since $\Pi(F=0)$ in (14) is independent to β, we have $F^* = \hat{F}$ when β is sufficiently high. ■

Proof. (of Proposition 3) In the always-fund case, the firm's ex-ante expected payoff Π is:

$$\begin{aligned}\Pi &= \theta \Pr(S_H|g)\left[\left(P_g X_H + (1-P_g) X_L\right)(1-\alpha(S_H)) - (1-P_g)F\right] \\ &+\theta \Pr(S_L|g)\left(P_g X_H + (1-P_g) X_L\right)(1-\alpha(S_L)) \\ &+(1-\theta)\Pr(S_H|b)\left[\left(P_b X_H + (1-P_b) X_L\right)(1-\alpha(S_H)) - (1-P_b)F\right] \\ &+(1-\theta)\Pr(S_L|b)\left(P_b X_H + (1-P_b) X_L\right)(1-\alpha(S_L)).\end{aligned}$$

Substituting $\alpha(S_H)$ and $\alpha(S_L)$ from (2) into the above equation, we can rewrite Π to be

$$\begin{aligned}\Pi = \underbrace{\theta\left[P_g X_H + (1-P_g) X_L - I\right] + (1-\theta)\left[P_b X_H + (1-P_b) X_L - I\right]}_{\text{The project's expected return}} \\ -\underbrace{(1-\beta)F \cdot \left[\theta \Pr(S_H|g)(1-P_g) + (1-\theta)\Pr(S_H|b)(1-P_b)\right]}_{\text{Expected penalty net of reimbursement}}. \qquad (15)\end{aligned}$$

From (15), one can easily see that in the always-fund case the optimal penalty F^* must be zero, because penalty F only decreases the firm's expected payoff Π. ■

Proof. (of Lemma 1) We first analyze the case when the investor only funds the project upon S_H and does not fund the project upon S_L. With the direct manipulation cost, the expected payoff function of a good firm given the manipulation level λ_g, denoted by $\Pi_g(\lambda_g)$, is

$$\Pi_g(\lambda_g) = (q+\lambda_g)\left[\left(P_g(X_H - X_L) + X_L\right)(1-\alpha(S_H)) - (1-P_g)F\right] - C(\lambda_g). \qquad (16)$$

The good firm chooses its optimal manipulation level (denoted by λ_g^*) to maximize its payoff. From the first-order condition, we have the good firm's optimal manipulation

$$\lambda_g^* = \frac{\left[P_g(X_H - X_L) + X_L\right](1-\alpha(S_H)) - (1-P_g)F}{K}. \qquad (17)$$

By similar analysis, we also solve the bad firm's optimal manipulation level (denoted by λ_b^*), which is

$$\lambda_b^* = \frac{\left[P_b(X_H - X_L) + X_L\right](1-\alpha(S_H)) - (1-P_b)F}{K}. \qquad (18)$$

From (17) and (18), it is easy to see that $\lambda_g^* > \lambda_b^*$.

Taking the derivative of λ_g^*/λ_b^* with respect to β, we have

$$\begin{aligned}\frac{d\lambda_g^*}{d\beta} &= \frac{\partial\lambda_g^*}{\partial\alpha\left(S_H\right)}\frac{d\alpha\left(S_H\right)}{\partial\beta} = -\frac{P_g(X_H-X_L)+X_L}{K}\cdot\frac{d\alpha\left(S_H\right)}{d\beta},\\ \frac{d\lambda_b^*}{d\beta} &= \frac{\partial\lambda_b^*}{\partial\alpha\left(S_H\right)}\frac{d\alpha\left(S_H\right)}{d\beta} = -\frac{P_b(X_H-X_L)+X_L}{K}\cdot\frac{d\alpha\left(S_H\right)}{d\beta}.\end{aligned}$$

Taking the derivative of $\alpha\left(S_H\right)$ with respect to β, we have

$$\begin{aligned}\frac{d\alpha\left(S_H\right)}{d\beta} &= -\frac{\left(1-\frac{V(S_H)-X_L}{X_H-X_L}\right)F}{V(S_H)}+\frac{\partial\alpha}{\partial V\left(S_H\right)}\frac{\partial V\left(S_H\right)}{\partial\theta_H}\cdot\frac{d\theta_H}{d\beta}\\ &= -\frac{\left(1-\frac{V(S_H)-X_L}{X_H-X_L}\right)F}{V(S_H)}-\frac{\left(I-\frac{\beta F X_H}{X_H-X_L}\right)\left(P_g-P_b\right)\left(X_H-X_L\right)}{\left[V\left(S_H\right)\right]^2}\left[\frac{\partial\theta_H}{\partial\lambda_g^*}\frac{d\lambda_g^*}{d\beta}+\frac{\partial\theta_H}{\partial\lambda_b^*}\frac{d\lambda_b^*}{d\beta}\right].\end{aligned}$$

Substituting $\frac{d\alpha(S_H)}{d\beta}$ into $\frac{d\lambda_g^*}{d\beta}$ and $\frac{d\lambda_b^*}{d\beta}$, we have

$$\begin{aligned}\frac{d\lambda_g^*}{d\beta} &= \frac{P_g(X_H-X_L)+X_L}{K}\cdot\left[\frac{\left(1-\frac{V(S_H)-X_L}{X_H-X_L}\right)F}{V(S_H)}+\frac{\left(I-\frac{\beta F X_H}{X_H-X_L}\right)\left(P_g-P_b\right)\left(X_H-X_L\right)\left[\frac{\partial\theta_H}{\partial\lambda_g^*}\frac{d\lambda_g^*}{d\beta}+\frac{\partial\theta_H}{\partial\lambda_b^*}\frac{d\lambda_b^*}{d\beta}\right]}{\left[V\left(S_H\right)\right]^2}\right]\\ \frac{d\lambda_b^*}{d\beta} &= \frac{P_b(X_H-X_L)+X_L}{K}\cdot\left[\frac{\left(1-\frac{V(S_H)-X_L}{X_H-X_L}\right)F}{V(S_H)}+\frac{\left(I-\frac{\beta F X_H}{X_H-X_L}\right)\left(P_g-P_b\right)\left(X_H-X_L\right)\left[\frac{\partial\theta_H}{\partial\lambda_g^*}\frac{d\lambda_g^*}{d\beta}+\frac{\partial\theta_H}{\partial\lambda_b^*}\frac{d\lambda_b^*}{d\beta}\right]}{\left[V\left(S_H\right)\right]^2}\right]\end{aligned}$$

where $\frac{\partial\theta_H}{\partial\lambda_g^*}=\frac{\theta(1-\theta)\left(1-q+\lambda_b^*\right)}{\left[(q+\lambda_g^*)\theta+\left(1-q+\lambda_b^*\right)(1-\theta)\right]^2}$, and $\frac{\partial\theta_H}{\partial\lambda_b^*}=\frac{-\theta(1-\theta)\left(q+\lambda_g^*\right)}{\left[(q+\lambda_g^*)\theta+\left(1-q+\lambda_b^*\right)(1-\theta)\right]^2}$.

Following the Cramer's rule, we have

$$\begin{aligned}\frac{d\lambda_g^*}{d\beta} &= \frac{\frac{\left(P_g+\frac{X_L}{X_H-X_L}\right)F[X_H-V(S_H)]}{KV(S_H)}}{1+AK[\left(P_b+\frac{X_L}{X_H-X_L}\right)\left(q+\lambda_g^*\right)-\left(P_g+\frac{X_L}{X_H-X_L}\right)(1-q+\lambda_b^*)]}>0,\\ \frac{d\lambda_b^*}{d\beta} &= \frac{\frac{\left(P_b+\frac{X_L}{X_H-X_L}\right)F[X_H-V(S_H)]}{KV(S_H)}}{1+AK[\left(P_b+\frac{X_L}{X_H-X_L}\right)\left(q+\lambda_g^*\right)-\left(P_g+\frac{X_L}{X_H-X_L}\right)(1-q+\lambda_b^*)]}>0,\end{aligned}$$

where $A\equiv\frac{(X_H-X_L)^2\left(I-\frac{\beta F X_H}{X_H-X_L}\right)(P_g-P_b)\theta(1-\theta)}{K^2[V(S_H)]^2\left[(q+\lambda_g^*)\theta+\left(1-q+\lambda_b^*\right)(1-\theta)\right]^2}>0$. Comparing $\frac{d\lambda_g^*}{d\beta}$ and $\frac{d\lambda_b^*}{d\beta}$, we have

$$\frac{d\lambda_g^*}{d\beta}-\frac{d\lambda_b^*}{d\beta}=\frac{\frac{(P_g-P_b)F[X_H-V(S_H)]}{KV(S_H)}}{1+AK[\left(P_b+\frac{X_L}{X_H-X_L}\right)\left(q+\lambda_g^*\right)-\left(P_g+\frac{X_L}{X_H-X_L}\right)(1-q+\lambda_b^*)]}>0.$$

That is, we have the result that $\frac{d\lambda_g^*}{d\beta} > \frac{d\lambda_b^*}{d\beta} > 0$. In the same way, we can show the above results also apply to the case when the investor funds the project upon both S_H and S_L. ■

Proof. (of Lemma 2) We first analyze the case when the investor only funds the project upon S_H and does not fund the project upon S_L. From (17) and (18), taking the derivative of λ_g^*/λ_b^* with respect to F, we have

$$\frac{d\lambda_g^*}{dF} = \underbrace{-\frac{1-P_g}{K}}_{\text{dampening effect (-)}} + \underbrace{\frac{-\left[P_g X_H + (1-P_g) X_L\right]}{K} \frac{d\alpha(S_H)}{dF}}_{\text{encouraging effect (+)}} \tag{19}$$

$$\frac{d\lambda_b^*}{dF} = \underbrace{-\frac{1-P_b}{K}}_{\text{dampening effect (-)}} + \underbrace{\frac{-\left[P_b X_H + (1-P_b) X_L\right]}{K} \frac{d\alpha(S_H)}{dF}}_{\text{encouraging effect (+)}}. \tag{20}$$

Taking the derivative of $\alpha(S_H)$ with respect to F, we have

$$\begin{aligned} \frac{d\alpha(S_H)}{dF} &= -\frac{\left(1-\frac{V(S_H)-X_L}{X_H-X_L}\right)\beta}{V(S_H)} + \frac{\partial\alpha}{\partial V(S_H)} \frac{\partial V(S_H)}{\partial\theta_H} \cdot \frac{d\theta_H}{dF} \\ &= -\frac{\left(1-\frac{V(S_H)-X_L}{X_H-X_L}\right)\beta}{V(S_H)} - \frac{\left(I-\frac{\beta F X_H}{X_H-X_L}\right)(P_g-P_b)(X_H-X_L)}{\left[V(S_H)\right]^2} \left[\frac{\partial\theta_H}{\partial\lambda_g^*}\frac{d\lambda_g^*}{dF} + \frac{\partial\theta_H}{\partial\lambda_b^*}\frac{d\lambda_b^*}{dF}\right]. \end{aligned} \tag{21}$$

Similarly, substituting (21) into (19) and (20), after some simplification, we have

$$\begin{aligned} \frac{d\lambda_g^*}{dF} &= -\frac{1-P_g-\beta\left(\frac{X_H}{V(S_H)}-1\right)\left(P_g+\frac{X_L}{X_H-X_L}\right)}{K} \\ &\quad + \frac{\left(P_g+\frac{X_L}{X_H-X_L}\right)(X_H-X_L)^2\left(I-\frac{\beta F X_H}{X_H-X_L}\right)(P_g-P_b)}{K\left[V(S_H)\right]^2}\left[\frac{\partial\theta_H}{\partial\lambda_g^*}\frac{d\lambda_g^*}{dF} + \frac{\partial\theta_H}{\partial\lambda_b^*}\frac{d\lambda_b^*}{dF}\right], \\ \frac{d\lambda_b^*}{dF} &= -\frac{1-P_b-\beta\left(\frac{X_H}{V(S_H)}-1\right)\left(P_b+\frac{X_L}{X_H-X_L}\right)}{K} \\ &\quad + \frac{\left(P_b+\frac{X_L}{X_H-X_L}\right)(X_H-X_L)^2\left(I-\frac{\beta F X_H}{X_H-X_L}\right)(P_g-P_b)}{K\left[V(S_H)\right]^2}\left[\frac{\partial\theta_H}{\partial\lambda_g^*}\frac{d\lambda_g^*}{dF} + \frac{\partial\theta_H}{\partial\lambda_b^*}\frac{d\lambda_b^*}{dF}\right]. \end{aligned}$$

Following the Cramer's rule, we have

$$\frac{d\lambda_g^*}{dF} = \frac{A\left(q+\lambda_g^*\right)\frac{(P_g-P_b)X_H}{X_H-X_L} - \frac{1-P_g-\beta\left(\frac{X_H}{V(S_H)}-1\right)\left(P_g+\frac{X_L}{X_H-X_L}\right)}{K}}{1+AK[\left(P_b+\frac{X_L}{X_H-X_L}\right)\left(q+\lambda_g^*\right)-\left(P_g+\frac{X_L}{X_H-X_L}\right)(1-q+\lambda_b^*)]}, \tag{22}$$

$$\frac{d\lambda_b^*}{dF} = \frac{A\left(1-q+\lambda_b^*\right)\frac{(P_g-P_b)X_H}{X_H-X_L} - \frac{1-P_b-\beta\left(\frac{X_H}{V(S_H)}-1\right)\left(P_b+\frac{X_L}{X_H-X_L}\right)}{K}}{1+AK[\left(P_b+\frac{X_L}{X_H-X_L}\right)\left(q+\lambda_g^*\right)-\left(P_g+\frac{X_L}{X_H-X_L}\right)(1-q+\lambda_b^*)]}, \tag{23}$$

where $A \equiv \frac{(X_H-X_L)^2\left(I-\frac{\beta F X_H}{X_H-X_L}\right)(P_g-P_b)\theta(1-\theta)}{K^2[V(S_H)]^2\left[\left(q+\lambda_g^*\right)\theta+\left(1-q+\lambda_b^*\right)(1-\theta)\right]^2} > 0$. Comparing (22) and (23), we have

$$\frac{d\lambda_g^*}{dF} - \frac{d\lambda_b^*}{dF} = \frac{A\left(2q-1+\lambda_g^*-\lambda_b^*\right)\frac{(P_g-P_b)X_H}{X_H-X_L} + \frac{(P_g-P_b)\left[1+\frac{\beta(X-V(S_H))}{V(S_H)}\right]}{K}}{1+AK[\left(P_b+\frac{X_L}{X_H-X_L}\right)\left(q+\lambda_g^*\right)-\left(P_g+\frac{X_L}{X_H-X_L}\right)(1-q+\lambda_b^*)]} > 0.$$

That is, we have the result that $\frac{d\lambda_b^*}{dF} < \frac{d\lambda_g^*}{dF}$. From (22), it is easy to see that if P_g is high enough and P_b is low enough, the numerator in (22) is positive, and we have $\frac{d\lambda_g^*}{dF} > 0$. In addition, when $\beta \to 1$, $\frac{1-P_g-\beta\left(\frac{X_H}{V(S_H)}-1\right)\left(P_g+\frac{X_L}{X_H-X_L}\right)}{K} \to \frac{X_H[V(S_H)-P_g(X_H-X_L)-X_L]}{KV(S_H)(X_H-X_L)} < 0$, and we also have $\frac{d\lambda_g^*}{dF} > 0$. From the property of continuity, we have that when β is sufficiently high, $\frac{d\lambda_g^*}{dF} > 0$. That is, when P_g is sufficiently high and P_b is sufficiently low, or when β is high enough, the higher penalty's encouraging effect dominates the dampening effect, and λ_g^* increases in the penalty. To be more specific, when

$$P_g + \beta\left(\frac{X_H}{V(S_H)}-1\right)\left(P_g+\frac{X_L}{X_H-X_L}\right) + AK\left(q+\lambda_g^*\right)\frac{(P_g-P_b)X_H}{X_H-X_L} > 1,$$

we have $\frac{d\lambda_g^*}{dF} > 0$. Given $\frac{d\lambda_b^*}{dF} < \frac{d\lambda_g^*}{dF}$, to show $\frac{d\lambda_b^*}{dF} < 0$, we need to show that there does not exist such a case where $\frac{d\lambda_g^*}{dF} > \frac{d\lambda_b^*}{dF} > 0$. Suppose $\frac{d\lambda_g^*}{dF} > \frac{d\lambda_b^*}{dF} > 0$ is true, from (20), we have $-\frac{P_b(X_H-X_L)+X_L}{K}\cdot\frac{d\alpha}{dF} - \frac{(1-P_b)}{K} > 0$, which is

$$-\frac{d\alpha}{dF} > \frac{(1-P_b)}{P_b(X_H-X_L)+X_L}. \tag{24}$$

Substituting (21) into (24), after some simplification we have

$$\begin{aligned} & \frac{V(S_H)(X_H - X_L)(1 - P_b)}{\beta\left[X_H - V\left(S_H\right)\right]\left[P_b(X_H - X_L) + X_L\right]} \\ < \quad & 1 + \frac{\left(I - \frac{\beta F X_H}{X_H - X_L}\right)\left(P_g - P_b\right)\left(X_H - X_L\right)^2 \left[\frac{\partial \theta_H}{\partial \lambda_g^*}\frac{d\lambda_g^*}{dF} + \frac{\partial \theta_H}{\partial \lambda_b^*}\frac{d\lambda_b^*}{dF}\right]}{V\left(S_H\right)\beta\left[X_H - V\left(S_H\right)\right]}. \end{aligned} \tag{25}$$

Since $X_H > V(S_H) > I > P_b(X_H - X_L) + X_L$, it can be verified that $\frac{V(S_H)(X_H - X_L)(1-P_b)}{\beta[X_H - V(S_H)][P_b(X_H - X_L)+X_L]} > 1$, and therefore (25) does not hold. That is, there does not exist such a case where $\frac{d\lambda_g^*}{dF} > \frac{d\lambda_b^*}{dF} > 0$ and therefore we have $\frac{d\lambda_b^*}{dF} < 0$. In the same way, we can show the above results also apply to the case when the investor funds the project upon both S_H and S_L.

In the lemma, we also consider the case in which λ_g^* is a corner solution because λ_g cannot be higher than $1 - q$. Thus we have $\frac{d\lambda_g^*}{dF} \geq 0$ when P_g is sufficiently high and P_b is sufficiently low, or when β is sufficiently high, and $\frac{d\lambda_g^*}{dF} \leq 0$ otherwise. ■

Proof. (of Lemma 3) To achieve a separating equilibrium, first, we need that the bad firm always quits the game because its payoff upon S_H is negative even with the highest posterior belief. That is,

$$\Pi_b\left(S = S_H\right) = \left[P_b X_H + \left(1 - P_b\right) X_L\right]\left[1 - \alpha\left(S_H | \theta_H = 1\right)\right] - (1 - P_b)F \leq 0,$$

where $\alpha\left(S_H | \theta_H = 1\right) = \frac{I - (1-P_g)\beta F}{P_g + (X_H - X_L) + X_L}$. That is, we need

$$F \geq \frac{P_g X_H + \left(1 - P_g\right) X_L - I}{\frac{(1-P_b)[P_g X_H + (1-P_g) X_L]}{P_b(X_H - X_L) + X_L} - \beta\left(1 - P_g\right)}.$$

Notice that $\frac{P_g X_H + (1-P_g) X_L - I}{\frac{(1-P_b)[P_g X_H + (1-P_g) X_L]}{P_b(X_H - X_L) + X_L} - \beta(1-P_g)}$ is exactly the least-cost separating level $\hat{F}$ in the main setting.

Second, we need that given this penalty level $\hat{F}$, the good firm always obtains S_H with certainty. That is, at $\hat{F}$, $(\lambda_g^* | F = \hat{F}) = \frac{[P_g(X_H - X_L) + X_L](1 - \alpha(S_H|\theta_H = 1)) - (1-P_g)\hat{F}}{K} \geq (1 - q)$.Substituting $\alpha\left(S_H | \theta_H = 1\right)$, we have the condition in the lemma. ■

Proof. (of Proposition 4) In the signal-contingent case with the direct manipulation cost, when F is low, both good and bad firms tend to manipulate upward. The firm's ex-ante expected payoff, denoted by $\Pi\left(\lambda_g^*, \lambda_b^*\right)$, is given by

$$
\begin{aligned}
& \Pi\left(\lambda_g^*, \lambda_b^*\right) = \theta \Pi_g\left(\lambda_g^*\right) + (1-\theta)\Pi_b\left(\lambda_b^*\right) \\
= \; & \theta\left[\left(q+\lambda_g^*\right)\left[\left(P_g(X_H - X_L) + X_L\right)\left(1-\alpha\left(S_H\right)\right) - (1-P_g)F\right] - C(\lambda_g^*)\right] + \\
& (1-\theta)\left[(1-q+\lambda_b^*)\left[\left(P_b(X_H - X_L) + X_L\right)\left(1-\alpha\left(S_H\right)\right) - (1-P_b)F\right] - C(\lambda_b^*)\right].
\end{aligned}
$$

Substituting $\alpha(S_H)$ from (2) into the above equation, we have

$$
\begin{aligned}
\Pi\left(\lambda_g^*, \lambda_b^*\right) = & \underbrace{\theta\left(q+\lambda_g^*\right)\left[P_g(X_H - X_L) + X_L - I\right] + (1-\theta)(1-q+\lambda_b^*)\left[P_b(X_H - X_L) + X_L - I\right]}_{\text{The project's expected return}} \\
& - \underbrace{(1-\beta)F \cdot \left[\theta\left(q+\lambda_g^*\right)(1-P_g) + (1-\theta)\left(1-q+\lambda_b^*\right)(1-P_b)\right]}_{\text{Expected penalty net of reimbursement}} - \underbrace{\left[\theta C\left(\lambda_g^*\right) + (1-\theta)C\left(\lambda_b^*\right)\right]}_{\text{Expected cost of manipulation}}
\end{aligned}
\tag{26}
$$

From (26), taking the derivative of Π with respect to F, we have

$$
\frac{d\Pi}{dF} = \frac{\partial \Pi}{\partial \lambda_g^*}\frac{d\lambda_g^*}{dF} + \frac{\partial \Pi}{\partial \lambda_b^*}\frac{d\lambda_b^*}{dF} - (1-\beta)\left[\theta\left(q+\lambda_g^*\right)(1-P_g) + (1-\theta)\left(1-q+\lambda_b^*\right)(1-P_b)\right], \tag{27}
$$

where

$$
\begin{aligned}
\frac{\partial \Pi}{\partial \lambda_g^*} &= \theta\left[P_g(X_H - X_L) + X_L - I - (1-P_g)(1-\beta)F - K\lambda_g^*\right] \\
&= \frac{\theta\left(1-\theta_H\right)\left(P_g - P_b\right)}{V\left(S_H\right)}\left[(I-\beta F)(X_H - X_L) - \beta F X_L\right] > 0,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \Pi}{\partial \lambda_b^*} &= (1-\theta)\left[P_b(X_H - X_L) + X_L - I - (1-P_b)(1-\beta)F - K\lambda_b^*\right] \\
&= \frac{(1-\theta)\theta_H\left(P_g - P_b\right)}{V\left(S_H\right)}\left[\beta F X_L - (I-\beta F)(X_H - X_L)\right] < 0.
\end{aligned}
$$
[12]

Substituting $\frac{\partial \Pi}{\partial \lambda_g^*}$ and $\frac{\partial \Pi}{\partial \lambda_b^*}$ into (27), we have

$$
\frac{\partial \Pi}{\partial \lambda_g^*}\frac{d\lambda_g^*}{dF} + \frac{\partial \Pi}{\partial \lambda_b^*}\frac{d\lambda_b^*}{dF} = \frac{\left[(I-\beta F)(X_H - X_L) - \beta F X_L\right]\left(P_g - P_b\right)}{V\left(S_H\right)}\left[\theta\left(1-\theta_H\right)\frac{d\lambda_g^*}{dF} - (1-\theta)\,\theta_H\frac{d\lambda_b^*}{dF}\right]. \tag{28}
$$

From Lemma 2, we have $\frac{d\lambda_b^*}{dF} < \frac{d\lambda_g^*}{dF}$, and $(1-\theta)\,\theta_H > \theta\,(1-\theta_H) > 0$, which implies $\left[\theta\,(1-\theta_H)\,\frac{d\lambda_g^*}{dF} - (1-\theta)\,\theta_H \frac{d\lambda_b^*}{dF}\right] > 0$. That is, $\frac{\partial \Pi}{\partial \lambda_g^*}\frac{d\lambda_g^*}{dF} + \frac{\partial \Pi}{\partial \lambda_b^*}\frac{d\lambda_b^*}{dF} > 0$. Substituting (28) , $\frac{d\lambda_g^*}{dF}$ and $\frac{d\lambda_b^*}{dF}$ into (27), we have $\frac{d\Pi}{dF}$ equals

$$
\begin{aligned}
&-(1-\beta)\left[\theta\left(q+\lambda_g^*\right)(1-P_g) + (1-\theta)\,(1-q+\lambda_b^*)(1-P_b)\right] \\
&+\frac{\left[(I-\beta F)\,(X_H - X_L) - \beta F X_L\right](P_g - P_b)}{V\,(S_H)}\left[\theta\,(1-\theta_H)\,\frac{d\lambda_g^*}{dF} - (1-\theta)\,\theta_H \frac{d\lambda_b^*}{dF}\right].
\end{aligned} \tag{29}
$$

The first item in (29) is negative which is the direct loss brought by the penalty, and the second item in (29) is positive which is the effects through the firm's manipulation decision. When β is high, the first item in (29) is less negative and would be outweighed by the second item, which results in positive $\frac{d\Pi}{dF}$. That is, when β is sufficiently high, $\frac{d\Pi}{dF} > 0$ when F is low and so $F^* > 0$.

When P_g is high, the first item in (29) is less negative and the second item in (29) is more positive, which results in higher $\frac{d\Pi}{dF}$. In addition, from the condition in Lemma 3, when P_g is sufficiently high, a separating equilibrium is achieved at $F = \hat{F}$, and the firm's ex-ante expected payoff is

$$
\begin{aligned}
\Pi\left(F = \hat{F}\right) &= \theta\{[P_g X_H + (1-P_g)\,X_L](1-\alpha\,(S_H)) - (1-P_g)\hat{F} - C\,(1-q)\} \\
&= \theta\left[P_g X_H + (1-P_g)\,X_L - I\right] - (1-\beta)\,\theta(1-P_g)\hat{F} - \theta C\,(1-q)\,.
\end{aligned}
$$

Whereas the firm's expected payoff at $F = 0$ is

$$
\begin{aligned}
\Pi\,(F=0) &= \underbrace{\theta\left(q+\lambda_g^*\right)\left[P_g(X_H - X_L) + X_L - I\right] + (1-\theta)(1-q+\lambda_b^*)\left[P_b(X_H - X_L) + X_L - I\right]}_{\text{The project's expected return}} \\
&\quad -\underbrace{\left[\theta C\left(\lambda_g^*\right) + (1-\theta) C\left(\lambda_b^*\right)\right]}_{\text{Expected cost of manipulation}}.
\end{aligned}
$$

When P_g is sufficiently high and P_b is sufficiently low, we have

$$
(1-\theta)(1-q+\lambda_b^*)\left[I - P_b(X_H - X_L) - X_L\right] + (1-\theta) C\left(\lambda_b^*\right) > (1-\beta)\,\theta(1-P_g)\hat{F} \tag{30}
$$

$\Pi\left(F = \hat{F}\right)$ is higher than $\Pi\,(F=0)$ and the optimal penalty level must be positive.

When the condition in Lemma 3 is satisfied, one can easily see that in the case of the separating equilibrium, the ex-ante payoff Π decreases in the penalty F. To show that the firm's ex ante payoff reaches the maximum either at $F=0$ or at $F=\hat{F}$, we need to show that the ex-ante payoff Π monotonically increases in the penalty F in the case of mixed-strategy equilibrium. Though it is analytically intractable to show this, our numerical examples show that Π monotonically increases in the penalty F in the case of mixed-strategy equilibrium and the firm's ex ante payoff reaches the maximum either at $F=0$ or at $F=\hat{F}$ when the condition in Lemma 3 is satisfied. ■

Proof. (of Proposition 5) In the always-fund case, a firm's ex-ante expected payoff is

$$\Pi\left(\lambda_g^*,\lambda_b^*\right)=\underbrace{\left[\theta P_g+(1-\theta)P_b\right]\left(X_H-X_L\right)+X_L-I}_{\text{Expected project return}}-\underbrace{\left[\theta C\left(\lambda_g^*\right)+(1-\theta)C\left(\lambda_b^*\right)\right]}_{\text{Expected cost of manipulation}} \tag{31}$$
$$-\underbrace{(1-\beta)\,F\left[\theta\left(q+\lambda_g^*\right)(1-P_g)+(1-\theta)\,(1-q+\lambda_b^*)(1-P_b)\right]}_{\text{Expected penalty net of reimbursement}}.$$

Taking the derivative of Π with respect to F, we have

$$\frac{d\Pi}{dF}=\underbrace{-\,(1-\beta)\left[\theta\left(q+\lambda_g^*\right)(1-P_g)+(1-\theta)\,(1-q+\lambda_b^*)(1-P_b)\right]}_{<0}+\frac{\partial\Pi_s}{\partial\lambda_g^*}\frac{d\lambda_g^*}{dF}+\underbrace{\frac{\partial\Pi_s}{\partial\lambda_b^*}\frac{d\lambda_b^*}{dF}}_{>0}, \tag{32}$$

where $\frac{\partial\Pi}{\partial\lambda_g^*}=-\theta\left[F\left(1-P_g\right)(1-\beta)+K\lambda_g^*\right]<0$, and $\frac{\partial\Pi}{\partial\lambda_b^*}=-(1-\theta)\left[F\left(1-P_b\right)(1-\beta)+K\lambda_b^*\right]<0$. When β is high, the first item in (32) is less negative. However, $\frac{d\lambda_g^*}{dF}$ could be positive when β is high as shown in Lemma 2, which implies that penalty could encourage good firm's manipulation and incur more good firm's manipulation cost. When q is high, the penalty has minimal effect on good firm's manipulation and the encouragement effect is negligible, i.e., $\frac{\partial\Pi}{\partial\lambda_g^*}\frac{d\lambda_g^*}{dF}$ is negligible. Since the penalty always discourages bad firm's manipulation and saves bad firm's manipulation cost $\frac{\partial\Pi}{\partial\lambda_b^*}\frac{d\lambda_b^*}{dF}>0$, we have that the optimal penalty is positive $F^*>0$ when both β and q are sufficiently high. ■

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