IMPLICIT AGGREGATION AND CONTOLLABILITY OF NON-CONTRACTIBLE INFORMATION *

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Abstract

We examine the interaction between implicit incentives, such as career concerns, arising from non-contractible information and explicit incentive contracts. Renegotiation of long-term contracts implicitly aggregates the same non-contractible information that gives rise to career concerns such that the resulting aggregate can be effectively contracted on. This implicit aggregation of the non-contractible information is determined by the (contractible) performance measures used in explicit contracts and is thus fixed, but the incentive weights on the implicit aggregate can be adjusted along with the weights on the contractible performance measures. Career concerns also implicitly aggregate the non-contractible information in a fixed way, but the corresponding incentive weights are fixed externalities in the contracting process. Total managerial incentives include both the fixed effects of career concerns and the controllable incentives on the effectively contractible aggregate. When the controllable incentives span the fixed effects of career concerns, the latter have no real effects with regard to total managerial incentives—they would optimally be the same with or without career concerns. In particular, this happens when the non-contractible information that gives rise to career concerns is effectively contractible. Long-term contracts with renegotiation dominate time-consistent short-term contracts because the latter can only generate fixed incentives on the same implicit aggregate of non-contractible information, and these fixed incentives are a contracting externality similar to career concerns.

KEYWORDS: dynamic agency, renegotiation, career concerns, non-contractible information.

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I Introduction

Non-contractible information implicitly enters managerial incentives (Hayes and Shaefer 2000) through a variety of mechanisms: career concerns (Holmström 1999), contract renegotiation (Hermalin and Katz 1991), or subjective performance evaluation (MacLeod 2003). However, explicit incentive contracts are widespread and ultimately what matters are the total incentives. How do implicit incentives based on non-contractible information interact with explicit incentives based on contractible information to produce total (effective) incentives?

In this study, implicit incentives originate from the effect of non-contractible information on future contracting terms. Favorable performance may lead to lower future compensation when performance targets are revised upward (ratchet effect), or to higher future compensation when beliefs about managerial ability are revised upward (career concerns). We will use career concerns as a generic term when referring to these types of implicit incentives despite their different origins.

The effect of non-contractible information on future contracting terms may also be based on promises made in the form of long-term term contracts. In this case, contract renegotiation implicitly aggregates non-contractible information into an effectively contractible aggregate, as opposed to career concerns also implicitly aggregating non-contractible information, but into an aggregate that is not effectively contractible because its incentive effects are fixed. Thus, the implicit managerial incentives arising from non-contractible information consist of fixed incentives due to career concerns and controllable incentives due to contract renegotiation. Left unchecked, career concerns arising from non-contractible information give rise to (potentially inefficient) non-congruent incentives and an associated risk premium. But the ultimate effect of career concerns depends on the total implicit incentives and on how these can be controlled via explicit contractual incentives.

Explicit incentives add to implicit incentives and complete the total managerial incentives. But explicit contractual incentives serve a dual role: they provide explicit managerial incentives and generate controllable implicit incentives through contract renegotiation. The principal thus gains an additional set of (effectively) contractible performance measures that are valuable because because they allow the principal to fine-tune risk premia in incentive contracts and improve the

congruity of performance measurement in multi-task settings (Feltham and Xie 1994).

Career concerns are the implicit incentives generated when future compensation depends on assessed managerial ability, which in turn depends on current output. Hence, absent explicit incentive compensation as in Fama (1980) or Holmström (1999), the agent has implicit incentives to increase output due to its impact on perceived ability.[1,2] These implicit incentives are fixed by the career concerns mechanism (labor market) and are externalities from the point of view of explicit incentive contracting. The labor market's beliefs about managerial ability are an implicit aggregation of the observable information that provides managerial incentives depending on how much of the expected value of managerial ability can be captured by the manager. Both the implicit aggregation and the corresponding incentives of career concerns are fixed incentive effects determined by the labor market.

Renegotiation of long-term contracts also creates implicit incentives based on observing non-contractible information at renegotiation time. Observation of the non-contractible information alters future performance expectations and partially 'locks-in' at renegotiation time some of the manager's future compensation. Thus, ex ante, the manager has incentives to affect the non-contractible information. Consequently, expectations of future compensation implicitly aggregate the non-contractible information. However, whereas this implicit aggregation is determined by the available (contractible) performance measures, and therefore fixed, the principal controls the incentive weights. By offering initial incentive rates on the contractible performance measures that are later renegotiated, the principal can determine some of the 'locked-in' compensation at renegotiation time, and thus control the incentive weights on the implicit aggregation.

Within a LEN setting similar to that commonly employed in the career concerns literature, we first demonstrate that renegotiation of a long-term contract effectively allows contracting on

[1]Uncertainty concerning managerial ability has been shown to affect pay for performance sensitivity in both adverse selection and career concerns settings. For example, when managers have private information about their own ability, output is used to provide incentives and to screen managers for ability, as in Arya and Mittendorf (2005) or Dutta (2008). In contrast, a career concerns setting is characterized by symmetric uncertainty about managerial ability—both the manager and the labor market learn about managerial ability from observing output.

[2]Dewatripont, Jewitt, and Tirole (1999) rank information systems, and Arya and Mittendorf (2011) consider the desirability of aggregated versus disaggregated performance information when incentives can only be provided through career concerns.

an implicit aggregation of the unverifiable information and we characterize a minimal effectively contractible aggregation.[3] As a particular case, we obtain conditions under which all the unverifiable information is effectively contractible through renegotiation, thus eliminating any real effects of career concerns as in Gibbons and Murphy (1992).[4] Finally, we derive conditions under which career concerns have no real effects even when the unverifiable information is not effectively contractible. This is the case when contract renegotiation allows the principal sufficient control over incentives (through the effectively contractible aggregation) to achieve the same total incentives with and without fixed implicit incentives.

Non-contractible information and renegotiation of long-term contracts are the two key features of the model because, when all performance information is contractible, short-term contracts with interim participation constraints are equivalent to renegotiating a long-term contract. By contrast, with non-contractible information, there is value to long-term contracting.

We contribute to the literature on career concerns and managerial incentives as follows. First, we characterize an implicit aggregation of the non-contractible information that is effectively contractible through renegotiation of long-term contracts. This extends the analysis in Hermalin and Katz (1991) to cases where there are multiple signals and the non-contractible information can only be partially contracted on.

In our study, the implicit information aggregation that is effectively contractible is a forecast of future contractible performance measures based on the observed unverifiable information. The effective contractibility of these forecasts through contract renegotiation relies on the confirmatory role of (backward-looking) contractible information. This provides another instance where accounting-based performance measures play a confirmatory role, a point emphasized previously

[3]The career concerns model introduced by Holmström (1999) and widely adopted in the literature relies on normally distributed information with a simple linear separable structure but exogenously specifies career concerns without including optimal incentive contracts. Gibbons and Murphy (1992) and Meyer and Vickers (1997) include optimal contracts along with career concerns but within a LEN framework that is consistent with the normally distributed information in Holmström (1999).

[4]Following Gibbons and Murphy (1992), implicit incentives have been examined in a contracting setting assuming the performance information is also directly contractible, noting the equivalence—within a LEN framework of linear contracts, exponential agent utility, and normally distributed performance measures—of short-term contracts and renegotiation-proof long-term contracts, see Meyer and Vickers (1997), Christensen, Feltham, and Şabac (2003), Şabac (2008).

in agency models with disclosure of (non-contractible) private managerial information (Dye 1983, Stocken 2000, Gigler and Hemmer 2001, Şabac and Tian 2015). Our setting is different, but, as in the disclosure literature, observing the non-contractible information alters the distribution of the contractible information, allowing the latter to play a confirmatory role.

Second, we show that observing the effectively contractible aggregate of the non-contractible information can be substituted for observing the non-contractible information without affecting the agency (i.e., is an incentive sufficient aggregate) if, and only if, career concerns have no real effects. An incentive sufficient aggregate requires less than explicit contractibility of all information as assumed by Gibbons and Murphy (1992), or effective contractibility of non-contractible information, as obtained in Hermalin and Katz (1991), but with the same effect of neutralizing (spanning) fixed implicit incentives.

When all information is contractible, career concerns based on it can be neutralized: the explicit contractual incentives are adjusted so that the total incentives, including those from career concerns, are the same as the optimal incentives absent career concerns. In this case, career concerns have no real effects on managerial actions or the principal's surplus—career concerns only affect how total incentives are split between implicit and explicit incentives.[5]

Consequently, given explicit incentive contracts, career concerns have real effects on total incentives and managerial actions only when based on non-contractible information. For example, any 'soft' firm- or industry-specific information will be unverifiable and non-contractible because only information verifiable in court will be included in a contract as inclusion of unverifiable information would leave the contract unenforceable. Kaarbøe and Olsen (2006, 2008), Autrey, Dikolli, and Newman (2007, 2010) address the efficacy of alternative information structures combining contractible and non-contractible information when career concerns are present.

Third, we contribute to the career concerns literature by providing necessary conditions for the robustness of results to the introduction of long term contracts: the non-contractible information

[5]The effect of output on contemporaneous compensation—explicit incentives—and the effect of current output on future compensation—implicit incentives—combine to produce total (effective) incentives. In Gibbons and Murphy (1992) ability is transferable and constant; as a consequence implicit incentives are strong early in an agent's career but weaken over time—to compensate, explicit incentives are strengthened as the agent's career matures.

should be informative about managerial ability beyond what is already embedded in forecasts of future (contractible) managerial performance. Otherwise, career concerns have no real effects, and thus results driven by the inefficiencies of career concerns incentives no longer hold.

When all information is contractible, the restriction to short-term contracts subject to interim participation constraints commonly used in career concerns models is innocuous because there is no value to long-term contracting.[6] But in the presence of non-contractible information, the restriction to short-term contracts—employed in the studies above combining contractible and non-contractible information—is no longer innocuous because the renegotiation driving career concerns can also be a means to effectively contract on unverifiable information if contracts are long term (Hermalin and Katz 1991).[7] That is, allowing renegotiation of long-term contracts matters significantly for incentives arising from career concerns and, thus, whether the results in the above mentioned papers continue to hold.

Our findings apply more broadly to incentive externalities that arise from renegotiating long-term contracts after observing unverifiable information. These are implicit incentives that the principal cannot control. If output can be contracted on, then it can be used both to assess managerial ability and to provide incentives, as in Gibbons and Murphy (1992) or Meyer and Vickers (1997). If managerial ability is transferable and there is competition in the labor market, the agent reaps all the benefits from leaving a positive impression. However, even if ability is firm-specific and thus not transferable, the contracts could still depend on the principal's beliefs about the manager's ability. This occurs in models with renegotiation and sharing of the surplus by the principal and the agent based on bargaining power (Meyer and Vickers 1997) and in models with renegotiation of long-term contracts or short-term fair contracts (Christensen et al. 2003 and Indjejikian and Nanda 1999).

[6]Short-term contracts do not generally sustain an equilibrium where the manager stays with the firm for multiple periods. A multi-period agency where the agent stays for several periods can be examined either by imposing interim participation constraints for the manager with short-term contracts, or by using long-term contracts (Christensen et al. 2003).

[7]If short-term contracts are negotiated every period and if current performance is informative about future performance, then current performance will affect the terms of subsequent contracts. With short-term contracts, the way current performance enters future contracts cannot be controlled by the principal, but when all performance information is contractible any expected effect on future compensation can be neutralized.

II Principal-agent model with non-contractible information

We use a two-period version of the single-period multi-task LEN model of Holmström and Milgrom (1991) and Feltham and Xie (1994). A risk neutral principal owns a production technology that requires productive effort on m tasks $a_t = (a_{t1}, a_{t2}, \ldots, a_{tm}) \in \mathbb{R}^m$ from a risk and effort averse agent in each of the two periods $t = 1, 2$.[8] The agent has exponential utility of terminal wealth with multiplicatively separable effort cost $u(w, a_1, a_2) = -\exp(-r(w - \kappa(a_1) - \kappa(a_2)))$, where w is the agent's terminal wealth, $\kappa(a_t)$ is a strictly convex function of the agent's action representing the agent's personal effort cost in period t, and r is the agent's risk aversion.[9]

The single-dimensional output for a given level of agent effort a_t is $\Pi_t = b_t(a_t) + \zeta_t$, $t = 1, 2$, where $\zeta = (\zeta_1, \zeta_2)$ is a vector of zero-mean joint normally distributed noise terms independent of the agent's actions a_t. The principal's expected benefit of agent effort is a weakly concave function $b_t(a_t)$ of the agent's actions in the two periods. The agent's actions a_t are unobservable and, hence, non-contractible.

A vector y_t of n performance measures is reported in each period, i.e., $y_t = (y_{t1}, y_{t2}, \ldots, y_{tn})$, $t = 1, 2$. The performance measures are joint normally distributed with $y_t = M_t a_t + \varepsilon_t$, where $\varepsilon_t \sim N(0, \Sigma_{\varepsilon t})$. Here, M_t is an $n \times m$ matrix of sensitivities of the n performance measures to managerial effort on the m tasks, and $\Sigma_{\varepsilon t}$ is the $n \times n$ variance-covariance matrix of the n performance measures. The vectors of noise terms ε_t have mean zero, and their distribution is independent of a_t. The output Π_t may or may not be observable and contractible; if it is observed, it is contractible and assumed to be part of the performance measure y_t.

A set of $k \leq n$ non-contractible signals $z = Ma_1 + \delta$ is observed at the same time as y_1. Here, M is the $k \times m$ matrix of sensitivities of the k non-contractible signals to managerial effort on the

[8] In what follows, we use the following vector and matrix algebra notation. Vectors are thought of as column vectors in all cases, and the scalar product of two vectors a and b in $\mathbb{R}^m$ is denoted by $a \cdot b$. For a matrix $M = [m_{ij}]_{1 \leq i \leq n, 1 \leq j \leq m}$ with n rows and m columns, we do not distinguish between the matrix and the associated linear operator $M : \mathbb{R}^m \longrightarrow \mathbb{R}^n$ defined by $Mb = (\sum_{1 \leq j \leq m} m_{ij} b_j)_{1 \leq i \leq n}$. We denote the transpose matrix as M^*, the same as the adjoint operator. Throughout the paper we use $A^* a \cdot b = a \cdot Ab$ and $(AB)^* = B^* A^*$.

[9] We assume a single consumption date and no discounting for simplicity. Our results carry over with minor modifications to a model with time-additive utility, multiple consumption dates, and discounting, while their qualitative nature remains unchanged; for details, see Dutta and Reichelstein (1999), Christensen, Feltham, Hofmann, and Şabac (2003), and Şabac (2007, 2008).

m tasks in the first period. The distribution of δ is independent of a_t, and $\delta \sim N(0, \Sigma_\delta)$. The noise terms ε_t and δ may be correlated with each other, or with the output noise ζ_t.

To reconcile the model with traditional career concerns models, these noise terms may include an unobservable and persistent component θ termed "managerial ability." This is the usual interpretation, but more generally θ can be any uncertain persistent characteristic that is not contractible and introduces incentive externalities into the agency. The contractible performance measures y_t and the non-contractible signals z may be correlated with and, thus, be informative about θ.

The contracts offered to the agent are restricted to be linear functions of the contractible performance measures with coefficients possibly depending on past observable information. The contracts are determined either by short-term linear contracts subject to interim participation constraints or by long-term linear contracts subject to renegotiation. The linearity of conditional expectations with normal distributions, exponential utility, and linear initial contracts ensure (as we will see below) that the agent's final wealth w will be normally distributed. Hence, the agent's preferences have a mean-variance representation given by the agent's certainty equivalent

$$\mathrm{CE}_t(w|a) = \mathrm{E}_t[w|a] - \tfrac{1}{2} r \mathrm{var}_t(w) - \kappa(a_1) + \kappa(a_2) \,, \tag{1}$$

where E_t and var_t denote expectation and variance, respectively, conditional on information available at date at date $t \in \{0, 1\}$.

Short-term contracts

A short-term contract w_t specifies a payment $w_t = f_t + v_t \cdot y_t$ at date t, so with short-term contracts the agent's terminal wealth is $w = w_1 + w_2$, see Figure 1.

If neither the principal nor the agent can commit for several periods, there are no equilibria where the agent stays for more than one period when managerial ability is non-transferable (Christensen et al. 2003). This problem is avoided if the agent commits to stay for multiple periods, but restrictions must be imposed on the principal at each contracting date, so the agent does not

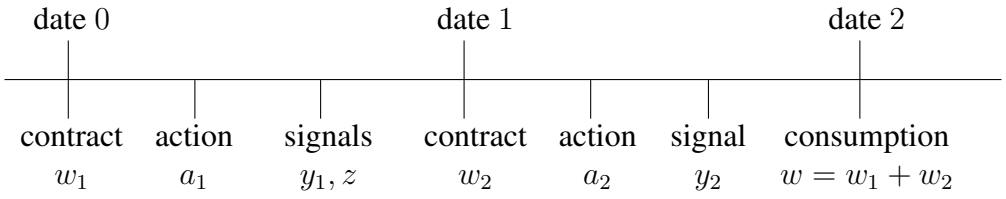

Figure 1: Time line for short-term contracts.

commit to slavery. As in the dynamic agency and career concerns literature, we assume the agent commits to stay with the firm for multiple periods, and we specify the participation constraints exogenously. First, at the start of the second period,

$$\mathrm{CE}_1(w_2|\hat{a}_1, \hat{a}_2) \geq B\ \mathrm{E}_1[\theta|\hat{a}_1]\ , \tag{2}$$

where $\hat{a}_t$ represent the actions the agent expects (or is expected by the principal) to take in equilibrium, the subscript on E_1 denotes expectations conditional on date $t = 1$ information, and θ is managerial ability. The parameter B exogenously specifies the impact of perceived managerial ability on the manager's reservation certainty equivalent. Second, the agent's participation constraint at the initial date is

$$\mathrm{CE}(w_1 + w_2|\hat{a}_1, \hat{a}_2) \geq 0\ , \tag{3}$$

The manager's expected ability plays no role at the initial date because it has an *ex ante* expected value of zero. The participation constraints (2) and (3) determine the agent's fixed wages and are both binding in equilibrium. [10]

[10]The participation constraints at the interim and initial dates can be generalized to

$(1 - B_2)\mathrm{CE}_1(w_2|\hat{a}_1, \hat{a}_2) \geq B_1\mathrm{E}_1[\theta|\hat{a}_1] + B_2\mathrm{E}_1[\Pi_2 - w_2|\hat{a}_1, \hat{a}_2]$ and
$(1 - B_2)\mathrm{CE}(w_1 + w_2|\hat{a}_1, \hat{a}_2) \geq B_2\mathrm{E}[\Pi_1 + \Pi_2 - (w_1 + w_2)|\hat{a}_1, \hat{a}_2]$, respectively.

Here B_2 exogenously specifies the division of expected surplus between the agent and the principal. These generalized participation constraints include, in addition to those mentioned in the text, additional settings from the literature. First, $B_1 = 0, B_2 = 1$ corresponds to Gibbons and Murphy (1992), where the agent captures the entire surplus; second, $B_1 = 0, B_2 = b \in [0, 1]$ corresponds to Meyer and Vickers (1997), where the principal and the agent

Our participation constraints include as particular cases several settings used in the literature in the context of career concerns. The case $B = 1$ corresponds to the career concerns model of Holmström (1999), where the agent captures the expected value of his ability. Specifically, in these types of models, the agent is risk-neutral and the wage in each period is equal to the expectation of managerial ability at the start of the period, $w_t = \mathrm{E}_{t-1}[\theta|\hat{a}_t]$; the only performance information available to the labor market is the output in each period, $\Pi_t = a_t + \theta + \zeta_t$. Thus, if we further impose the restriction $v_t = 0$, that is, the agent only receives a fixed wage, we get exactly a two-period version of Holmström's model. A multi-task version of the Holmström career concerns model is analyzed by Dewatripont et al. (1999), who allow $w_t = \mathrm{E}[\theta|z_{t-1}, \hat{a}_t]$ for a vector of tasks $\hat{a}_t$, an arbitrary vector of non-contractible signals z_{t-1}, and a scalar managerial ability parameter θ.

The case $B = 0$ corresponds to the fair contracts in Indjejikian and Nanda (1999) and Christensen et al. (2003), where the principal captures all the surplus. In this case, the agent's reservation certainty equivalent is zero at each date $t = 1, 2$. While we distinguish between the performance measures y_t and the output Π_t, we can include as particular cases the studies above where the output is the only contractible performance measure, that is $y_t = \Pi_t$, as well as a more general case where the output Π_t is only one of the available performance measures.

share the surplus. Specifically, in these models, the agent is risk-averse, the output is the only available performance measure, and is contractible. In Gibbons and Murphy (1992), with $B_1 = 0, B_2 = 1$, we get $\mathrm{E}_1[w_2] = \mathrm{E}_1[\Pi_2]$ and $\mathrm{E}[w_1 + w_2] = \mathrm{E}[\Pi_1 + \Pi_2]$ because the agent's expected wage cannot exceed the principal's expected benefit and, in this case equals the expected benefit due to assumed perfect competition between principals. In Meyer and Vickers (1997, p. 559, p. 562–563), the agent's second-period reservation certainty equivalent is bounded from below by a share b of the *total certainty equivalent* $\mathrm{TCE}_2 = \mathrm{CE}_1(w_2) + \mathrm{E}_1[\Pi_2 - w_2]$, that is, the total between the agent's certainty equivalent and the principal's expected benefit net of compensation costs; in other words, this is the "total pie" available for sharing and b can be thought of as the agent's exogenous bargaining power. At the initial date, Meyer and Vickers (1997) assume a fixed reservation certainty equivalent, that is, $b = 0$. Since the parameter B_2 in our model, which corresponds to the bargaining power b in their model, is exogenous, we assume for consistency the same value of B_2 in both participation constraints. This also makes the model consistent with the one in Gibbons and Murphy (1992), where $B_2 = 1$ at both dates. The general formulation of the interim participation constraint combines managerial human capital and external labor market influences through the parameter B_1 and sharing of the surplus between principal and agent through the parameter B_2. However, as implicit incentives arising from surplus sharing are qualitatively similar to those arising from career concerns, we restrict attention only to the latter by setting $B_2 = 0$.

Long-term contracts

A long-term linear contract at date $t = 0$ is a take-it-or-leave-it offer by the principal $w^I = f_I + v_{I1} \cdot y_1 + v_{I2} \cdot y_2$. Once the initial contract is accepted, we assume for simplicity that the principal and the agent are committed to employment in both periods.[11] In exchange for the commitment to employment for both periods, the principal commits to making a "fair" linear renegotiation offer at date $t = 1$ to the agent, $w^R = f_R + v_{R2} \cdot y_2$, where f_R and v_{R2} may depend on $t = 1$ information, see Figure 2.

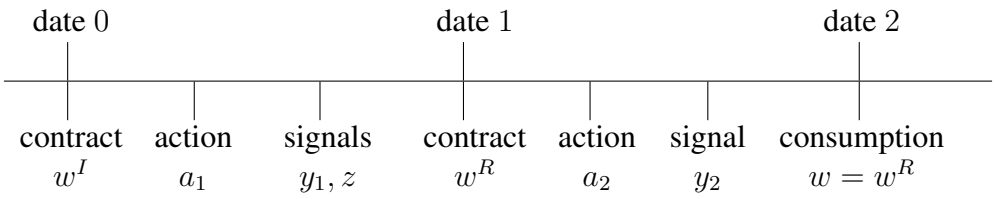

Figure 2: Time line for renegotiation of a long-term contract.

The renegotiation offer introduces career concerns similar to those present with short-term contracts. That is, the agent will receive a renegotiation offer that includes, in addition to the reservation certainty equivalent provided by the initial contract, a fraction $B \geq 0$ of his revised expected ability. We assume w^R satisfies

$$\mathrm{CE}_1(w^R|\hat{a}_1, \hat{a}_{R2}) \geq \mathrm{CE}_1(w^I|\hat{a}_1, \hat{a}_{I2}) + B\mathrm{E}_1[\theta|\hat{a}_1]. \tag{4}$$

The initial contract determines in part the renegotiation offer, but it need not be renegotiation-proof.[12] When accepting the initial contract, the agent rationally anticipates the renegotiation offer

[11] This commitment assumption reflects common executive compensation practices that include deferred compensation forfeited by managers when leaving before the end of the contract term (which commits the manager) and 'golden parachutes' that specify payments to managers if their employment is terminated early (which commit the firm). For an analysis without such commitment, see, for example, Christensen et al. (2003).

[12] The renegotiation-proofness principle applies when all the information is contractible, and $B = 0$ (see Şabac 2007). When there is non-contractible information, there will be non-trivial renegotiation of initial contracts (see, e.g., Christensen et al. 2013). Non-trivial renegotiation of initial contracts may also occur if there are career

w^R that determines the agent's equilibrium payoff. Consequently, we state the initial participation constraint at $t = 0$ in terms of the renegotiation offer:

$$\mathrm{CE}(w^R|\hat{a}_1, \hat{a}_2) \geq 0 \,. \tag{5}$$

The contracts are optimal and satisfy the interim participation constraints *given a set of actions and associated incentive rates that implement those actions*, but without making any claims as to the optimality of those induced actions from the principal's perspective. The time-consistent equilibrium in which the induced actions are the optimal ones at each contracting stage given available information is then simply a particular case of the more general scenario with exogenously chosen implementable actions. Thus, in all cases, the incentive rates will be the optimal ones, conditional on exogenously given implementable actions, see, e.g. Christensen, Şabac, and Tian (2010).

III Effective incentives and implicit aggregation

We now derive the optimal short-term contracts and long-term contracts with renegotiation. This allows us to determine the total, or effective incentives on the contractible information (explicit) and on the non-contractible information (implicit). We then separate the implicit incentives into incentive externalities that are not controllable by the principal and controllable implicit incentives.

Effective incentives

In what follows, the conditional expectations at the interim date will be used extensively:

$$\begin{aligned} \mathrm{E}_1[y_2|\hat{a}_1, \hat{a}_2] &= \alpha_1(\hat{a}_1, \hat{a}_2) + \Lambda y_1 + \Gamma z \\ \mathrm{E}_1[\theta|\hat{a}_1] &= \alpha_2(\hat{a}_1) + \phi \cdot y_1 + \psi \cdot z \,, \end{aligned} \tag{6}$$

concerns even when all information is contractible.

where $\alpha_i(\cdot)$ is a catch-all constant. The coefficients Λ and ϕ capture the correlation between the first-period performance measures y_1 and second-period performance y_2 and managerial ability θ, respectively, while controlling for the information in z. Similarly, the coefficients Γ and ψ capture the correlation between the non-contractible information z and second-period performance y_2 and managerial ability θ, respectively, while controlling for the information in y_1.

Because short-term contracts with interim participation constraint (2) or renegotiation of long-term contracts subject to the renegotiation constraint (4) introduce implicit incentives, the agent faces effective (or total) incentives that consist of explicit and implicit incentives. We begin by determining the effective incentives and the induced actions for given explicit incentives (all necessary proofs are in the Appendix).

Lemma 1 *For given incentive rates v_1, v_2 in a sequence of short-term contracts w_1, w_2, the agent's total compensation is*

$$\begin{aligned} w_1 + w_2 &= f_1 + v_1 \cdot y_1 + f_2 + v_2 \cdot y_2 \\ &= f_1 - v_2 \cdot \alpha_1 + B\alpha_2 + \kappa(\hat{a}_2) + \tfrac{1}{2} r \operatorname{var}_1(w_2) \\ &\qquad + (v_1 + B\phi - \Lambda^* v_2) \cdot y_1 + (B\psi - \Gamma^* v_2) \cdot z + v_2 \cdot y_2 \ . \end{aligned} \tag{7}$$

The actions induced by the short-term contracts with explicit incentive rates v_1, v_2 are

$$\begin{aligned} \nabla_{a_1} \kappa(a_1) &= M_1^* v_1^e + M^* v_z^e = M_1^*(v_1 + B\phi - \Lambda^* v_2) + M^*(B\psi - \Gamma^* v_2) \\ \nabla_{a_2} \kappa(a_2) &= M_2^* v_2^e = M_2^* v_2 \ . \end{aligned} \tag{8}$$

The implicit incentives arise from the fixed second-period wage f_2, that is set to satisfy the interim participation constraint (2). With short-term contracts, the effective incentive v_2^e on y_2 is the same as the explicit incentive v_2, $v_2^e = v_2$, and there are no incentive externalities with regards to a_2. By contrast, the effective incentive on y_1 is $v_1^e = v_1 + B\phi - \Lambda^* v_2$ and consists of the explicit incentive v_1, and an implicit incentive including two first-period incentive externalities: a fixed effect of career concerns $B\phi$, and a ratchet effect $-\Lambda^* v_2$ due to the inter-temporal correlation

of the performance measures. Thus, if $B > 0$ and $\phi > 0$, career concerns increase the agent's effective first-period incentives, whereas—with positively correlated performance measures—the ratchet effect reduces the effective first-period incentives.

We refer to these as incentive externalities because at the start of the first period when w_1 is set, they are either exogenously fixed by the contracting environment (career concerns) or depend on second-period decisions the principal cannot commit to and must rationally anticipate (in this case the ratchet effect is due to second-period incentives).

The effective incentive on z is entirely an implicit incentive, because z is not contractible but the second-period wage *implicitly* depends on z *ex ante*. The implicit incentive $v_z^e = B\psi - \Gamma^* v_2$ also consists of two incentive externalities: a fixed effect of career concerns $B\psi$ and a ratchet effect $-\Gamma^* v_2$ due to the inter-temporal correlation of z with the second-period performance measures. As with the effective first-period incentives above, if $B > 0$ and $\psi > 0$, career concerns increase the implicit incentives on the non-contractible information z, whereas—with positively correlated second-performance and non-contractible information—the ratchet effect reduces the implicit incentives on z.

We now turn to long-term contracts subject to the renegotiation and career concerns.

Lemma 2 *For given incentive rates v_{I1}, v_{I2} and v_{R2} in a sequence of renegotiated long-term contracts w^I, w^R, subject to constraint* (4)*, the agent's final total compensation is*

$$\begin{aligned} w^R = {} & f_I + v_{I2} \cdot \alpha_1 - v_{R2} \cdot \alpha_1 + B\alpha_2 \\ & - \kappa(\hat{a}_{I2}) - \tfrac{1}{2} r \operatorname{var}_1(w^I) + \kappa(\hat{a}_{R2}) + \tfrac{1}{2} r \operatorname{var}_1(w^R) + v_{R2} \cdot y_2 \\ & + [v_{I1} + B\phi + \Lambda^*(v_{I2} - v_{R2})] \cdot y_1 + [B\psi + \Gamma^*(v_{I2} - v_{R2})] \cdot z \,. \end{aligned} \tag{9}$$

The actions induced by the long-term contracts with explicit incentive rates v_{I1}, v_{I2} and v_{R2} are

$$\begin{aligned} \nabla_{a_1} \kappa(a_1) &= M_1^* v_1^e + M^* v_z^e \\ &= M_1^* \left[v_{I1} + B\phi + \Lambda^*(v_{I2} - v_{R2}) \right] + M^* \left[B\psi + \Gamma^*(v_{I2} - v_{R2}) \right] \\ \nabla_{a_2} \kappa(a_2) &= M_2^* v_2^e = M_2^* v_{R2} \,. \end{aligned} \tag{10}$$

Thus, the effective incentive on y_2 is the same as the explicit incentive v_{R2}, $v_2^e = v_{R2}$, whereas the effective incentive on y_1 is $v_1^e = v_{I1} + B\phi + \Lambda^*(v_{I2} - v_{R2})$. The effective (implicit) incentive on z is $v_z^e = B\psi + \Gamma^*(v_{I2} - v_{R2})$. The incentive externalities in the effective incentives on first-period performance and non-contractible information are the same as those present with short-term contracts: a fixed effect of career concerns, $B\phi$ or $B\psi$, and a ratchet effect $-\Lambda^* v_{R2}$ or $-\Gamma^* v_{R2}$.

With long-term contracts, however, the effective incentives include one more term, $\Lambda^* v_{I2}$ or $\Gamma^* v_{I2}$, that is not an externality because the principal controls v_{I2}. Each term represents the portion of expected total compensation that renegotiation 'locks in' upon observation of first-period performance y_1 or non-contractible information z, respectively. The principal does not have a commitment problem with respect to v_{I2} because that is the second-period incentive rate offered in the initial contract. The fact that v_{I2} will be replaced in renegotiation by v_{R2} makes it a 'free parameter' for the principal in the first period. Most importantly for our purposes, v_{I2} allows the principal some degree of control over the implicit incentives on z.[13]

The principal is only constrained by incentive externalities at the initial date when setting incentives for first-period effort a_1 as shown in (8) and (10). With short-term contracts, the principal can attain any effective incentive rate on the first-period performance y_1 by freely choosing v_1, see (8). By contrast, with short-term contracts, the effective incentive on the non-contractible information z is entirely determined by the two incentive externalities, and is fixed.

With long-term contracts, the principal can also attain any effective incentive rate on the first-period performance y_1 by freely choosing v_{I1}, see (10). However, the space of attainable effective incentive rates on the k non-contractible signals z is

$$\mathcal{V}_z^e \equiv \left\{ v_z^e \in \mathbb{R}^k | v_z^e = B\psi + \Gamma^* x, \forall x \in \mathbb{R}^n \right\}. \tag{11}$$

The principal chooses the effective incentive rates v_z^e in the space $\mathcal{V}_z^e$ through the choice at $t =$

[13] When z is contractible, v_{I2} is redundant because first-period incentives can be fully controlled through the incentive rate v_{I1} on first-period performance, which in this case also includes z. Similarly, with short-term contracts, any fixed effects of career concerns, or the ratchet effect, can be fully neutralized through the incentive rate v_1 on first-period performance, which also includes z.

0 of the difference between the incentive rates on the second-period contractible signals y_2 in the initial contract, v_{I2}, and the renegotiated contract, v_{R2}, i.e., $x = v_{I2} - v_{R2}$. The principal controls the incentive rate v_{I2}, and thus only the term $\Gamma^* v_{I2}$ in the total effective incentive in (11). Consequently, the space of controllable incentives on z is

$$\mathcal{V}_z \equiv \left\{ v_z \in \mathbb{R}^k | v_z = \Gamma^* x, \forall x \in \mathbb{R}^n \right\}. \tag{12}$$

It follows that the principal can always effectively 'undo' the incentive externality due to the ratchet effect $-\Gamma^* v_{R2}$, so the attainable effective incentives are constrained only by the incentive externalities arising from career concerns, $\mathcal{V}_z^e = B\psi + \mathcal{V}_z$.

Effectively contractible implicit aggregation

The effective (implicit) incentives on the non-contractible information can be equivalently characterized by an effectively contractible implicit aggregation. Indeed, rewriting the variable pay that depends on z in equation (9), we have $v_z^e \cdot z = [B\psi + \Gamma^*(v_{I2} - v_{R2})] \cdot z = (B\psi - \Gamma^* v_{R2}) \cdot z + v_{I2} \cdot \Gamma z$. As the principal can freely choose the incentive rate v_{I2} on the n-dimensional vector Γz, the aggregate Γz is effectively contractible.

Definition 1 *An aggregate Υz of the non-contractible signals is* effectively contractible *if, conditional on the non-contractible information z being observable,*

(a) The implementable actions (a_1, a_2) are the same as if Υz were directly contractible.

(b) Any implementable actions (a_1, a_2) can be induced at the same cost as if Υz were directly contractible.

In addition, an effectively contractible aggregate Υz is a an incentive sufficient aggregate *if the same actions can be implemented at the same cost when Υz is observed instead of z.*

To illustrate, we assume two tasks and two performance measures in each period. The non-contractible signal is also two-dimensional, so that $m = n = k = 2$. The performance measures

are $y_t = M_t a_t + \varepsilon_t$ and the non-contractible information is $z = Ma_1 + \theta e + \varsigma$, where $e = [1\ ,1]^*$ and θ is managerial ability as before. The variance-covariance matrix of (y_2, θ, z) is

$$\Sigma(y_2, \theta, z) = \begin{bmatrix} \sigma^2_{\varepsilon_{21}} & 0 & 0 & \gamma_{11} & \gamma_{12} \\ 0 & \sigma^2_{\varepsilon_{22}} & 0 & \gamma_{21} & \gamma_{22} \\ 0 & 0 & \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \gamma_{11} & \gamma_{21} & \sigma^2_\theta & \sigma^2_\theta + \sigma^2_{\varsigma_1} & \sigma^2_\theta \\ \gamma_{12} & \gamma_{22} & \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta + \sigma^2_{\varsigma_2} \end{bmatrix}.$$

That is, we assume managerial ability does not affect the contractible performance measures, θ is uncorrelated with y_1 and y_2, and y_1 is uncorrelated with y_2, but z (or more precisely ς) is correlated with y_2[14]. It follows that $\mathrm{E}_1[y_2] = \alpha_1 + \Gamma z$, $\mathrm{E}_1[\theta] = \alpha_2 + \psi \cdot z$, and the space of attainable incentive rates on z is $\mathcal{V}^e_z = \{v^e_z \in \mathbb{R}^2 | v^e_z = B\psi + \Gamma^* x, \forall x \in \mathbb{R}^2\}$, where

$$\psi = \frac{\sigma^2_\theta}{\sigma^2_\theta\sigma^2_{\varsigma_1} + \sigma^2_\theta\sigma^2_{\varsigma_2} + \sigma^2_{\varsigma_1}\sigma^2_{\varsigma_2}} \begin{bmatrix} \sigma^2_{\varsigma_2} \\ \sigma^2_{\varsigma_1} \end{bmatrix} \text{ and}$$

$$\Gamma = \frac{1}{\sigma^2_\theta\sigma^2_{\varsigma_1} + \sigma^2_\theta\sigma^2_{\varsigma_2} + \sigma^2_{\varsigma_1}\sigma^2_{\varsigma_2}} \begin{bmatrix} (\gamma_{11} - \gamma_{12})\sigma^2_\theta + \gamma_{11}\sigma^2_{\varsigma_2} & -(\gamma_{11} - \gamma_{12})\sigma^2_\theta + \gamma_{12}\sigma^2_{\varsigma_1} \\ (\gamma_{21} - \gamma_{22})\sigma^2_\theta + \gamma_{21}\sigma^2_{\varsigma_2} & -(\gamma_{21} - \gamma_{22})\sigma^2_\theta + \gamma_{22}\sigma^2_{\varsigma_1} \end{bmatrix}.$$

To see that contract renegotiation may only allow effective contracting on a particular aggregation of z, we assume $\gamma_{11} = \gamma_{22} = \gamma$, and $\gamma_{12} = \gamma b$, $\gamma_{21} = \gamma/b$. Then Γ has rank one (the rows of Γ are linearly dependent) and can be written as

$$\Gamma = \Gamma^1\Upsilon = \frac{\gamma\left(\sigma^2_\theta + \sigma^2_{\varsigma_2} - b\sigma^2_\theta\right)}{\sigma^2_\theta\sigma^2_{\varsigma_1} + \sigma^2_\theta\sigma^2_{\varsigma_2} + \sigma^2_{\varsigma_1}\sigma^2_{\varsigma_2}} \begin{bmatrix} 1 \\ 1/b \end{bmatrix} \begin{bmatrix} 1 & \dfrac{b\sigma^2_\theta + b\sigma^2_{\varsigma_1} - \sigma^2_\theta}{\sigma^2_\theta + \sigma^2_{\varsigma_2} - b\sigma^2_\theta} \end{bmatrix},$$

where $\Upsilon = \left[1\ , (b\sigma^2_\theta + b\sigma^2_{\varsigma_1} - \sigma^2_\theta)/(\sigma^2_\theta + \sigma^2_{\varsigma_2} - b\sigma^2_\theta)\right]$. That is, each of the two components of Γz is a multiple of Υz. In this case, $q = 1 < k = n = 2$ implies the conditional expectations of

[14] Allowing ability to affect the contractible performance measures complicates the analysis without providing additional insights.

both second-period performance measures y_{21}, y_{22} depend on $z = (z_1, z_2)$ through the same linear aggregation Υz or some multiple thereof, i.e. the two components (signals) of z have the same relative weights when forecasting the two components of y_2 (both rows in Γ equal a multiple of Υ). Then the aggregate Υz is effectively contractible—the agency is as well of as if Υz were directly contractible.

Effective contractibility only captures the principal's scope of control over the agent's incentives, without the stronger requirement that the effectively contractible aggregate be substituted for the non-contractible information without affecting the agency. If assessing expected managerial ability places the same relative weights on the two signals in z, i.e. ψ is proportional to Υ, then the agent's incentives to work on bolstering assessed ability can be effectively countered by putting a negative weight on Υz. In this case Υz is an incentive sufficient aggregate because observing z does not add anything to observing only the aggregate Υz. In contrast, the effectively contractible aggregate may not be substituted for the non-contractible information when the latter creates career concerns which the former cannot offset, and is thus not an incentive sufficient aggregate.

To to illustrate this last point, let $b = 1$, such that $\Upsilon = \left[1\ , \sigma^2_{\varsigma_1}/\sigma^2_{\varsigma_2}\right] = \left[\sigma^2_{\varsigma_2}\ , \sigma^2_{\varsigma_1}\right]/\sigma^2_{\varsigma_2}$ and it is easily seen that $\pi = B\sigma^2_{\varsigma_2}(\sigma^2_\theta/\gamma)$ can be chosen such that $B\psi = \Upsilon^*\pi$. Thus, when $b = 1$, ψ is spanned by Υ, and Υz is an incentive sufficient aggregate.

The effective incentives on z consist of the incentive externality $B\psi$ and the implicit incentives controllable by the principal, $\Gamma^*(v_{I2} - v_{R2})$, which make up the subspace spanned by the rows of Υ. When $b = 1$ this subspace is the one-dimensional line along the vector ψ (ψ and Υ are collinear but of different length) as depicted in Figure 3.

This has two consequences: 1) the principal cannot effectively contract on z; 2) because this subspace includes the fixed incentive externality $B\psi$, career concerns have no real effects—they can always be undone by the principal. When $b = 1$, the space of effective incentives is $\mathcal{V}^e_z = \left\{v^e_z \in \mathbb{R}^2 | v^e_z = B\psi + \left(\Gamma^q\Upsilon\right)^* x, \forall x \in \mathbb{R}^1\right\} = \left\{v^e_z \in \mathbb{R}^2 | v^e_z = v\psi, \forall v \in \mathbb{R}^1\right\}$. That is, any effective contract can be written as $f + v_1 \cdot y_1 + v\psi \cdot z + v_{R2} \cdot y_2$, and the principal can choose any weight v on $\psi \cdot z$. In this case the effective incentives on z are fully controllable by the principal, $\mathcal{V}^e_z = \mathcal{V}_z$.

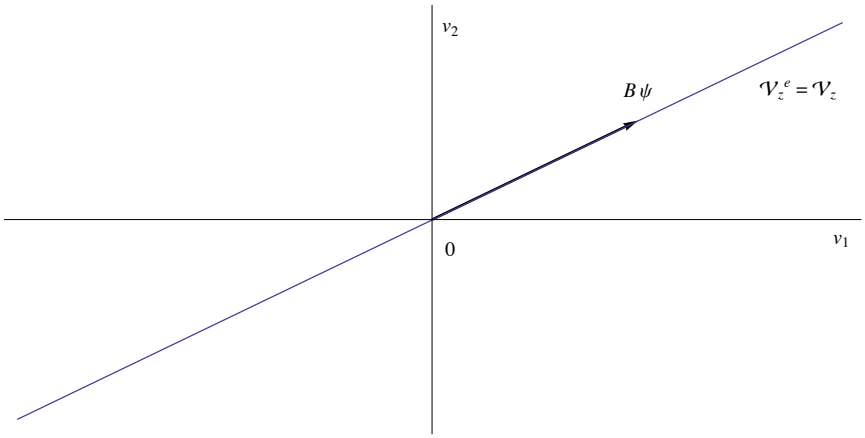

Figure 3: Attainable effective incentives on z when ψ is spanned by the rows of Υ.

In other words, the agency problem with career concerns is equivalent to one where the principal can contract on a linear aggregation of the components of z and there are no career concerns—observing z in addition to Υz does not add anything and Υz is an incentive sufficient aggregate. This linear aggregation, however, is fixed and entirely determined by the contractible performance measures and how they relate with the non-contractible signals—similar in nature to contracting on stock price, which is another particular aggregation of the observable signals.

The subspace $\mathcal{V}_z = \{v_z \in \mathbb{R}^2 | v_z = \Upsilon^* x, \forall x \in \mathbb{R}^1\}$ represents the portion of effective incentives that the principal can control and is depicted by the line through the origin denoted $\mathcal{V}_z$ in Figure 4. When $b \neq 1$, ψ is not spanned by the rows of Υ, and the space of attainable effective incentive rates on the non-contractible signals z is

$$\mathcal{V}_z^e = \left\{v_z^e \in \mathbb{R}^2 | v_z^e = B\psi + \Upsilon^* x, \forall x \in \mathbb{R}^1\right\} = B\psi + \mathcal{V}_z.$$

That is, $\mathcal{V}_z^e$ is the one-dimensional row space of the aggregation matrix $\Upsilon = [I^q, \Omega]$ translated by the incentive externality $B\psi$. In Figure 4, $B\psi$ is not in the subspace $\mathcal{V}_z$ spanned by the rows of Υ and $\mathcal{V}_z^e$ is a line parallel to $\mathcal{V}_z$. The spaces $\mathcal{V}_z$ and $\mathcal{V}_z^e$ also represent the agent's implicit incentives when observing only Υz or z, respectively. It is then evident we cannot replace z by the contractible aggregate Υz without affecting the agency because the impact of the incentive

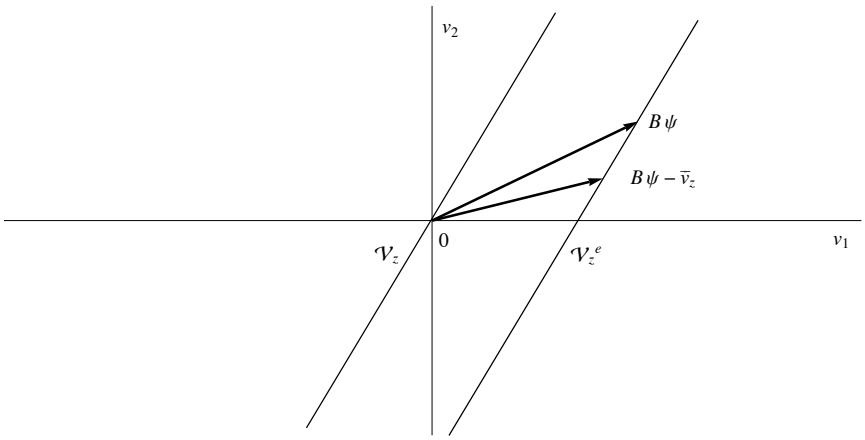

Figure 4: Attainable effective incentives on z when ψ is not spanned by the rows of Υ.

externality cannot be neutralized (observing z adds to the agent's incentives)—the aggregate Υz is effectively contractible but not incentive sufficient. Thus, career concerns have real effects in this example.

Whereas Γz is an n-dimensional effectively contractible aggregate, Γ has rank $q \leq k$, and the space of attainable incentives on z, $\mathcal{V}_z^e$, has dimension $q \leq k \leq n$. This suggests the principal has q degrees of freedom in controlling incentives on z and there may be a q-dimensional effectively contractible aggregate that matches the principal's scope of control. That is, there may be a 'minimal' aggregation of the information in z that is effectively contractible.[15] We next characterize such an aggregate of z, including conditions such that it is an incentive sufficient aggregate.

Assume without loss of generality that the non-contractible signals are ordered such that the first q columns of Γ are linearly independent, i.e., $\Gamma = \left[\Gamma^q, \Gamma^{k-q}\right]$, where Γ^q has rank q. Let Ω be the $(q \times (k-q))$-matrix that determines the columns of Γ^{k-q} as linear combinations of the columns in Γ^q, i.e., $\Gamma^{k-q} = \Gamma^q \Omega$. As a consequence, $\Gamma = \Gamma^q \left[I^q, \Omega\right]$, where I^q denotes the q-dimensional identity matrix. We define $\Upsilon := [I^q, \Omega]$ so that $\Gamma = \Gamma^q \Upsilon$.

Proposition 1 *Assume the number of non-contractible signals z is less than or equal to the number of contractible signals y_t in each period, $k \leq n$, and that Γ has rank $q \leq k$.*

[15] Minimal aggregation of performance measures is also considered by Şabac and Yoo (2016), but in the context of contractible information in a multi-task agency.

A. *The q-dimensional aggregate Υz of the non-contractible signals is effectively contractible with long-term contracts, where $\Upsilon = [I^q, \Omega]$.*

B. *The aggregate Υz is an incentive sufficient aggregate if, and only if, the incentive externality caused by career concerns $B\psi$ (equivalently the vector ψ) is spanned by the rows of $\Upsilon = [I^q, \Omega]$, i.e., there exists a $(q \times 1)$ -vector π such that $B\psi = \Upsilon^*\pi$. The total surplus is independent of career concerns (B), if, and only if, Υz is an incentive sufficient aggregate.*

Part A of Proposition 1 characterizes an implicit aggregation of the non-contractible information that is effectively contractible through contract renegotiation. In general, Γz is the portion of forecasted second-period performance that is based on the non-contractible information z. Observing the aggregate Γz instead of z is just as good in forecasting second-period performance because $\mathrm{E}[y_2|\hat{a}_1, \hat{a}_2, y_1, z] = \alpha_1(\hat{a}_1, \hat{a}_2) + \Lambda y_1 + \Gamma z = \mathrm{E}[y_2|\hat{a}_1, \hat{a}_2, y_1, \Gamma z]$. By a similar argument, the aggregate Υz can always be substituted for z in forecasting y_2 because Υz is a sufficient statistic for Γz (by construction, since $\Gamma = \Gamma^q\Upsilon$). By a slight abuse of language we will also refer to Υz as a forecast of future performance based on observed non-contractible information.[16]

If Γ has full rank k, then z itself is effectively contractible and also an incentive sufficient aggregate. Indeed, since $(k \times k)$-matrix $\Gamma\Gamma^*$ is invertible, the space of attainable effective incentive rates $\mathcal{V}_z^e$ is the full k-dimensional linear space $\mathbb{R}^k$, because for any $v_z^e \in \mathbb{R}^k$, we can find incentive rates on the second-period contractible signals y_2 in the initial contract v_{I2} such that the effective incentive rates on z is v_z^e, i.e., $v_{I2} = (\Gamma\Gamma^*)^{-1}\Gamma(v_z^e - B\psi) + v_{R2}$. In other words, the principal can obtain any desired effective incentive on z, v_z, by suitably choosing explicit incentives on the contractible second-period performances y_2 in the initial contract, i.e., v_{I2}, such that the effective incentive (which includes career concerns and explicit incentives), i.e., $v_z^e = B\psi + \Gamma^*(v_{I2} - v_{R2})$, is equal to v_z. If $n = k = 1$, then Γ is a scalar and (unless y_2 and z are uncorrelated) z is always effectively contractible.

[16]The forecasted second-period performance Γz is effectively contractible because the incentive on second-period performance v_{I2} in the initial contract sets a payment—based on expected future performance conditional on observing z at the intermediate date—as part of the final renegotiated contract. By having unrestricted control of v_{I2}, the principal has unrestricted control of implicit incentives on the forecast Γz, or equivalently of implicit incentives $v = v_{I2}\Gamma^q$ on Υz (because $v_{I2}\Gamma = v_{I2}\Gamma^q\Upsilon$).

Conversely, if z is effectively contractible, then the attainable incentives on z are the full space $\mathbb{R}^k$, $\mathcal{V}_z^e = \mathcal{V}_z = \mathbb{R}^k$, and that implies Γ has full rank k (see (11) and (12)). Consequently, z is effectively contractible, if, and only if Γ has full rank.

To illustrate the effective contractibility of z, assume in the example introduced earlier that $\gamma_{11} = \gamma_{22} = \gamma \neq 0$, while $\gamma_{12} = \gamma_{21} = 0$, such that the implicit aggregation is characterized by

$$\Gamma = \frac{\gamma}{\sigma_\theta^2\sigma_{\varsigma_1}^2 + \sigma_\theta^2\sigma_{\varsigma_2}^2 + \sigma_{\varsigma_1}^2\sigma_{\varsigma_2}^2} \begin{bmatrix} \sigma_\theta^2 + \sigma_{\varsigma_2}^2 & -\sigma_\theta^2 \\ -\sigma_\theta^2 & \sigma_\theta^2 + \sigma_{\varsigma_1}^2 \end{bmatrix}.$$

In this case the rows of Γ are linearly independent such that Γ has full rank equal to the number of non-contractible signals ($k = 2$) and z is effectively contractible with renegotiation of long-term contracts. That is, the space of attainable effective incentive rates on the k non-contractible signals z is $\mathcal{V}_z^e = \{v_z^e \in \mathbb{R}^2 | v_z^e = \Upsilon^* x, \forall x \in \mathbb{R}^2\} = \mathbb{R}^2$ (note that Υ is the identity matrix in this case).

On the other hand, if Γ has rank $q < k$, the space $\mathcal{V}_z^e$ has dimension q and, thus, places non-trivial restrictions on the attainable effective incentive rates on the non-contractible signals z. Specifically, the space of attainable effective incentive rates on z is the q-dimensional row space of Γ translated by the incentive externality vector $B\psi$, and is thus a q-dimensional affine subspace of $\mathbb{R}^k$ (and does not contain the null vector, as depicted in Figure 4 above).

Part B of Proposition 1 characterizes a necessary and sufficient condition for the effectively contractible implicit aggregate Υz to be informationally sufficient in the agency: that career concerns have no real effects. The intuition is that incentive externalities based on non-contractible information, such as career concerns, have real effects if, and only if, the incentive externalities cannot be spanned within the principal's effective scope of control over the implicit incentives based on the same non-contractible information. In other words, career concerns matter only when they arise from non-contractible information that cannot be brought under the principal's scope of control via implicit incentives.[17] Otherwise, career concerns do not matter, and the agency prob-

[17]From equation (9) it can be seen that the incentive externalities arising from career concerns $B\psi$ cannot be brought under the principal's control when there is no v_{I2} such that $B\psi = \Gamma^* v_{I2} = \Upsilon^*(\Gamma^q)^* v_{I2}$. That is, v_{I2} cannot be chosen such that $v_z^e \cdot z = [B\psi + \Gamma^*(v_{I2} - v_{R2})] \cdot z = \Gamma^* v \cdot z = v \cdot \Gamma z$, for an arbitrary v, and thus observing z

lem is equivalent to one where the non-contractible information is substituted by a contractible incentive sufficient aggregate—instead of z the contracting parties observe Υz.

If the incentive externality caused by career concerns $B\psi$ is in the row space of Γ, then the space of attainable incentive rates on z is the q-dimensional row space of Γ, which is a subspace of $\mathbb{R}^k$ (and contains the null vector, as depicted in Figure 3 above). The point is that when the latter condition is satisfied, then career concerns can be offset. That is, a v_{I2} can be chosen such that $B\psi \cdot z + v_{I2} \cdot \Gamma z = 0$. Thus, even though the non-contractible signals z are not effectively contractible with long-term contracts, it is possible to effectively contract on an aggregate of these signals with long-term contracts, and it may be possible that career concerns have no real effects. At the same time, $\psi \cdot z$ represents the portion of assessed managerial ability that is based on the non-contractible information z. When ψ is spanned by the rows of Υ (or equivalently Γ), the expected managerial ability can equally well be inferred from the aggregate Υz as from the non-contractible information z itself. Indeed, if $\psi = \Upsilon^* \pi$, we have that $\psi \cdot z = \pi \cdot \Upsilon z$ and $\mathrm{E}[\theta|\hat{a}_1, \hat{a}_2, y_1, z] = \mathrm{E}[\theta|\hat{a}_1, \hat{a}_2, y_1, \Upsilon z]$. In this case, Υz is simultaneously sufficient for forecasting second-period performance and for estimating managerial ability. Consequently, observing the aggregate Υz can be substituted for observing z without affecting the agency, that is Υz is an incentive sufficient aggregate. Moreover, as the forecast Υz is effectively contractible and sufficient for estimating managerial ability, it can also be used to undo any incentive externalities arising from career concerns.

In general, the implemented actions and the total surplus only depend on the effective incentive rates, v_z^e, on the non-contractible signal, z, the incentive rates on the first-period contractible signals, y_1, and on the incentive rates on the second-period contractible signals, y_2, in the renegotiated contract at $t = 1$, where the latter are independent of the incentive rates in the initial contract offered at $t = 0$. In other words, the principal's decision problem at $t = 0$ can be represented as choosing a long-term linear contract, $w = f + v_1 y_1 + v_z z + v_2 y_2$, subject to the constraints that v_2 must equal v_{R2}, and v_z must be in the space $\mathcal{V}_z^e$ (see (11)). The principal's choice of f and v_1 is otherwise only constrained by the participation and incentive compatibility constraints.

gives rise to externalities.

Equivalently, the effective contractibility of Υz means that the principal's decision problem at $t = 0$ can be represented as choosing a long-term linear contract, $w = f + B\psi z + v_1 y_1 + v\Upsilon z + v_2 y_2$, subject only to the constraint that v_2 must equal v_{R2}. The principal's choice of f, v_1, and v is otherwise only constrained by the participation and incentive compatibility constraints. When Υz is incentive sufficient, the principal's decision problem at $t = 0$ can be represented as choosing a long-term linear contract, $w = f + v_1 y_1 + v\Upsilon z + v_2 y_2$, subject subject to the same constraints—in this case there are no career concerns.

Long-term vs. short-term contracts

Not surprisingly, when all information at date $t = 1$ is contractible, the incentive externalities can all be undone and the short-term and long-term contract settings are equivalent.

Corollary 1 *Assume all available performance information consists of the contractible performance measures y_1 and y_2, and consider the two contract settings with either a sequence of short-term contracts subject to the participation constraint* (2) *or a long-term contract subject to the renegotiation constraint* (4)*. Then, any actions (a_1, a_2) that can be implemented in one of the two contract settings can be implemented in the other setting with the same effective incentive rates for the two performance measures y_1 and y_2, and at the same cost. The total surplus is the same in both contract settings, and it is independent of career concerns (B).*

Corollary 1 generalizes similar results in Gibbons and Murphy (1992) and Meyer and Vickers (1997), who only have contractible performance measures and focus on short-term contracts. It establishes that in both the short- and long-term contract settings, the incentive externalities arising from career concerns and surplus sharing do not play a substantive role when all information is contractible. The optimal effective incentives for given actions and, consequently, the total surplus, are independent of the parameter B. The main insight is that, whereas the short- and long-term contracts may have different explicit incentives, the effective incentives are the same, and only these determine the total surplus.

When the information available to the labor market is also directly contractible and, thus, can be included in incentive contracts, inefficient incentives arising from the agent's career concerns (such as myopia) can be fully mitigated by the principal via explicit incentives. Then, career concerns have no real effects—it is only a matter of how effective incentives are allocated between explicit and implicit incentives. On the other hand, the labor market may rely on performance-relevant information that is not directly contractible and, thus, cannot be directly included in incentive contracts. Non-contractible information opens the door for career concerns to have real effects, and renegotiation of long-term contracts facilitates implicit contracting on such information.

We now compare the short- and long-term contract settings when contracts cannot be written on z. When all the observable information is contractible, the same actions could be induced with the same effective incentives and risk premia by either short-term or long-term contracts. This does not hold when there are non-contractible signals. Both types of contracts yield implicit incentives on z, but the degree to which the principal can control these incentives differs.

The implicit incentive on z with short-term contracts, i.e., $v_z^e = B\psi - \Gamma^* v_2$, is a single vector in the space $\mathcal{V}_z^e$ (see (11)) of attainable incentives under long-term contracting and cannot be controlled by the principal. The second-period incentive v_2 is set after z is observed and, thus, is entirely determined by the posterior beliefs at $t = 1$. In other words, the principal cannot control either the relative weights in the aggregation of the signals z or the intensity applied to this aggregation. With short-term contracts, the non-contractible signals create an incentive externality which cannot be mitigated by the principal. This leads to the following result.

Corollary 2 *The non-contractible signals z are not effectively contractible with short-term contracts, the total surplus depends on career concerns, and the principal strictly prefers long-term contracts to short-term contracts (except in knife-edge cases).*

To illustrate, consider again the example above with $\gamma_{11} = \gamma_{22} = \gamma \neq 0$ and $\gamma_{12} = \gamma_{21} = 0$, such that Γ has full rank $k = 2$, and z is effectively contractible with long-term contracts. Under short-term contracting, the implicit incentive on z is $v_z^e = B\psi - \Gamma^* v_2$. This is an incentive externality that the principal cannot control. Thus, z is not effectively contractible with short-term

contracts. Consequently, long-term contracts clearly dominate short-term contracts. Effectively contracting on z enables the principal to: 1) undo any incentive externality on z arising from career concerns; 2) improve congruity as the principal can offer any relative incentive weights on the performance measures when the manager undertakes two tasks (as long as the vectors M_1 and M are not collinear); and 3) improve risk sharing.[18]

Conditional on z being observed, the possibility of writing long-term contracts is generally advantageous. This simply derives from the fact that writing long-term contracts allows for more possibilities and is independent of whether z is effectively contractible or not. In Figure 4, the vector $\bar{v}_z^e = B\psi - \bar{v}_z$ represents the fixed incentive on z that the principal cannot control with short-term contracts, whereas $\mathcal{V}_z^e$ is the space of attainable incentives with long-term contracts. Whether it is optimal to observe z in the first place is another matter as the implicit incentives on z can work to the advantage as well as to the disadvantage of the agency.

Observing z has a negative effect if contracts are short-term and $M = M_2 = 0$ (such that $v_{R2} = 0$). In this case, the updating of beliefs pertaining to θ introduces uninsurable career risk, $B\psi \cdot z$, without supplying incentives for first-period effort. Consequently, mandatory disclosure of z is detrimental to economic efficiency under these assumptions as in Autrey, Dikolli, and Newman (2007). However, if z is effectively contractible through long-term contracts (Γ or Υ is of full rank k), then this conclusion is reversed, and thus the ranking of information systems will depend on whether short- or long-term contracts are used.[19]

Regardless of whether career concerns have real effects, control over implicit incentives is valuable in providing insurance against the randomness in y_t (when $M_t \neq 0$). These incentives can be optimally controlled only with long-term contracts by effectively contracting on an aggregate of z and allowing some separate control over insurance and incentives. With short-term contracts, the

[18] If all signals, z and the performance measures y_t in each period, are single dimensional, then z is effectively contractible if $\text{cov}(y_2, z) \neq 0$ because the rank of Γ is one. Thus, when signals are single-dimensional, career concerns play a role only if $\text{cov}(y_2, z) = 0$ or if long-term contracting is disallowed.

[19] The value of information rationing under renegotiation as a partial remedy for lack of commitment has been extensively studied but is outside the scope of our study. See, for example, Demski (1998), Demski and Frimor (1999), Indjejikian and Nanda (1999), Christensen, Demski, and Frimor (2002), Christensen et al. (2003, 2005), Arya and Mittendorf (2011), and Christensen et al. (2013). A ranking of information systems under renegotiation of long-term contracts when all information is contractible is presented in Şabac (2015).

effective incentive on z is fixed as the single point (vector) $\bar{v}_z^e$ on the line $\mathcal{V}_z^e$, whereas with long-term contracts, the principal can choose from all points (vectors) along the line $\mathcal{V}_z^e$ (see Figure 4). This is similar to the problem of controlling two tasks with a single performance measure, where issues of congruity arise. Adding a second performance measure may remove those problems. The same happens here, but one task is motivating a_t, while the other task is adjusting the risk premium in response to renegotiation and too high an incentive in the second period.

We conclude that long-term contracts with career concerns at renegotiation time dominate short-term contracts either because they allow a larger space of implementable actions, or because the same actions can be implemented at a lower risk premium. In both cases the main factor is that long-term contracts allow a better control of the effective (implicit) incentives on the non-contractible signals z through renegotiation of the initial contract.

Taken together, Corollaries 1 and 2 imply that, for career concerns to have real effects with respect to the agent's actions and the principal's outcome, there has to be non-contractible information that is not effectively contractible, or there have to be exogenous constraints that require short-term contracts. Otherwise, career concerns only impact explicit incentives without having any real effects.

IV Conclusion

Managerial incentives are a combination of explicit incentives arising from performance-based bonus plans, stock and option awards, and implicit incentives arising from career concerns, promotions, or turnover. Performance-relevant information affects incentives in direct and indirect ways: verifiable and contractible performance measures form the basis of explicit incentive plans (contracts), whereas unverifiable and thus non-contractible information only provides implicit incentives. In this study, we focused on renegotiation of long-term contracts and career concerns as two channels through which non-contractible performance information can provide incentives. By controlling explicit incentive contracts, the principal partially controls implicit incentives based on

unverifiable information.

Two key ingredients make such control possible: long-term contracts and sufficiently many contractible performance measures that are correlated with the non-contractible information. The principal effectively controls incentives on an implicit aggregate of the non-contractible information, making that aggregate effectively contractible. In our study, the effectively contractible implicit aggregate is a forecast of future performance based on the non-contractible information. The verifiable performance measures play a confirmatory role with respect to these forecasts and facilitate implicit contracting on them. The verifiable performance measures and long-term contracts act as implicit information aggregators in a stewardship setting that is analogous to stock prices acting as implicit information aggregators in a valuation setting.

We characterized conditions such that externally determined (exogenous) implicit incentives can be offset by controlling incentives on the implicit aggregation. In such cases, career concerns do not play a substantive role, even though the non-contractible information is not effectively contractible.[20]Thus, for career concerns or similar incentive externalities to play a substantive role, contracts must be either short-term or, with long-term contracts, the external implicit incentives must fall outside the principal's scope of control of the non-contractible information, and this generally requires multiple unverifiable signals. Long-term contracts dominate short-term contracts and, hence, future analyses of career concerns should either allow for long term-contracts, or provide compelling reasons why long-term contracts are not feasible.

An important implication of our analysis is that the key pay/performance relationship is that between total incentive compensation and the effectively contractible performance information. Career concerns enter into incentives only as a fixed effect depending on the effectively non-contractible information available. Furthermore, career concerns may or may not have a real effect on how optimal incentives are structured, depending on whether their fixed effects are spanned by the effective incentives controlled by the principal.

[20]We have also established necessary and sufficient conditions for the unverifiable information to be effectively contractible through renegotiation. They resemble the sufficient conditions in Hermalin and Katz (1991), although the models are different. In these cases, career concerns play no substantive role.

Another implication is that there are two sides to performance measure controllability. One side, the basis of the controllability principle (Antle and Demski 1988) refers to whether the manager's actions have any impact on the performance measures, so that the manager 'controls' those performance measures. The other side is whether the principal has control over the incentives that a performance measure provides. This is particularly important with non-contractible information, where the possibilities range from no control to effective contractibility.

A Appendix: Proofs

Proof of Lemma 1. Because the interim participation constraint (2) is binding, the second-period fixed wage is

$$f_2 = -v_2 \cdot \mathrm{E}_1[y_2|\hat{a}_1, \hat{a}_2] + B\mathrm{E}_1[\theta|a_1] + \kappa(\hat{a}_2) + \tfrac{1}{2}r \ \mathrm{var}_1(w_2). \tag{13}$$

Substituting in the conditional expectations (6) and gathering the terms that depend on y_1 we have

$$f_2 = -v_2 \cdot \alpha_1 + B\alpha_2 + \kappa(\hat{a}_2) + \tfrac{1}{2}r \ \mathrm{var}_1(w_2) + (B\phi - \Lambda^* v_2) \cdot y_1 + (B\psi - \Gamma^* v_2) \cdot z \ .$$

Substituting in the agent's total compensation for the two periods yields (7).

We next determine the agent's optimal choice of effort $a_t = (a_{t1}, a_{t2}, \ldots, a_{tm})$ in response to the effective incentives determined above. Since productive effort is not observable and not directly contractible, the agent's action choice maximizes his certainty equivalent of compensation, which in this case is equivalent to a_t satisfying the first-order condition

$$\nabla_{a_t} \mathrm{CE}_t(w_1 + w_2|a) = \nabla_{a_t} \mathrm{E}_t[w_1 + w_2|a] - \nabla_{a_t} \kappa(a) = 0 \ , \tag{14}$$

where ∇_{a_t} denotes the gradient vector of partial derivatives $(\partial/\partial a_{t1}, \partial/\partial a_{t2}, \ldots, \partial/\partial a_{tm})$. In writing the agent's incentive compatibility constraint, we have used the fact that the action does

not affect the variance of compensation. From (14) it follows that the actions induced by the sequence of short-term contracts that satisfy the interim participation constraint (2) and have given explicit incentive rates v_1, v_2 are those in (8). □

Proof of Lemma 2. For given incentive rates v_{I1}, v_{I2} and v_{R2}, the renegotiation constraint (4) is binding and determines the fixed wage in the renegotiation offer $w^R = f_R + v_{R2} \cdot y_2$:

$$f_R = f_I - \kappa(\hat{a}_{I2}) - \tfrac{1}{2} r \operatorname{var}_1(w^I) + \kappa(\hat{a}_{R2}) + \tfrac{1}{2} r \operatorname{var}_1(w^R) + v_{I1} \cdot y_1 \\ + v_{I2} \cdot \mathrm{E}_1[y_2|\hat{a}_1, \hat{a}_{I2}] - v_{R2} \cdot \mathrm{E}_1[y_2|\hat{a}_1, \hat{a}_{R2}] + B\mathrm{E}_1[\theta|\hat{a}_1] \,. \quad (15)$$

Substituting in the conditional expectations (6) and gathering the terms that depend on y_1 we have

$$\begin{aligned} f_R = f_I &+ v_{I2} \cdot \alpha_1(\hat{a}_1, \hat{a}_{I2}) - v_{R2} \cdot \alpha_1(\hat{a}_1, \hat{a}_{R2}) + B\alpha_2 \\ &- \kappa(\hat{a}_{I2}) - \tfrac{1}{2} r \operatorname{var}_1(w^I) + \kappa(\hat{a}_{R2}) + \tfrac{1}{2} r \operatorname{var}_1(w^R) \\ &+ [v_{I1} + B\phi + \Lambda^*(v_{I2} - v_{R2})] \cdot y_1 + [B\psi + \Gamma^*(v_{I2} - v_{R2})] \cdot z \,. \end{aligned} \quad (16)$$

The renegotiation offer is then $w^R = f_R + v_{R2} \cdot y_2$ and is given by(9). It follows that the actions induced by a long-term contract subject to the renegotiation constraint (4) are given by (10). □

Proof of Proposition 1. We first show that Υz is effectively contractible. Since $\Gamma = \Gamma^q \Upsilon$, the row space of Γ is the same as the q-dimensional row space of $\Upsilon = [I^q, \Omega]$ because

$$\begin{aligned} \mathcal{V}_z &= \left\{ v \in \mathbb{R}^k | v = (\Gamma^q \Upsilon)^* \, x, \forall x \in \mathbb{R}^n \right\} = \left\{ v \in \mathbb{R}^k | v = \Upsilon^* \left(\Gamma^q\right)^* x, \forall x \in \mathbb{R}^n \right\} \\ &= \left\{ v \in \mathbb{R}^k | v = \Upsilon^* x, \forall x \in \mathbb{R}^q \right\}, \end{aligned}$$

where the last equality follows from the fact that Γ^q has full column rank q. It follows that, for any incentive weight on z that the principal can fully control, $v = \Gamma^* v_{I2}$, there exists $x \in \mathbb{R}^q$ such that $v = \Upsilon^* x$. In other words, $v \cdot z = \Gamma^* v_{I2} \cdot z = \Upsilon^* x \cdot z$, or equivalently $v \cdot z = v_{I2} \cdot \Gamma z = x \cdot \Upsilon z$. Thus, Υz is effectively contractible.

To determine whether Υz is a sufficient incentive aggregate, we need to determine whether the

effective incentives attainable on z are affected by replacing z with Υz. First, we note that the posterior beliefs about y_2 at renegotiation time do not change if we replace z by Υz. To see this, it suffices to prove that $\mathrm{E}[y_2|z, y_1] = \mathrm{E}[y_2|\Upsilon z, y_1]$ because then the conditional variances are also the same, $\mathrm{var}(y_2|z, y_1) = \mathrm{var}(y_2 - \mathrm{E}[y_2|z, y_1]) = \mathrm{var}(y_2 - \mathrm{E}[y_2|\Upsilon z, y_1]) = \mathrm{var}(y_2|\Upsilon z, y_1)$.

Since $\mathrm{E}[y_2|z, y_1] = \alpha_1 + \Lambda y_1 + \Gamma z$, we can write $y_2 = \alpha_1 + \Lambda y_1 + \Gamma z + (y_2 - \mathrm{E}[y_2|z, y_1])$. Taking conditional expectations with respect to $(\Upsilon z, y_1)$ yields

$$\mathrm{E}[y_2|\Upsilon z, y_1] = \mathrm{E}[\alpha_1 + \Lambda y_1 + \Gamma z|\Upsilon z, y_1] + \mathrm{E}[(y_2 - \mathrm{E}[y_2|z, y_1])\,|\Upsilon z, y_1]\,.$$

Because $\Gamma = \Gamma^q \Upsilon$, it follows that $\mathrm{E}[\alpha_1 + \Lambda y_1 + \Gamma z|\Upsilon z, y_1] = \alpha_1 + \Lambda y_1 + \Gamma^q \Upsilon z = \mathrm{E}[y_2|z, y_1]$. To complete the proof that $\mathrm{E}[y_2|z, y_1] = \mathrm{E}[y_2|\Upsilon z, y_1]$, note that $\mathrm{E}[(y_2 - \mathrm{E}[y_2|z, y_1])\,|\Upsilon z, y_1] = 0$ by the law of iterated expectations. Indeed, writing the conditional expectations with respect to the σ-algebras generated by the available information, we note that $\sigma(\Upsilon z, y_1) \subseteq \sigma(z, y_1)$ holds generally for any matrix Υ and implies that $\mathrm{E}[\mathrm{E}[y_2|\sigma(z, y_1)]|\sigma(\Upsilon z, y_1)] = \mathrm{E}[y_2|\sigma(\Upsilon z, y_1)]$.

It follows that, absent incentive externalities, $B = 0$, the effective incentives attainable on z are the same with observing Υz as with observing z. Indeed, $\mathcal{V}_z^e$ is the row space of Γ, and that is the same as the q-dimensional row space of the aggregation matrix $\Upsilon = [I^q, \Omega]$. It follows that $\mathcal{V}_z^e$ is the same with the non-contractible information z as it is with a contractible aggregate Υz. Consequently, Υz is an incentive sufficient aggregate.

Consider next the setting in Proposition 1 when the incentive externality from career concerns $B\psi$ is non-zero, but is spanned by the rows in Υ, i.e., there exists a vector $\pi \in \mathbb{R}^q$ such that $B\psi = \Upsilon^* \pi$. Then, $\mathcal{V}_z^e$ is the q-dimensional row space of the aggregation matrix $\Upsilon = [I^q, \Omega]$, since

$$\begin{aligned}\mathcal{V}_z^e &= \left\{v_z^e \in \mathbb{R}^k | v_z^e = \Upsilon^* \pi + (\Gamma^q \Upsilon)^* x, \forall x \in \mathbb{R}^n\right\} = \left\{v_z^e \in \mathbb{R}^k | v_z^e = \Upsilon^* (x + \pi), \forall x \in \mathbb{R}^q\right\} \\ &= \left\{v_z^e \in \mathbb{R}^k | v_z^e = \Upsilon^* x, \forall x \in \mathbb{R}^q\right\},\end{aligned}$$

where the second equality follows from the fact that Γ^q has full column rank q, and the last equality follows from the fact that the sum of any two vectors in $\mathbb{R}^q$ also is in $\mathbb{R}^q$. In other words, any impact

of incentive externalities due to career concerns can be neutralized if these are linearly dependent on the aggregate Υz of the non-contractible signals. Thus, the principal's decision problem at $t = 0$ can be represented as choosing a long-term linear contract, $w = f + v_1 y_1 + v\Upsilon z + v_2 y_2$, subject only to the constraint that v_2 must equal v_{R2}. Everything is as if there are no incentive externalities, and Υz is an incentive sufficient aggregate.[21]

Now turn to the setting in Proposition 1 when the incentive externality caused by career concerns $B\psi$ is not spanned by the rows of Υ. Then, $\mathcal{V}_z^e$ is the q-dimensional row space of the aggregation matrix $\Upsilon = [I^q, \Omega]$ translated by the incentive externality because

$$\begin{aligned}\mathcal{V}_z^e &\equiv \left\{v_z^e \in \mathbb{R}^k | v_z^e = B\psi + \Gamma^* x, \forall x \in \mathbb{R}^n\right\} = \left\{v_z^e \in \mathbb{R}^k | v_z^e = B\psi + \left(\Gamma^q \Upsilon\right)^* x, \forall x \in \mathbb{R}^n\right\} \\ &= \left\{v_z^e \in \mathbb{R}^k | v_z^e = B\psi + \Upsilon^* \left(\Gamma^q\right)^* x, \forall x \in \mathbb{R}^n\right\} = \left\{v_z^e \in \mathbb{R}^k | v_z^e = B\psi + \Upsilon^* x, \forall x \in \mathbb{R}^q\right\}.\end{aligned}$$

In this case, Υz is not an incentive sufficient aggregate. Indeed, when directly contracting only on Υz, the space of attainable effective incentive rates is the q-dimensional linear subspace spanned by the rows of $\Upsilon = [I^q, \Omega]$ and differs from the affine space described above.

Replacing z with any other contractible aggregate $\Upsilon' z$ is equivalent to having effective incentives on z in the linear subspace spanned by the rows of Υ', which can never equal the affine space $\mathcal{V}_z^e$ characterized above. Thus, there can be no incentive sufficient aggregate of z. In other words, career concerns and surplus sharing have real effects in this setting because the principal cannot neutralize the impact of the incentive externalities. □

Proof of Corollary 1. Let a_1, a_2 be two implementable actions under short-term contracts subject to the interim participation constraint (2). Let v_1, v_2 denote the associated explicit incentives in the two short-term contracts. The effective incentive on y_2 is then also v_2. It follows that a long-term contract subject to renegotiation will induce the same action a_2 with the same effective incentive v_2

[21] Saying that the aggregate Υz of the non-controllable signals z is effectively contractible is not the same as saying that the first q non-contractible signals, corresponding to the first linearly independent columns of Γ, are effectively contractible. Instead, the q-dimensional linear subspace $\mathcal{V}_z^e$ of $\mathbb{R}^k$ may be "tilted" relative to $\mathbb{R}^q$, because the aggregation matrix $\Upsilon = [I^q, \Omega]$ aggregates all the $k > q$ non-contractible signals using both the identity matrix, I^q, for the first q non-contractible signals, z^q, and redundancy matrix, Ω, for the last $k - q$ non-contractible signals, z^{k-q}, i.e., $\Upsilon z = I^q z^q + \Omega z^{k-q}$.

as long as $v_2 = v_{R2}$. Obviously, the converse is true because $v_2 = v_{R2}$ will induce the same a_2 with short-term contracts as with a long-term contract subject to renegotiation where the second-period incentive rate is v_{R2}.

Consider now the effective incentive for the first-period action under short-term contracts, $v_1 + B\phi - \Lambda^* v_2$, and assume that the second-period incentive in the renegotiation offer under long-term contracting is as determined before, $v_2 = v_{R2}$. Substituting $v_2 = v_{R2}$ into the effective incentive for the first-period action under long-term contracts gives $v_{I1} + B\phi + \Lambda^*(v_{I2} - v_{R2})$. Setting $v_{I2} = 0$ and $v_{I1} = v_1$ will give the same effective incentive on y_1 and the same induced action a_1 with a long-term contract subject to the renegotiation constraint (4). To prove the converse, one simply works through the effective incentives in reverse order—we leave the details to the reader.

To show that the principal and the agent each get the same payoff under both contract regimes, we note that (a_1, a_2) are induced with the same effective incentive rates. Obviously, the agent's effort costs are the same, and the risk premia are also the same since the agent's initial participation constraints are the same in both settings. □

References

Antle, R. and J. S. Demski (1988, October). The controllability principle in responsibility accounting. *The Accounting Review 63*(4), 700–718.

Arya, A. and B. Mittendorf (2005). Offering stock options to gauge managerial talent. *Journal of Accounting and Economics 40*, 189–210.

Arya, A. and B. Mittendorf (2011). The benefits of aggregate performance metrics in the presence of career concerns. *Management Science 57*, 1424–1437.

Autrey, R. L., S. S. Dikolli, and D. P. Newman (2007). Career concerns and mandated disclosure. *Journal of Accounting and Public Policy 26*, 527–554.

Autrey, R. L., S. S. Dikolli, and D. P. Newman (2010). Performance measure aggregation, career incentives, and explicit incentives. *Journal of Management Accounting Research 22*, 115–

131.

Christensen, P. O., J. S. Demski, and H. Frimor (2002). Accounting policies in agencies with moral hazard and renegotiation. *Journal of Accounting Research 40*, 1071–1090.

Christensen, P. O., G. A. Feltham, C. Hofmann, and F. Şabac (2003). Timeliness, accuracy, and relevance in dynamic incentive contracts. Working paper, University of British Columbia.

Christensen, P. O., G. A. Feltham, and F. Şabac (2003). Dynamic incentives and responsibility accounting: A comment. *Journal of Accounting and Economics 35*, 423–436.

Christensen, P. O., G. A. Feltham, and F. Şabac (2005). A contracting perspective on earnings quality. *Journal of Accounting and Economics 39*, 265–294.

Christensen, P. O., H. Frimor, and F. Şabac (2013). The stewardship role of analyst forecasts, and discretionary versus non-discretionary accruals. *European Accounting Review 22*(2), 257–296.

Christensen, P. O., F. Şabac, and J. Tian (2010). Ranking performance measures in multi-task agencies. *The Accounting Review 85*(5), 1545–1575.

Demski, J. S. (1998, Fall). Performance measure manipulation. *Contemporary Accounting Research 15*(3), 261–285.

Demski, J. S. and H. Frimor (1999). Performance measure garbling under renegotiation in multiperiod agencies. *Journal of Accounting Research 37*, 187–214.

Dewatripont, M., I. Jewitt, and J. Tirole (1999). The economics of career concerns, part I: Comparing Information Structures. *Review of Economic Studies 66*, 183–198.

Dutta, S. (2008, March). Managerial expertise, private information, and pay-performance sensitivity. *Management Science 54*(3), 429–442.

Dutta, S. and S. Reichelstein (1999). Asset valuation and performance measurement in a dynamic agency setting. *Review of Accounting Studies 4*, 235–258.

Dye, R. A. (1983, Autumn). Communication and post-decision information. *Journal of Accounting Research 21*(2), 514–533.

Fama, E. F. (1980, April). Agency problems and the theory of the firm. *Journal of Political Economy 88*(2), 288–307.

Feltham, G. A. and J. Xie (1994). Performance measure congruity and diversity in multi-task principal/agent relations. *The Accounting Review 69*, 429–453.

Gibbons, R. and K. J. Murphy (1992). Optimal incentive contracts in the presence of career concerns: Theory and evidence. *Journal of Political Economy 100*, 468–504.

Gigler, F. B. and T. Hemmer (2001, October). Conservatism, optimal disclosure policy, and the timeliness of financial reports. *The Accounting Review 76*(4), 471–493.

Hayes, R. M. and S. Schaefer (2000, Summer). Implicit contracts and the explanatory power of top executive compensation for future performance. *The RAND Journal of Economics 31*(2), 273–293.

Hermalin, B. E. and M. L. Katz (1991, November). Moral hazard and verifiability: The effects of renegotiation in agency. *Econometrica 59*(6), 1735–1753.

Holmström, B. (1999). Managerial incentive problems: A dynamic perspective. *Review of Economic Studies 66*, 169–182.

Holmström, B. and P. Milgrom (1991). Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law Economics and Organization 7*, 24–52.

Indjejikian, R. and D. Nanda (1999). Dynamic incentives and responsibility accounting. *Journal of Accounting and Economics 27*, 177–201.

Kaarbøe, O. M. and T. E. Olsen (2006). Career concerns, monetary incentives, and job design. *Scandinavian Journal of Economics 108*(2), 299–316.

Kaarbøe, O. M. and T. E. Olsen (2008, Spring). Distorted performance measures and dynamic incentives. *Journal of Economics & Management Strategy 17*(1), 143–183.

MacLeod, W. B. (2003, March). Optimal contracting with subjective evaluation. *The American Economic Review 93*(1), 216–240.

Meyer, M. A. and J. Vickers (1997). Performance comparisons and dynamic incentives. *Journal of Political Economy 105*(3), 547–581.

Şabac, F. (2007, May). Dynamic agency with renegotiation and managerial tenure. *Management Science 53*(5), 849–864.

Şabac, F. (2008). Dynamic incentives and retirement. *Journal of Accounting and Economics 46*(1), 172–200.

Şabac, F. (2015). Ranking performance measures when contracts are renegotiated. Working paper, University of Alberta.

Şabac, F. and J. J. Tian (2015, July). On the stewardship value of soft managerial reports. *The Accounting Review 90*(4), 1683–1706.

Şabac, F. and J. Yoo (2016). Performance measure aggregation in multi-task agencies. Working paper.

Stocken, P. C. (2000, Summer). Credibility of voluntary disclosure. *The RAND Journal of Economics 31*(2), 359–374.