Biased Boards

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Abstract

We study a corporate board tasked with advising and monitoring a firm's CEO in an investment setting. The board has both compensation and non-pecuniary incentives—we label the latter *board bias*. The optimal board bias is jointly determined by the CEO's initial information advantage and by whether the board has commitment power when dealing with the CEO. We show that the optimal board bias is weakly "friendly" (partially aligned with the CEO) if the board lacks commitment power and the CEO's information advantage is high. In contrast, it is weakly "antagonistic" (counter to the CEO's bias) if the board has commitment power, or if it lacks such power and the CEO's information advantage is small. For given board bias, commitment power improves communication, thereby reducing the need for the board to exert costly (advising and monitoring) effort. Endogenizing the board bias shows that commitment power may be associated with greater board effort. We also show that the shareholders may be better off with a board that lacks commitment power.

1 Introduction

The dual role of the corporate board as advisor and monitor of management has come under scrutiny in the wake of the recent corporate scandals, and the balance appears to be tilting towards the monitoring role (Faleye et al., 2011; Adams et al., 2015). At the same time, the incentive side remains less well-understood: what are the key drivers of directors' behavior? While board compensation has become more significant, non-pecuniary concerns appear to remain important drivers of directors' decision-making. The typical concern is that of board capture, i.e., the board siding with management, although there also is anecdotal evidence that some CEOs view their boards as overly conservative and mainly interested in protecting their reputational capital.[1]

We develop a model to ask how shareholders should assemble and incentivize the board in a setting in which: (a) the firm faces a strategy choice modeled as an investment (or M&A) decision that should be tailored to the realization of some state variable; (b) the CEO is endowed with private, imperfect, information about the state but at the margin prefers a larger project scale; (c) the board can learn about the state through costly information gathering or through communicating with the CEO; and (d) the directors' preferences over the scale of the investment also may deviate from those of the shareholders. Information gathering encapsulates the dual role of boards in that the information gathered would uncover (monitoring) and improve upon (advising) the CEO's information: The more precise the CEO's private signal, the less important is board advising—but the monitoring aspect of information gathering remains.

[1]Yermack (2004), Linck et al. (2009), Cook & Co (2014) study directors' pay. Linck et al. (2009) also document increased liability risks post Sarbanes-Oxley and a rebalancing in board composition in favor of lawyers and financial experts replacing executives from other firms. On board capture, see Macey (2008, Ch.5), Nguyen (2016); on director conservatism, Deloitte (2015, p.13): "The increased scrutiny has reduced the risk appetites of many companies. 'There is an element of overgovernance,' one CEO said. 'The board has taken a risk-averse view and management are reporting to it.' ... One CEO commented that a very good reason for boards to focus on risk was to avoid the stigma of becoming high-profile failures."

The shareholders in our model assemble and contract with the board but do not interfere in the actual decision-making. We take the directors' preferences to be (at least partly) under the control of the shareholders at the board composition stage. For instance, directors who are personally linked to the CEO or derive career-concerns benefits from larger projects may have a similar bias regarding the investment scale as the CEO. In contrast, directors who represent debtholders (e.g., Kroszner and Strahan 2001), or are accountants or academics concerned about the downside risk of investments for reputational reasons, may prefer a smaller-than-NPV-maximizing scale of investment. We refer to a board that is perfectly (imperfectly) aligned with the shareholders as "unbiased" ("biased"), and to a biased board as "antagonistic" (to the CEO) if it prefers a smaller-scale investment than the shareholders, and as "friendly" if it is at least partially aligned with the CEO (i.e., also engages in empire building).

Why would shareholders ever appoint a biased board—and if so, should the bias be antagonistic or friendly? Conditional on successful information gathering by the board, the shareholders would best be served by an *unbiased* board. If the board's information gathering effort remains unsuccessful, a *friendly* board would foster communication between the CEO and the board. The probability of an information gathering success, however, hinges on the board's effort, which is maximized by an *antagonistic* board. In a simple binary-state model, we find a key factor moderating this tradeoff is whether the board has commitment power when entering into communication with the CEO.

With *commitment*, before soliciting a report, the board devises an incentive-compatible "menu" of investment levels that ensures the CEO picks the investment that corresponds to his signal (Holmstrom 1977).[2] Under *noncommitment* communication takes the form of cheap talk: the board reacts to the CEO's report in a sequentially rational manner (Crawford and Sobel 1982).[3] All else

[2]See also Melumad and Shibano (1991), Alonso and Matoushek (2008).

[3]See also Dessein (2002), Harris and Raviv (2005, 2008, 2010), Adams and Ferreira (2007), Baldenius et al. (2014), Chakraborty and Yilmaz (2015).

equal, commitment power on the part of the board improves the information flow. Cheap-talk communication in our setting is "bang-bang:" frictionless up to a certain level of CEO/board misalignment, uninformative babbling beyond that level. Board commitment allows for a form of *constrained communication* even in cases where cheap talk would falter: specifically, the menu of permissible investment levels leaves the CEO indifferent upon observing a low state. With communication and information gathering effort as alternative channels for the board to acquire information, one would expect commitment power to be associated with reduced board effort. A key insight of the paper is that this intuition overlooks the endogenous nature of board bias.

Our predictions for the optimal board bias are jointly determined by the board's ability to commit and the CEO's initial information advantage. The shareholders prefer a weakly friendly board, emphasizing communication, if the board lacks commitment power and the CEO's signal is sufficiently precise. In contrast, the shareholders prefer a weakly antagonistic board, emphasizing information gathering, if: (a) the board lacks commitment power and the CEO's prior signal is rather noisy, or (b) the board has commitment power (in which case the CEO's signal precision matters only quantitatively).

For extreme (low or high) CEO biases, the board's ability to commit is immaterial, and the board should be unbiased. For mildly biased CEOs, the board can elicit a truthful report even with cheap talk. For highly biased CEOs, the board would find it too costly to elicit a nontrivial report from the CEO even with commitment (and, a fortiori, without); instead it invests based on its prior about the state.[4] In either case, both the information gathering and the communication channel are unaffected by changes in the board bias, at the margin. Nominating a biased board would distort decisions without any countervailing informational benefits.[5]

[4]With cheap talk nontrivial information transmission would simply be infeasible then.

[5]To be precise, the argument in the main text only rules out "small" board bias levels, close to zero. In the model section we show that also discrete jumps in board bias away from

Only for intermediate CEO bias levels does commitment matter: If the board were unbiased, commitment would facilitate constrained communication in settings where cheap talk would collapse. Consider a special case in which information gathering is prohibitively costly, reducing the board's role to eliciting a report from the CEO and choosing the investment. The commitment setting then calls for an unbiased board as there is no agency problem left at the board level. Under noncommitment, adapting arguments in Dessein (2002), the optimal board bias is either zero or just friendly enough to elicit a truthful report from the CEO—the information gain outweighs the investment distortion for small board bias.[6] Moreover, a replication result obtains for this special case: a board given the optimal friendly bias to compensate for its lack of commitment generates the same expected shareholder value as a board with commitment power.

Allowing for the board to exert information gathering effort pushes the optimal board bias farther away from that of the CEO. Creating discord between the CEO and the board stimulates board effort. If the board can commit, the shareholders assemble it (weakly) antagonistically. In contrast, with noncommitment the predictions are parameter-specific. As the only direct consequence of noncommitment is a drop in the communication efficiency for given board bias, this information loss can be mitigated through either information acquisition channel: a friendly board would improve communication whereas an antagonistic board would exert more information gathering effort. We show that if the CEO's signal is noisy, so that relatively little can learned from the CEO, the shareholders favor an antagonistic board emphasizing information gathering. If the CEO's information is very precise, a friendly board emphasizing communication is optimal. Such a friendly board (lacking commitment) exerts less information gathering effort than an antagonistic board that has commitment power.

zero—say, to induce nontrivial communication from the CEO to the board, or to foster board information gathering—also leave the shareholders worse off because of the attendant bias cost.

[6]In the absence of information gathering, our noncommitment setting becomes a variant of Dessein's (2002) model. It is easy to show that in our binary state model it is always optimal for the board for retain the decision rights rather than delegating them to the CEO.

We also show that the replication result, which suggested zero value (to the shareholders) of having a board endowed with commitment power, breaks down if the board can gather information on its own. For given board bias, commitment comes at an opportunity cost of reduced board effort as communication and information gathering are substitutes. The magnitude of this opportunity cost depends on the precision of the CEO's private signal: high signal precision diminishes the advisory aspect of information gathering. The shareholders then benefit on balance from the board's commitment power. The reverse may hold for CEO signals noisy enough so as to call for an antagonistic board even under noncommitment. A board that strongly disagrees with the CEO *and* anticipates handicapped communication for lack of commitment has strong information gathering incentives. At the same time, a noisy CEO signal increases the value of information gathering by the board, which now serves both monitoring and advising roles. These arguments combined can leave the shareholders strictly better off with a board that lacks commitment power.

Methodologically our paper builds on Holmstrom (1977) for communication with commitment, and on Crawford and Sobel (1982) for cheap talk. Much of the subsequent literature has studied the optimal assignment of decision rights.[7] By considering both types of communication, together with an alternative information acquisition channel, we derive novel insights into the direction of the optimal board bias and the value of commitment in communication.

Our notion of friendly boards builds on the insight in Dessein (2002) that intermediaries whose preferences are partially aligned with the CEO foster communication. In contrast, friendly boards in Adams and Ferreira (2007) have higher costs of wresting control from the CEO. Board monitoring in our model aims at reducing pre-decision information asymmetry, as in Kumar and Sivaramakrishnan (2008), Baldenius et al. (2014). Elsewhere, monitoring is taken to

[7]See Melumad and Shibano (1991), Alonso and Matoushek (2008) for the commitment case, and Dessein (2002), Harris and Raviv (2005, 2008, 2010), Adams and Ferreira (2007), Baldenius et al. (2014), Chakraborty and Yilmaz (2015) for the cheap-talk case.

uncover manipulation (Laux and Laux, 2009) or to reduce the noise in performance metrics (Drymiotes, 2007; Drymiotes and Sivaramakrishnan, 2012).[8]

Other recent papers have dealt with board bias in rather different settings. Most closely related is Chakraborty and Yilmaz (2015) who study the interaction of board bias and the allocation of decision rights ("advisory" versus "supervisory" boards). While their model also builds on Dessein's (2002) results, the other aspect of board bias in our model—as a source of monitoring incentives—is absent from their model. Levit (2012) considers a Dye (1985)-type model in which the CEO can gather private information at some cost, which it can choose to disclose. Levit shows that installing an antagonistic board can mitigate the ensuing problem of underprovision of CEO effort.[9] Lastly, our predictions that higher-skilled CEOs—in our setting, CEOs endowed with more precise information—face friendlier boards is similar to Hermalin and Weisbach's (1998) conclusion, albeit due to a very different mechanism.

The remainder of the paper is organized as follows. Section 2 lays out the model. Section 3 considers a special case in which information gathering is infeasible. Section 4 addresses the full-fledged model. Section 5 summarizes the main findings and discusses the value of board commitment to the shareholders.

2 Model

A firm is to make a strategy choice, modeled as an investment decision. The model entails three players: shareholders, the CEO, and the board of directors. The CEO is endowed with information regarding the optimal scale of the investment. The shareholders are passive; their role is confined to assembling the board and setting its compensation contract. The board is tasked with making the investment decision—for that it seeks ways to learn about the environment.

[8] Also related is Friedman (2013, 2014) on the CEO/CFO interplay in contracting settings.

[9] This is related to Li (2001) who has demonstrated a role for an antagonistic committee to overcome underprovision of effort problems arising from free-riding in teams.

Technology. For given economic state, ω, and scale of the investment undertaken, y, the realized firm value is simply the net present value of the investment,

$$\pi = \omega y - \frac{y^2}{2}.$$

For any state, ω, the NPV-maximizing investment level thus equals $y^*(\omega) = \omega$. For simplicity, we assume a symmetric, binary distribution, $\omega \in \{0, 1\}$, $Pr(\omega = 1) = \frac{1}{2}$, while allowing for continuous investments, $y \in \mathbb{R}$. Denote by

$$\Lambda_\emptyset = Var(\omega) = \frac{1}{4}$$

the unconditional variance, or *prior information loss*.

Information and decision rights. At the outset, the CEO privately learns a binary signal s about the state ω. Without loss of generality, $s \in \{0, 1\}$. The probability the signal is correct is $Pr(s = \omega \mid \omega) = q \in \left(\frac{1}{2}, 1\right]$, where q captures the CEO's precision. Denote by

$$\Lambda_s = \mathbb{E}_s[Var(\omega \mid s)] = q(1 - q)$$

the expected posterior variance conditional on the s being available, or *expected posterior information loss*. As shorthand, let $q(1 - q) \equiv Q$. Also, denote by

$$\Delta \equiv \mathbb{E}[\omega \mid s = 1] - \mathbb{E}[\omega \mid s = 0] = 2q - 1$$

the value of the CEO's signal in terms of its updating impact about the state.

The shareholders and the board only know the prior of ω. By engaging in *information gathering effort*, however, the board may learn (probabilistically) the state realization. For simplicity, we assume that if the board's effort is successful, it uncovers the state ω perfectly. The information gathering effort, $e \in [0, 1]$, is normalized to equal the probability that the board learns ω. Therefore, information gathering by the board in our model has features of both monitoring (uncovering the CEO's private information) *and* advising (removing the residual uncertainty in the CEO's information).

Regardless of whether information gathering has uncovered ω—which we label an (information gathering) *success*—the decision rights over the investment level y rest with the board. If the board has learned ω, it will choose its preferred level of investment. If information gathering was unsuccessful, the board may elicit a report r about s from the CEO.[10] We normalize the message space to coincide with the signal space, i.e., $r \in \{0, 1\}$. For the communication subgame in case of unsuccessful information gathering, we consider two scenarios regarding the board's commitment power (see below for more detail).

Payoffs. Because the focus of the paper is on optimal incentive provision for the board, we will sidestep any issues of explicit agency problems and compensation at the CEO level. Instead, we assume in reduced form that the CEO prefers a larger scale investment than the shareholders ("empire building"). Specifically, the CEO aims to choose y so as to maximize

$$\pi + by = \frac{1}{2}(\omega + b)^2 - \frac{1}{2}(y - \omega - b)^2, \tag{1}$$

if stated in terms of a quadratic loss function. We refer to $b \geq 0$ as *CEO bias*.[11]

The board's information gathering effort choice is a function of its pecuniary and non-pecuniary incentives. The shareholders compensate the board with a fixed payment F and an equity stake α.[12] The shareholders' residual claim is:

$$\begin{aligned} V &= (1 - \alpha)\pi - F \\ &= (1 - \alpha)\left[\frac{1}{2}\omega^2 - \frac{1}{2}(y - \omega)^2\right] - F. \end{aligned} \tag{2}$$

In line with board compensation practice we assume throughout that $\alpha \in [0, 1]$ and $F \geq 0$.[13]

[10] Unlike in continuous state models (e.g., Dessein 2002), it is easy to show in binary state settings such as ours that the board, once granted decision making authority by the shareholders, never benefits from delegating this authority to the CEO.

[11] It is without loss of generality to assume positive b. All results would go through qualitatively, with suitably flipped interpretations, if $b \leq 0$.

[12] Compensation packages for directors of large U.S. corporations comprise on average 55% cash and 45% equity (Larcker and Tayan 2011, 108).

[13] A non-negative salary F would also emerge endogenously, if (i) the board were protected

As for non-pecuniary incentives, we allow not just the CEO's but also the board's preferences over the scale of the investment to deviate from those of the shareholders. Specifically, let $\bar{\beta} \in \mathbb{R}$ denote the board's bias over the desired investment level and $C(e) = \frac{ce^2}{2}$ the board's effort cost. The board's payoff reads:

$$
\begin{aligned}
U &= \alpha\pi + \bar{\beta}y + F - C(e) \\
&\equiv \alpha(\pi + \beta y) + F - \frac{ce^2}{2}, \quad \text{for} \quad \beta \equiv \frac{\bar{\beta}}{\alpha} \\
&= \alpha\left[\frac{1}{2}(\omega + \beta)^2 - \frac{1}{2}(y - \omega - \beta)^2\right] + F - \frac{ce^2}{2}, \qquad (3)
\end{aligned}
$$

also stated in terms of quadratic loss functions. For convenience, we will mostly work with the *scaled* bias variable $\beta \equiv \bar{\beta}/\alpha$. Throughout the paper, we impose the individual rationality constraint that U exceeds the board's reservation utility $\bar{U}$, which is normalized to zero.[14]

A key assumption is that the *board bias,* β, is a choice variable for the shareholders that may be positive or negative. This is a stylized approach to capturing observable director characteristics predictive of their respective preference over the project scale. For instance, if several board members are closely connected with the CEO (e.g., board interlocks) or derive career benefits from larger projects, one would expect $\beta > 0$. In contrast, if the board is dominated by directors representing debtholders or by accountants or academics concerned foremost with their reputational capital, it may overweight the project downside and prefer a smaller investment level than the shareholders, i.e., $\beta < 0$. We treat the board as one decision-maker rather than modeling explicitly the way individual preferences and efforts are aggregated (Li, 2001; Harris and Raviv, 2008).

by limited liability (i.e., total compensation must be nonnegative) and (ii) firm value were subject to some random shock, η, with sufficiently rich domain, e.g., $[\omega y - \frac{1}{2}y^2]\eta,\ \eta \in [0, \bar{\eta}]$, where the realization of η is realized after all actions were chosen.

[14]The expression for U suggests that by choice of the board bias β, the shareholders can (i) influence the board's actions and (ii) directly affect the board's expected payoff holding constant his actions. This may raise concerns in a contracting setting, as the shareholders may be able to extract the additional surplus thus (mechanically) generated. As we will show below, however, the board's individual rationality constraint will always be slack at the optimal solution. Hence such "money pump" concerns do not arise in our setting.

The sequence of events is as follows:

Date 0: Shareholders choose (α, F, β).

Date 1: The board chooses information gathering effort, e. Then, with probability e the board learns ω; with $1-e$ it remains uninformed.

Date 2: The board makes the investment decision. In case of unsuccessful information gathering the board elicits a report r from the CEO and makes the investment decision based on the CEO's report.

For the Date-2 communication and investing stage, we consider two alternative scenarios. In the *commitment* setting the board specifies a report-contingent investment rule, $y(r)$, as in Holmstrom (1977). In the *noncommitment* setting the board chooses the investment in a sequentially rational manner in response to the CEO's report; the communication between the CEO and the board takes the form of cheap talk (e.g., Crawford and Sobel 1982).

We now characterize the players' preferred investment levels given any available information, $\Omega \in \{\emptyset, r, s, \omega\}$. If it successfully uncovers ω, the board will choose $y = \omega + \beta$, perfectly adapting the investment to its preferences. If information gathering fails to uncover ω, then the players' preferred investment levels are, respectively: $y_S(\Omega) = \mathbb{E}[\omega \mid \Omega]$ for the shareholders (NPV maximization), $y_C(\Omega) = \mathbb{E}[\omega \mid \Omega] + b$ for the CEO, and $y_B(\Omega) = \mathbb{E}[\omega \mid \Omega] + \beta$ for the board. We denote by $\bar{y}$ the investment level the board chooses if information gathering has failed, and we investigate how $\bar{y}$ is affected by the CEO's report r on s.

To shed light on the role of the board bias in mediating the communication and investment game between the board and the CEO, we begin with a limit case in which the board cannot gather information on its own and hence can rely only on communication with the CEO when choosing the investment.

3 The Communication Subgame

Suppose information gathering by the board is ineffective or prohibitively costly, i.e., $c \to \infty$. We focus on how the board bias β affects the Date-2 communication and the resulting investment decisions for any CEO bias, b. We first treat β as exogenous and endogenize it later.

3.1 Exogenous Board Bias

We begin by taking as given the board bias, β. The shareholders play no role in this subsection. In the Date-2 communication subgame, the remaining players aim to minimize their respective expected investment-related losses. With b and β capturing the players' respective preferences over the scale of the investment, communication will become less informative, the greater the relative preference divergence $|b-\beta|$. We ask in this section with how quickly this preference divergence impedes communication depending on the board's commitment power.

If the board cannot precommit to a report-contingent investment schedule, the communication between the CEO and the board takes the form of cheap talk (Crawford and Sobel, 1982). We allow the CEO to play mixed reporting strategies. Let σ_s represents the probability of the CEO reporting $r = 1$ if he observes s. The belief function $\Gamma(r)$ denotes the board's inferred probability of the actual state being 1. A *Perfect Bayesian Equilibrium* of the subgame is a strategy-belief profile $(\sigma_s^*, \bar{y}^{nc}(r), \Gamma(r))$ (superscript "*nc*" denotes "noncommitment") consisting of the board's decision rule:

$$\bar{y}^{nc}(r) \in \underset{y \in R}{\operatorname{argmin}} \quad \Gamma(r)\,(y-1-\beta)^2 + (1-\Gamma(r))\,(y-\beta)^2\,,$$

the CEO's reporting strategy:

$$\begin{aligned}\sigma_s^* \in \underset{\sigma \in [0,1]}{\operatorname{argmin}}\ & \sigma\left[Pr(\omega = 1 \mid s)\,(\bar{y}^{nc}(1) - 1 - b)^2 + Pr(\omega = 0 \mid s)\,(\bar{y}^{nc}(1) - 0 - b)^2\right] \\ & +(1-\sigma)\left[Pr(\omega = 1 \mid s)\,(\bar{y}^{nc}(0) - 1 - b)^2 + Pr(\omega = 0 \mid s)\,(\bar{y}^{nc}(0) - 0 - b)^2\right]. \quad (4)\end{aligned}$$

The board's inference about the state, $\Gamma(r) = Pr(\omega = 1 \mid r)$, follows Bayes' rule.

It is well known that "babbling" (uninformative communication) is always *an* equilibrium of such a game. In keeping with the literature, however, we focus on the most informative equilibrium (all proofs are in Appendix A):

Lemma 1 (Noncommitment) *Suppose the board is uninformed about ω at Date 2 and cannot precommit to a report-contingent investment schedule. The most informative communication equilibrium then is given by:*

(a) If $|\beta - b| \leq \frac{\Delta}{2}$, the CEO reports truthfully, and the board invests according to $\bar{y}^{nc}(r) = \mathbb{E}[\omega \mid s = r] + \beta$.

(b) If $|\beta - b| > \frac{\Delta}{2}$, babbling is the unique communication equilibrium, and the board invests according to its prior: $\bar{y}^{nc}(r) \equiv \mathbb{E}[\omega] + \beta = \frac{1}{2} + \beta$.

As is generally the case in cheap-talk settings, the greater the preference divergence, the less information can be credibly conveyed between a sender and a receiver. Given the binary signal space, the change in the communication equilibrium is "bang-bang:" for $|\beta - b| \leq \frac{\Delta}{2}$ the CEO perfectly reports his information and the board can implement its preferred investment level given s at no cost. For $|\beta - b|$ exceeding $\frac{\Delta}{2}$ no information can be credibly conveyed (babbling). The more informative the CEO's signal, the wider the range of preference divergence parameters that allows for perfect communication (recall, $\Delta \equiv 2q - 1$).

With commitment on the part of the board, the communication subgame represents a special case of Holmstrom (1977). Given a binary state space, the board simply devise an investment pair $\langle y(0), y(1) \rangle$ so as to minimize its expected investment-related loss subject to truth-telling constraints for the CEO. Omitting irrelevant scalars, the board's subprogram $\mathcal{SP}^c$ (superscript "c" for "commitment" on the part of the board) can be written as follows: for (b, β),

$$\mathcal{SP}^c: \min_{\{y(1),y(0)\}} \sum_{s\in\{0,1\},\omega\in\{0,1\}} Pr(s,\omega)\,(y(s)-\omega-\beta)^2,$$

s.t.:

$$\mathbb{E}_\omega\left[(y(1)-\omega-b)^2 \mid s=1\right] \le \mathbb{E}_\omega\left[(y(0)-\omega-b)^2 \mid s=1\right], \qquad (TT_1)$$

$$\mathbb{E}_\omega\left[(y(0)-\omega-b)^2 \mid s=0\right] \le \mathbb{E}_\omega\left[(y(1)-\omega-b)^2 \mid s=0\right]. \qquad (TT_0)$$

Constraint (TT_s), $s=0,1$, ensures that the CEO truthfully reports his private signal s. We denote by $\mathbb{1}_X \in \{0,1\}$ an indicator function that takes the value of 1 if and only if statement "X" is true.

Lemma 2 (Commitment) *At Date 2, suppose the board is uninformed about ω but can precommit to a report-contingent investment schedule, $y(r)$. Then:*

(a) If $|b-\beta| < \frac{\Delta}{2}$, then $\bar{y}^c(r) = \beta + \mathbb{E}[\omega \mid s=r]$, and the CEO's report fully reveals s.

(b) If $|b-\beta| \in [\frac{\Delta}{2}, \Delta]$, then $\bar{y}^c(r=0) = b + \mathbb{E}[\omega \mid s = \mathbb{1}_{\beta>b}] - \frac{\Delta}{2}$ and $\bar{y}^c(r=1) = b + \mathbb{E}[\omega \mid s=\mathbb{1}_{\beta>b}] + \frac{\Delta}{2}$, and the CEO's report fully reveals s.

(c) If $|b-\beta| > \Delta$, then the board commits to ignoring any report issued by the CEO and invests according to its prior: $\bar{y}^c(r) = \mathbb{E}[\omega] + \beta = \frac{1}{2} + \beta$.

A board that has commitment power can always induce the CEO to report obediently, but the relative preference divergence determines the cost of maintaining truthtelling incentives. All else equal, the ability to commit to a report-contingent investment plan leaves the board weakly better off—but when will this improvement be strict? Contrasting Lemmas 1 and 2, we find the outcome is unaffected by the board's commitment power for levels of preference divergence that are either sufficiently small ($|b-\beta| < \frac{\Delta}{2}$) or high ($|b-\beta| > \Delta$); only for intermediate levels ($|b-\beta| \in [\frac{\Delta}{2}, \Delta]$) does board commitment affect the outcome.

As we will formally show below, it is never optimal for the shareholders to set $\beta \ge b$. Hence, for the sake of illustration we focus here already on the case

$\beta < b$ (hence $\mathbb{1}_{\beta > b} = 0$ in Lemma 2). The pressing reporting friction then is to incentivize the CEO to truthfully report a low signal realization, $s = 0$. Suppose $b - \beta < \frac{\Delta}{2}$, indicating a small relative preference divergence between the CEO and board. Then, even conditional on observing a low signal, if the CEO is presented with a choice between the board's preferred investment levels, the CEO will pick $y_B(0)$ because $y_B(1) - y_C(0) > y_C(0) - y_B(0)$. Truthful reporting is ensured then even absent board commitment, and the board can implement its preferred investment level of $y_B(s)$ for free. We refer to this as *perfect communication*, or Case (i). For $b - \beta > \Delta$, in contrast, the investment distortions required to ensure truthtelling outweigh the value of the information learned; hence communication has no impact on the decision. With slight abuse of terminology, we refer to this as *no communication*, or Case (iii).

For intermediate preference divergence ($\frac{\Delta}{2} \leq b - \beta \leq \Delta$), board commitment is necessary and sufficient for nontrivial communication. Without commitment Case (iii) obtains (babbling); with commitment the board can still elicit an informative report from the CEO, but doing so is not free anymore. Specifically, the board commits to an investment pair that leaves a CEO who has observed a low state indifferent in that $\bar{y}^c(1) - y_C(0) = y_C(0) - \bar{y}^c(0) = f$, for some constant f. Minimizing the board's expected investment-related loss over all f yields $f = \frac{\Delta}{2}$ as stated in Lemma 2b. The more informative the CEO's signal, the greater the spread in the investment levels offered (again, $\Delta \equiv 2q - 1$).[15] The cost to the board associated with these investment distortions is increasing in $|b - \beta|$. We refer to this as *constrained communication*, or Case (ii).[16]

For later use, we compute the players' expected losses conditional on the outcome of board information gathering for either commitment scenario, $j = c, nc$.

[15] If the CEO were to prefer a lower investment level than the board (i.e., if $b < \beta$), then the binding reporting incentive would be (TT_1) conditional on the CEO observing $s = 1$, which would give rise to optimal investment levels $\bar{y}^c(1) = y_C(1) + f$ and $\bar{y}^c(0) = y_C(1) - f$, where $y_C(1) = b + E[\omega \mid s = 1]$ and $f = \frac{\Delta}{2}$.

[16] Constrained communication can also be interpreted as "constrained delegation" whereby the CEO is given the investment levels to choose from.

If information gathering was *successful*, the board chooses $y(\omega) = \omega + \beta$, resulting in losses, respectively, of $L_B = 0$ for the board, and $L_S = \frac{1}{2}\mathbb{E}_\omega[(y(\omega) - \omega)^2] = \frac{\beta^2}{2}$ for the shareholders; both these loss terms are independent of the commitment setting. The players' losses conditional on *unsuccessful* information gathering are denoted by $\bar{L}^j_S(\beta, b) = \frac{1}{2}\sum_{s,\omega} Pr(s,\omega)\left(\bar{y}^j(s) - \omega\right)^2$ for shareholders, and $\bar{L}^j_B(\beta, b) = \frac{1}{2}\sum_{s,\omega} Pr(s,\omega)\left(\bar{y}^j(s) - \omega - \beta\right)^2$ for the board. See Tables 1 and 2 (and Figure 1) for a summary.

	Case (i): $\lvert b-\beta\rvert \in [0, \frac{\Delta}{2})$	Case (ii): $\lvert b-\beta\rvert \in [\frac{\Delta}{2}, \Delta]$	Case (iii): $\lvert b-\beta\rvert > \Delta$
$\bar{y}^c(s)$	$\mathbb{E}[\omega \mid s] + \beta$	$b + \mathbb{E}[\omega \mid \mathbb{1}_{\beta>b}] + (2s-1)\frac{\Delta}{2}$	$\frac{1}{2} + \beta$
$\bar{L}^c_B$	$\frac{1}{2}\Lambda_s$	$\frac{1}{2}\left[\Lambda_s + (\lvert b-\beta\rvert - \frac{\Delta}{2})^2\right]$	$\frac{1}{2}\Lambda_\emptyset$
$\bar{L}^c_S$	$\frac{1}{2}(\Lambda_s + \beta^2)$	$\frac{1}{2}\left[\Lambda_s + (b - \frac{\Delta}{2})^2\right] + \mathbb{1}_{\beta>b} \cdot b\Delta$	$\frac{1}{2}(\Lambda_\emptyset + \beta^2)$

Table 1: Outcome Given Unsuccessful Information Gathering: Commitment

	Case (i): $\lvert b-\beta\rvert \in [0, \frac{\Delta}{2}]$	Case (iii): $\lvert b-\beta\rvert > \frac{\Delta}{2}$
$\bar{y}^{nc}(s)$	$\mathbb{E}[\omega \mid s] + \beta$	$\frac{1}{2} + \beta$
$\bar{L}^{nc}_B$	$\frac{1}{2}\Lambda_s$	$\frac{1}{2}\Lambda_\emptyset$
$\bar{L}^{nc}_S$	$\frac{1}{2}(\Lambda_s + \beta^2)$	$\frac{1}{2}(\Lambda_\emptyset + \beta^2)$

Table 2: Outcome Given Unsuccessful Inform. Gathering: Noncommitment

In Cases $k \in \{i, iii\}$, the loss terms of the board and shareholders differ only by the bias term, $\frac{\beta^2}{2}$, reflecting that both board and shareholders equally internalize any prevailing information loss, $\frac{\Lambda_l}{2}$, $l = \emptyset, s$. In Case (ii), which can obtain only with board commitment, the CEO's private signal s affects the investment level, but eliciting this information comes at a cost in the form of investment distortions from the perspective of the board.

For exogenous bias levels b and β, the analysis so far confirms the standard intuition that commitment power improves communication. In our setting it does so strictly for a non-empty subset of parameters—specifically, for intermediate levels of preference divergence between the board and the CEO (see Figure 1). However, improved communication between CEO and board does not necessarily imply that the shareholders are better off with a board that can commit.

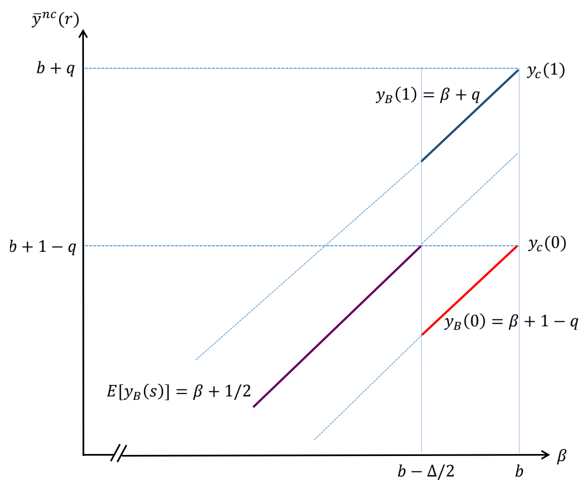

Fig.1a: Noncommitment: investments

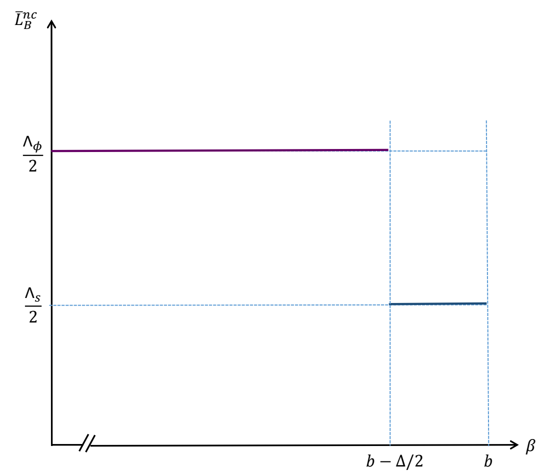

Fig.1b: Noncommitment: board loss

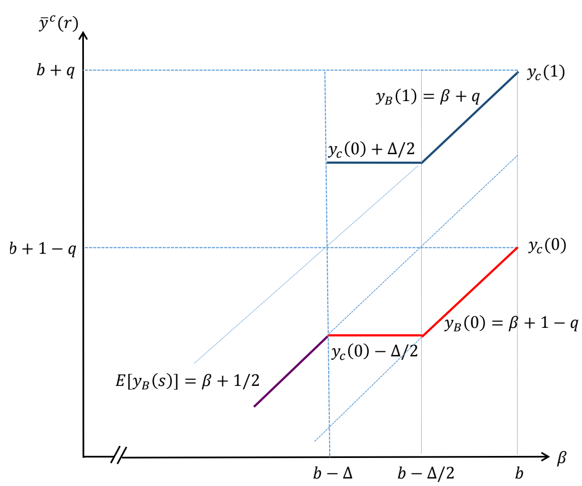

Fig.1c: Commitment: investments

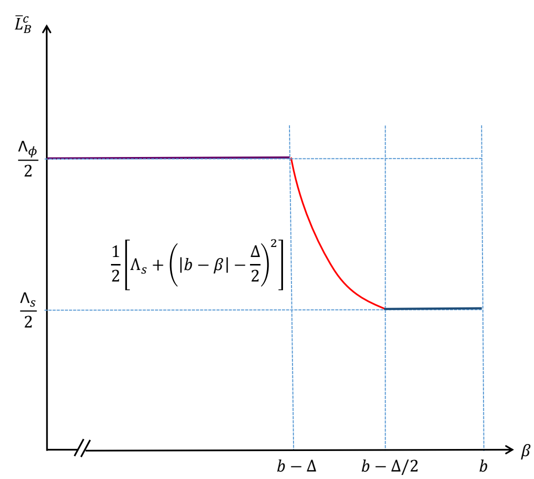

Fig.1d: Commitment: board loss

Fig.1: Investments and Board Loss Conditional on Failed Information Gathering

Lemma 3 *With exogenous board bias,* $\bar{L}_S^c \leq \bar{L}_S^{nc}$ *if* $b \leq \Delta$.

Board commitment benefits the shareholders provided the value of the CEO's signal, Δ, exceeds his bias, b. But what about highly biased CEOs? For board commitment to matter, $|b - \beta| \in \left[\frac{\Delta}{2}, \Delta\right]$ has to hold. Cheap talk then leads the board to act on its prior (y reflects β), whereas with commitment the investment levels is anchored on $y_C(0)$ to ensure truthtelling by the CEO (y reflects b). Suppose $\beta \in \left[b - \Delta, b - \frac{\Delta}{2}\right]$ and b becomes large (and so does β, in lockstep). With commitment the board prefers constrained to no communication: the value of the CEO's signal compensates the board for the bias cost, which for the board is a function of the *relative* divergence $|b - \beta|$. The shareholders however care about the *absolute* bias levels b and β. With a quadratic loss function, as both bias terms increase in lockstep, the shareholders eventually prefer the decision to reflect β rather than b, as $b > \beta$, and hence prefer a board without commitment power acting on its prior but based on a less extreme bias.[17]

We revisit the value of board commitment in Section 5 for the full model.

3.2 Endogenous Board Bias

We now allow for the shareholders to choose β in their best interest, while retaining the focus on the Date-2 communication subgame, i.e., $c \to \infty$ still holds for now. If the board can commit to a report-contingent investment pair, then the shareholders are best served at Date 2 by an unbiased board ($\beta^c = 0$) that fully internalizes their objective. Under noncommitment, Dessein (2002) has shown in a continuous state model that installing an intermediary with a bias parameter between zero (shareholders) and the sender's bias, improves the communication between the sender and receiver. The communication benefit outweighs the attendant bias cost, which is of second order for small levels of $\beta > 0$. Similar

[17]E.g., let $\beta = b - \frac{5}{6}\Delta$, thereby pegging β to b in a manner consistent with constrained communication under commitment. Then, it is easy to show that $\bar{L}_S^c > \bar{L}_S^{nc}$ for any $b > \frac{25}{24}\Delta$.

logic applies to our discrete state model, raising the question whether biasing the board can substitute for its lack of commitment (proof omitted):

Lemma 4 (Replication Result) *Suppose $c \to \infty$, i.e., information gathering is ineffective. Then the optimal board bias under commitment is $\beta^c(b) = 0$ for any b; while under noncommitment:*

$$\beta^{nc}(b) = \begin{cases} 0, & \textit{for } \; b \notin \left(\frac{\Delta}{2}, \Delta\right), \\ b - \frac{\Delta}{2}, & \textit{for } \; b \in \left(\frac{\Delta}{2}, \Delta\right). \end{cases}$$

Given these optimal board bias levels, the resulting loss to the shareholders is the same across commitment scenarios: $\bar{L}_S^{nc}(\beta^{nc}(b), b) = \bar{L}_S^{c}(\beta^c(b) = 0, b)$.

For intermediate CEO bias levels, under noncommitment the shareholders nominate a board that is biased toward the CEO by just enough to facilitate truthful communication. In our binary state model, the communication benefit is simply $\frac{1}{2}(\Lambda_\emptyset - \Lambda_s)$. The attendant bias cost to the shareholders equals $\frac{1}{2}\beta^2$ where $\beta = b - \frac{\Delta}{2}$. Equating the two yields a cutoff for the CEO bias equal to Δ, beyond which the shareholders give up on communication.

Lemma 4 shows that commitment is of no value to the shareholders if information gathering by the board is infeasible, provided the board bias can be chosen endogenously. By assembling a friendly board the shareholders can replicate their expected payoff from the commitment setting. This replication result is surprising as the board bias is a rather blunt instrument: it is chosen ex ante and cannot be conditioned on s. The investment menu set by the (unbiased) board under commitment in contrast has two entries: one for each signal the CEO may have observed. But given the symmetric prior distribution (both states are equally likely), the distortions built into the investment menu by a board with commitment power are the same for each signal, and equal to the distortion resulting from a friendly board without commitment power.[18]

[18]The symmetry of the prior distribution is important for this argument. If the two states

We now turn to the full-fledged model to explore the role of board bias in conjunction with equity incentives in motivating the board to gather information.

4 The Board as Advisor and Monitor

We now allow for nontrivial information gathering by the board, i.e., c is finite. The board's choice of information gathering effort will be driven by its incentives, (α, β, F). The CEO's reporting behavior remains as described in Lemmas 1 and 2. For now, we set the stage for the analysis at a generic level, i.e., without specific reference to the commitment assumptions in the communication subgame.

The board's incentives to gather information derive from the potential to tailor the scale of investment more closely to its own preferences. The board chooses its information gathering effort to maximize its expected payoff as per Date 1, which by (3) reads: for $j = c, nc$,

$$EU^j = \alpha\left(\frac{1}{2}\mathbb{E}_\omega[(\omega+\beta)^2] - eL_B - (1-e)\bar{L}_B^j(\beta, b)\right) + F - \frac{ce^2}{2}. \tag{5}$$

Recall that $L_B = 0$ because upon successful information gathering the board chooses its preferred investment scale. The board's optimal effort $e^j(\alpha, \beta)$ is thus determined by the first-order condition,

$$e^j(\alpha, \beta, b) = \frac{\alpha}{c}\bar{L}_B^j(\beta, b), \tag{6}$$

and hence increasing in the equity stake, α, and its "cost of ignorance," $\bar{L}_B^j(\cdot)$. Furthermore, there is *complementarity* between α and $\bar{L}_B^j(\cdot)$: the greater is $\bar{L}_B^j(\cdot)$, the more effectively an increase in α elicits board effort, at the margin.

We now turn to the shareholders' decision problem at Date 0, when assembling and contracting with the board. Anticipating the actions taken by the

were not equally likely to occur, program $\mathcal{SP}^c$ would entail minimizing the probability-weighted distortions (no longer equally weighted) across the two signals. The optimal board bias under noncommitment (commitment) would again be weakly positive (zero), but the replication result in Lemma 4 would generally break down.

board and the CEO, for any CEO bias b, the shareholders choose (α, F, β) to maximize their expected Date-0 utility, which by (2) reads: for $j = c, nc$,

$$EV^j(\alpha, \beta, F \mid b) = (1-\alpha)\left(\frac{1}{4} - e^j(\alpha,\beta,b)L_S(\beta) - [1 - e^j(\alpha,\beta,b)]\bar{L}^j_S(\beta,b)\right) - F. \quad (7)$$

At Date 0 the shareholders solve the following program: for any $j = c, nc$,

$$\mathcal{P}^j_0: \quad \max_{\alpha\in[0,1],\, \beta\in\mathbb{R},\, F\in\mathbb{R}_+} EV^j(\alpha, \beta, F \mid b)$$
$$\text{subject to:} \quad EU^j(\cdot) \geq \bar{U} = 0. \quad \text{(IR)}$$

Denote the solution to this program by (α^j, β^j, F^j), $j = c, nc$. For the remainder of the paper, we impose the joint parameter restrictions that $c \in \left[\frac{Q-2Q^2}{2+4Q}, \frac{Q^2}{1-2Q}\right]$ and $q < \bar{q} \equiv \frac{1}{2} + \frac{\sqrt{3}}{6}$. The bounds on c ensure interior board efforts and equity shares, $(e, \alpha) \in [0,1]^2$ for Programs $\mathcal{P}^c_0$ and $\mathcal{P}^{nc}_0$; the bound on q ensures this permissible range for c is non-empty (see Appendix B for details).

To simplify the solution of the full-fledged model, we state two preliminary results that hold regardless of the commitment setting.

Lemma 5 *The board bias is bounded by the CEO bias: $\beta^j < b$, $j = c, nc$.*

Only the *relative* preference divergence $|b - \beta|$ matters for the board's effort and communication incentives (recall $\bar{L}^j_B$ is symmetric in β around b for $j = c, nc$), whereas any *absolute* board bias is costly to the shareholders due to distorted investment decisions by the board (Tables 1 and 2).[19] Lemma 5 allows us to rewrite the preference divergence between the CEO and the board simply as $b - \beta$; as β increases, the two parties become better aligned.

Lemma 6 *The board's individual rationality constraint is slack in Programs $\mathcal{P}^j_0$, and $F^j = 0$, for $j = c, nc$.*

[19] Suppose the shareholders were to choose a board bias of $b + \delta$, where $\delta > 0$. Such an arrangement would be dominated by a board with $b-\delta$, because the board's information gathering effort would remain unaffected, at reduced investment-related loss terms to the shareholders, L_S and $\bar{L}^j_S$.

For any triple (α, β, F) satisfying limited liability, the board could simply choose zero effort and still earn a nonnegative expected payoff. We henceforth drop the fixed salary F from the notation.

Before solving $\mathcal{P}_0^c$ and $\mathcal{P}_0^{nc}$, it is helpful to gain some intuition for the trade-offs involved. The board bias β affects the shareholders' expected payoff through three channels: (a) directly through the investment decision made by a fully informed board, $\omega + \beta$; (b) through the board's investment decision upon unsuccessful information gathering, $\bar{y}^j$, by way of mediating the communication subgame as per Lemmas 1 and 2; and (c) through the board's information gathering effort, e^j, which in turn determines the weights on (a) and (b). The board's equity stake α trades off information gathering and equity dilution concerns, i.e., for any b, and $j = c, nc$, and given the equilibrium $\beta^j(b)$, by (7):

$$\alpha^j(b) \in \arg\max_{\alpha \in [0,1]} \quad (1-\alpha)\left[\frac{1}{4} - \bar{L}_S^j(\beta^j(b), b) + e^j(\alpha, \beta^j(b), b)\left[\bar{L}_S^j(\beta^j(b), b) - L_S(\beta^j(b))\right]\right].$$

We now solve the shareholders' optimization problems for each commitment scenario. Recall that $\beta^c = 0 \leq \beta^{nc}$ for $c \to \infty$. Given the preceding arguments, one would expect the potential for information gathering to push down the board bias: Greater discord between the board and CEO should increase the board's effort, by (6), and the shareholders should benefit from such additional effort, especially since it comes "for free" at the margin, by Lemma 6. The analysis to follow bears out this intuition.

4.1 Commitment

Absent information gathering concerns, by Lemma 4, the shareholders prefer an unbiased board if the board has commitment power. Yet as argued above, the board's incentive constraint (6) together with $\bar{L}_B^c$ (Table 1) suggest the board will gather (weakly) more information the greater its discord with the CEO. So the optimal β trades off: Within each of the three cases in Table 1 (perfect, constrained, or no communication), lowering $|\beta|$ minimizes the shareholders' loss *holding constant* the outcome of board information gathering, because the loss terms $(L_S, \bar{L}^c_S)$ are reaching their respective minima at $\beta = 0$. At the same time, lowering β elicits weakly greater information gathering effort, whereas raising β may improve CEO/board communication in the sense of inducing a "jump" across communication cases, e.g., from babbling (Case (iii)) to constrained (Case (ii)) or even perfect communication (Case (i)). That is, the direction of any deviation from $\beta = 0$ trades off information gathering and communication benefits.

Our next result describes the solution to Program $\mathcal{P}^c_0$:

Proposition 1 (Commitment) *If the board can commit to a report-contingent investment rule, then:*

(a) The optimal board bias $\beta^c(b)$ is continuous, single-troughed, and weakly <u>antagonistic</u>:

- *For $b < \frac{\Delta}{2}$, $\beta^c(b) = 0$, implementing Case (i);*
- *For $b \in [\frac{\Delta}{2}, \tilde{b}]$, $\beta^c(b) = \beta^{int}(b) < 0$, with $\beta^{int}(b)$ uniquely determined by*

$$\left(b - \frac{\Delta}{2} - \beta^{int}(b)\right)^2 \left(b - \frac{\Delta}{2} + 2\beta^{int}(b)\right) = -\left(b - \frac{\Delta}{2}\right) q(1-q), \quad (8)$$

  and $\tilde{b}$ uniquely determined by $\beta^{int}(\tilde{b}) = \tilde{b} - \Delta$, implementing Case (ii);
- *For $b \in (\tilde{b}, \Delta]$, $\beta^c(b) = b - \Delta < 0$, implementing Case (ii);*
- *For $b > \Delta$, $\beta^c(b) = 0$, implementing Case (iii).*

(b) The optimal equity stake $\alpha^c(b)$ is continuous and monotonically non-decreasing.

Figure 2 depicts a numerical example illustrating Proposition 1 and 2 and Corollaries 3 and 3′, below.

The optimal board bias with commitment is weakly antagonistic—why? First, we show in the proof that the shareholders never want to "jump" across the cases (i)–(iii) given in Table 1. The (absolute) board bias level required to induce such a discrete jump in the communication game between the CEO and the board would be so high that any benefits from enhanced communication or information gathering are outweighed by the attendant investment distortions.[20] Thus, we only need to consider "local" (within-case) changes in β. Recall from the incentive constraints (6) that the board's information gathering incentives are affected by its "cost of ignorance," $\bar{L}_B(\beta, b)$. For sufficiently small or large CEO biases ($b < \frac{\Delta}{2}$ or $b > \Delta$), introducing a small board bias has no impact on the board's effort, as $\bar{L}_B(\beta, b)$ then is independent of β, but it would increase the shareholders' bias cost. Hence, the board should be unbiased.

For intermediate CEO bias, $b \in [\frac{\Delta}{2}, \Delta]$, first note that it is never optimal to appoint a friendly board: Setting $\beta > 0$ would impede information gathering and introduce bias into the investment decision without any offsetting benefits, because $\bar{L}_S^c$ is independent of β in Case (ii). On the other hand, introducing a small *antagonistic* bias, $\beta < 0$, exposes the shareholders to only second-order investment distortions if the board becomes informed, while generating a first-order benefit through improved information gathering. An antagonistic board is therefore optimal for intermediate CEO bias levels. As b exceeds $\frac{\Delta}{2}$, the interior solution to the shareholders' optimization problem given Case (ii) trades off the above effects. As the CEO bias reaches some threshold $\tilde{b}$, however, this interior solution would result in a preference divergence $b - \beta^{int}(b)$ exceeding Δ, resulting in no-communication, Case (iii). For $b \in (\tilde{b}, \Delta]$, thus, the shareholders select the knife-edge board bias, $\beta^c(b) = b - \Delta$, that just ensures constrained communication, i.e., Case (ii).

The board's optimal equity stake trades off effort incentives and dilution

[20] Technically, the "no jumping across cases" result arises from the fact that e^c is continuous in β. Recall that in the case of ineffective information gathering ($c \to \infty$), the shareholders optimally assemble an unbiased board, that is, there is "no jumping cases" for ineffective information gathering. With information gathering becoming effective, the only reason the shareholders might want to "jump" across cases is for information gathering reasons. However due to the continuity of e^c in β, the shareholders will never want to "jump" across cases.

concerns. Both forces push for a positive relation between CEO agency problems and α^c: More severe agency problems at the CEO level dampen the shareholders' dilution cost and increase the board's cost of ignorance. The latter in turn makes equity a more powerful incentive instrument because of the complementarity of α and $\bar{L}_B^c$ in eliciting board effort.

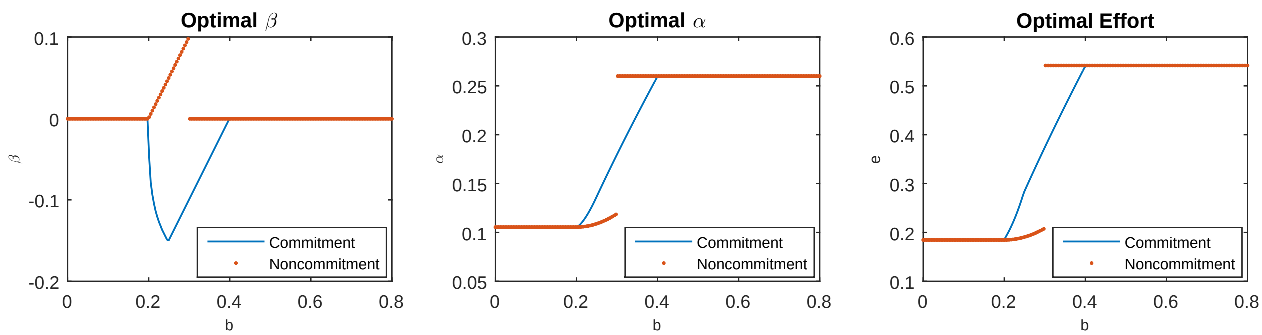

Fig.2a—"High-q": (β^j, α^j, e^j), $j = c, nc$, for $c = 0.06$ and $q = 0.7$.

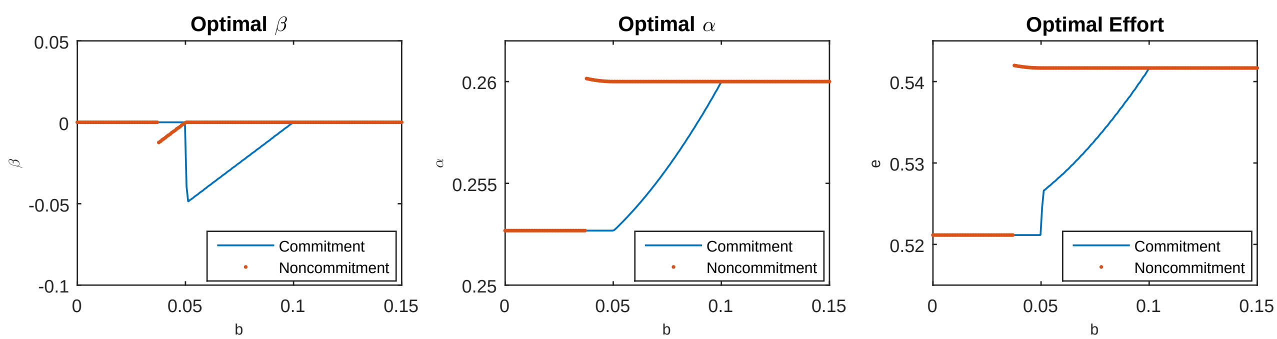

Fig.2b—"Low-q": (β^j, α^j, e^j), $j = c, nc$, for $c = 0.06$ and $q = 0.55$.

Fig.2: Investments and Board Loss Conditional on Failed Information Gathering

We now turn to cheap-talk communication.

4.2 Noncommitment

If the board cannot precommit to a report-contingent investment rule, Lemma 1 has shown that cheap-talk communication between the CEO and the uninformed board takes the form of either babbling (Case (iii)), or perfect reporting by the CEO (Case (i)). For any CEO bias, b, by carefully calibrating the board bias β, the shareholders can implement either of these two communication cases.

As shown in Section 3.2, for CEO bias in excess of $\frac{\Delta}{2}$, babbling would ensue if the board were unbiased ($\beta = 0$), but the shareholders can facilitate perfect communication by assembling a sufficiently friendly board ($\beta = b - \frac{\Delta}{2} > 0$). Likewise, for $b < \frac{\Delta}{2}$ perfect communication would ensue if the board were unbiased, but the shareholders could "block" communication by assembling a sufficiently antagonistic board, i.e., setting $\beta = b - \frac{\Delta}{2} - \epsilon < 0$, for any small $\epsilon > 0$ (we henceforth suppress ϵ). Applying the logic developed in Section 4.1, an antagonistic board may gather more information, a facet of the model sidestepped in our preliminary analysis of the subgame in Section 3.2.

Our next result is a stepping stone toward solving Program $\mathcal{P}_0^{nc}$:

Lemma 7 *With finite c, for any q there exists a unique threshold for the CEO bias, denoted $\hat{b}(q)$, with $\hat{b}(q) < \Delta$, such that the solution to Program $\mathcal{P}_0^{nc}$ entails perfect communication (Case (i)) for $b \leq \hat{b}(q)$, and babbling (Case (iii)) for $b > \hat{b}(q)$.*

While the shareholders can always "flip" between perfect communication and babbling by choosing β, doing so is costly. For highly biased CEOs, facilitating communication with a sufficiently friendly board would be too costly in terms of the resulting investment distortions. Similar logic rules out blocking communication for very low levels of b by nominating a very antagonistic board. Furthermore, comparing Lemma 7 with Lemma 4 (the limit case of $c \to \infty$), we find that the potential for the board to engage in information gathering shrinks the b-region over which the shareholders appoint a friendly board to foster communication, from $(\frac{\Delta}{2}, \Delta)$ to $(\frac{\Delta}{2}, \hat{b}(q))$, where $\hat{b}(q) < \Delta$. This illustrates an opportunity cost of having a friendly board in terms of reduced information gathering; in Lemma 4 this opportunity cost was zero by construction.

More generally, the direction of the optimal board bias trades off communication and information gathering considerations. The more can be learned from the CEO through communication (i.e., the greater is q), the higher one would expect $\hat{b}(q)$ to be, and vice versa. Using this intuition, we find:

Proposition 2 ***(Noncommitment)*** *If the board cannot commit to a report-contingent investment rule, then there exists a unique threshold for the CEO's precision $\hat{q}$, such that:*

(a) ***High** q**:*** *If $q \geq \hat{q}$, then $\hat{b}(q) \geq \frac{\Delta}{2}$ and:*

- *The optimal board bias $\beta^{nc}(b)$ is discontinuous at $\hat{b}(q)$, non-monotonic, and weakly friendly:*
  - *For $b < \frac{\Delta}{2}$, $\beta^{nc}(b) = 0$, implementing Case (i);*
  - *For $b \in \left[\frac{\Delta}{2}, \hat{b}(q)\right)$, $\beta^{nc}(b) = b - \frac{\Delta}{2} > 0$, implementing Case (i);*
  - *For $b \geq \hat{b}(q)$, $\beta^{nc}(b) = 0$, implementing Case (iii).*
- *The optimal equity stake $\alpha^{nc}(b)$ is monotonically non-decreasing with a discrete jump up at $\hat{b}(q)$.*

(b) ***Low** q**:*** *If $q < \hat{q}$, then $\hat{b}(q) < \frac{\Delta}{2}$ and:*

- *The optimal board bias $\beta^{nc}(b)$ is discontinuous at $\hat{b}(q)$, non-monotonic, and weakly antagonistic:*
  - *For $b < \hat{b}(q)$, $\beta^{nc}(b) = 0$, implementing Case (i);*
  - *For $b \in \left[\hat{b}(q), \frac{\Delta}{2}\right)$, $\beta^{nc}(b) = b - \frac{\Delta}{2} < 0$, implementing Case (iii);*
  - *For $b \geq \frac{\Delta}{2}$, $\beta^{nc}(b) = 0$, implementing Case (iii).*
- *The optimal equity stake $\alpha^{nc}(b)$ is non-decreasing for any $b \notin \left(\hat{b}(q), \frac{\Delta}{2}\right)$, with a discrete jump up at $\hat{b}(q)$, but strictly decreasing for any $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$.*

In comparison with Proposition 1, commitment power on the part of the board makes a difference only for intermediate levels of CEO bias, for which the shareholders calibrate the board bias level to balance investment bias costs against information gathering and communication benefits. For CEOs endowed with very precise information ("high q" case), there is little incrementally new information to be unearthed, and any information gathering by the board has the flavor of monitoring only. The optimal solution then emphasizes communication by way of a friendly board. If the CEO's signal is rather noisy ("low q" case), board information has monitoring and advising features, so the shareholders assemble an antagonistic board to elicit greater information gathering effort.

The intuition given for a positive association between CEO agency problems and the board's equity stake in Proposition 1 applies also for noncommitment, provided the CEO has sufficiently precise private information. For CEOs with noisy private information, however, the optimal equity stake $\alpha^{nc}(b)$ is locally *de*creasing. Specifically, for $q < \hat{q}$ and $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$, as b increases, so does $\beta^{nc} = b - \frac{\Delta}{2} < 0$, in lockstep, to block cheap talk communication as a way to foster board effort. Hence, the board's cost of ignorance under noncommitment remains unaffected but the dilution cost becomes more severe, locally: The board bias shrinks in absolute terms, resulting in investment decisions better aligned with NPV maximization.

To highlight the potential for information gathering, comparing our results with those in Section 3 (no information gathering) shows that the potential for information gathering pushes towards a more antagonistic board, all else equal. With commitment, it changes the optimal board bias from zero everywhere to one that is strictly antagonistic for intermediate CEO bias levels (zero otherwise). With noncommitment, the optimal board bias for $c \to \infty$ was weakly friendly everywhere, and strictly so for intermediate CEO bias levels. Making c finite either leaves this pattern qualitatively intact and only shrinks the parameter region for a friendly board (for CEOs with precise private signals, by Lemma 7), or it qualitatively flips into an optimal board bias that is weakly antagonistic (for CEOs with noisy private signals).

5 Discussion (and the Value of Commitment)

This section discusses the implications of the preceding results, compares the equilibrium contracts and outcomes across the commitment settings, and revisits the issue of the value of board commitment to the shareholders. By Propositions 1 and 2, the equilibrium board bias is jointly determined—in terms of direction and absolute magnitude—by the CEO's information advantage and the board's commitment power. Specifically, we predict a friendly board if the board cannot commit and the CEO has a significant information advantage. We predict an antagonistic board in one of two scenarios: either the board can commit (the CEO's signal precision then matters only quantitatively), or it cannot commit but the CEO's private signal is noisy. In any case, the optimal board bias is nonmonotonic in the severity of agency conflicts at the CEO level.

Because the CEO's precision level qualitatively affects the properties of the equilibrium (at least with cheap talk), we will deal with the high- and low-q cases separately. The precision of the CEO's private signal can reflect manager characteristics (e.g., the manager's skill level), firm characteristics (e.g., startup versus more mature firms), or a combination of the two (e.g., the fit between the CEO and the firm). To fix ideas, we will adopt the manager characteristics interpretation and refer to "high-skill" and "low-skill" managers going forward.

5.1 Highly Skilled CEOs (High-q)

Comparing the optimal board bias across the regimes for high q, we find (no proof required):

Corollary 1 *For $q \geq \hat{q}$, the optimal board bias $\beta^j(b)$ satisfies:*

(a) Commitment power makes the board bias more antagonistic: $\beta^c(b) \leq 0 \leq \beta^{nc}(b)$ *for any* b*, with both inequalities strict for* $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$.

(b) The ranking of efficiency of communication is a function of the CEO bias:

- *For* $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$*, commitment yields Case (ii) whereas noncommitment yields Case (i); hence,* $\bar{L}^c_B(\beta^c(b), b) > \bar{L}^{nc}_B(\beta^{nc}(b), b)$.
- *For* $b \in \left(\hat{b}(q), \Delta\right)$*, commitment yields Case (ii) whereas noncommitment yields Case (iii); hence,* $\bar{L}^c_B(\beta^c(b), b) < \bar{L}^{nc}_B(\beta^{nc}(b), b)$.

Does greater board antagonism wipe out the communication advantage customarily associated with commitment power? The answer is, sometimes but not always. For $b \in [\frac{\Delta}{2}, \hat{b}(q)]$ a friendly board indeed ensures that cheap talk perfectly transmits the high-skilled CEO's private information, whereas commitment achieves only constrained communication. For $b \in [\hat{b}(q), \Delta]$ the ranking of communication efficiency flips: cheap talk collapses as the board now is unbiased. Given the substitute nature of the two information acquisition channels, such reversal of the relative communication efficiency has implications for the board's equity stake and the induced effort level, in equilibrium.

Recall that the board's equity stake α trades off dilution and effort incentives, with α and the board's cost of ignorance $\bar{L}^j_B$ being complements in eliciting effort. One might expect dilution to be more of a concern, if (a) the CEO's bias is small and (b) the board can commit. As for the latter though, while the ability to commit leaves the *board* weakly better off, for given bias levels, this does not imply that it improves gross firm value, which is driving the *shareholders*' dilution concerns (see Lemma 3). Moreover, by Corollary 1(b), with the board's bias being endogenous, commitment power may increase the board's cost of ignorance and thereby boost the incentive benefits associated with α. If a friendly board anticipates perfect cheap-talk communication with a precisely-informed CEO, not just will the *level* of information gathering be small, but so will be the *incremental* effort engendered by an increase in α^{nc}. The equilibrium association between the board's commitment power and its equity stake hence is unclear:

Corollary 2 *For $q \geq \hat{q}$, the optimal equity stake $\alpha^j(b)$ satisfies:*

(a) $\alpha^j(b)$ is monotonically nondecreasing in b, $j = c, nc$.

(b) $\alpha^c(b) > \alpha^{nc}(b)$ for $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$, but $\alpha^{nc}(b) > \alpha^c(b)$ for $b \in \left(\hat{b}(q), \Delta\right)$.

Part (a) confirms the intuition that more severe agency problems at the CEO level alleviate the dilution concerns. The ranking of the equity stakes in part (b) is determined in a one-to-one fashion by the ranking of the $\bar{L}_B^j(\beta^j(b), b)$ terms, which in turn reflect the induced communication cases in Corollary 1(b). This illustrates the complementarity of equity grants and the board's cost of ignorance in eliciting effort. If in equilibrium, for a given b, commitment yields Case (n) and cheap talk yields Case (m), with $n, m \in \{i, ii, iii\}$, then $\alpha^c(b) > \alpha^{nc}(b)$ if $n > m$, and vice versa. For instance, for $b \in (\frac{\Delta}{2}, \hat{b}(q))$, cheap talk perfectly transmits the CEO's private information; commitment only achieves constrained communication (Case (ii)). Thus, equity is a more powerful incentive instrument for the (antagonistic) board under commitment.

A similar result obtains for the induced board effort for highly skilled CEOs:

Corollary 3 *If $q \geq \hat{q}$, then the equilibrium board effort is higher under commitment for $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$, and higher under noncommitment for $b \in \left(\hat{b}(q), \Delta\right)$.*

We turn now to the complementary case of a CEO endowed with rather noisy private information.

5.2 Low-skilled CEOs (Low q)

For high q, Corollary 1 has shown that commitment always resulted in a more antagonistic board but the ranking of the equilibrium communication efficiency was parameter-dependent. For low-skilled CEOs, in contrast, we find:

Corollary 1′ *For $q < \hat{q}$, the optimal board bias $\beta^j(b)$ satisfies:*

(a) The ranking of the $\beta^j(b)$ is a function of the CEO bias: $\beta^c(b) = 0 > \beta^{nc}(b)$ for $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$, whereas $\beta^{nc}(b) = 0 > \beta^c(b)$ for $b \in \left(\frac{\Delta}{2}, \Delta\right)$.

(b) Communication in equilibrium is always more efficient with commitment; i.e., if commitment given $\beta^c(b)$ yields Case (n) and noncommitment given $\beta^{nc}(b)$ yields Case (m), then $n \leq m$, for any b.

With low-skilled CEOs the board bias ranking may go either way, but the inherent communication advantage conferred by commitment power remains intact even once we endogenize the board bias. Thus, commitment power always alleviates the board's cost of ignorance. Given the CEO's postulated low signal precision, the board's information gathering adds relatively more value. Any board bias under cheap talk is intended to block—not facilitate—communication so that the equilibrium board bias is weakly antagonistic even under cheap talk, further deteriorating the communication efficiency. This has implications for the board's equilibrium equity stake and effort choice:

Corollary 2′ *For $q < \hat{q}$, the equilibrium equity stake $\alpha^j(b)$ satisfies:*

(a) $\alpha^c(b)$ is monotonically nondecreasing in b; but $\alpha^{nc}(b)$ is nonmonotonic, strictly decreasing locally for any $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$.

(b) Commitment power always reduces the board's equity stake, i.e., $\alpha^c(b) \leq \alpha^{nc}(b)$ for any b.

Lack of commitment exacerbates the board's cost of ignorance for any level of CEO bias, making equity a powerful incentive instrument (part (b)). Perhaps more surprisingly, for low-skilled CEOs $\alpha^{nc}(b)$ is locally *de*creasing (part (a)). This may seem odd given greater CEO bias in general mitigates dilution concerns. However, for $q < \hat{q}$ and $b \in (\hat{b}(q), \frac{\Delta}{2})$, as b increases, so does $\beta^{nc} = b - \frac{\Delta}{2} < 0$, in lockstep, to just ensure the board prefers babbling to perfect communication, and thereby to foster board effort. Hence, the board's cost of ignorance under noncommitment remains unaffected but the dilution cost becomes more severe, locally: The board bias approaches zero from below, resulting in investment decisions better aligned with NPV maximization.

Corollary 3′ *For $q < \hat{q}$, commitment power always reduces the board's equilibrium effort level, i.e., $e^c(\cdot) \leq e^{nc}(\cdot)$ for any b.*

As with a highly skilled CEO, the ranking of $\alpha^j(b)$ and $e^j(\cdot)$ is determined by the board's cost of ignorance under the respective commitment settings, i.e., by the induced communication cases. The reason is again the complementarity of the equity stake and cost of ignorance in eliciting board effort.

5.3 The Optimal *Unscaled* Board Bias

Our results on the optimal board bias so far have been cast in terms of β, the board bias scaled by the board's equity stake. Since the primitive measure of board bias for empirical researchers will be the *unscaled* (or "raw") board bias, $\bar{\beta}^j(b) \equiv \alpha^j(b) \cdot \beta^j(b)$, the question is, do our results carry over? Having characterized both $\alpha^j(\cdot)$ and $\beta^j(\cdot)$, the answer is yes:

Corollary 1″

(a) $\bar{\beta}^j(b) = 0$ if and only if $\beta^j(b) = 0$, $j = c, nc$.

(b) With noncommitment:

- High-q: $\bar{\beta}^{nc}(b)$ is positive and strictly increasing, for any $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$.
- Low-q: $\bar{\beta}^{nc}(b)$ is negative and strictly increasing, for any $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$.

(c) With commitment, $\bar{\beta}^c(b)$ is negative, continuous and single-troughed in b.

Scaling the equilibrium board bias by a strictly positive equity stake leaves unchanged its direction and therefore also the ranking across the commitment regimes. (Whenever the $\beta^j(b)$ are both nonzero, they are of opposite sign.) As for the *change* in the board bias as b changes, the effects of b on α^j and β^j in general reinforce each other.[21] Hence, all results for the scaled board bias carry over qualitatively to the raw board bias.

It remains to put together the above insights and evaluate the net effect of the board's commitment power on the shareholders' expected payoff.

5.4 The Value of Board Commitment

For the special case where the sole channel through which the board could learn about the environment was by communicating with the CEO, because $c \to \infty$, Section 3 has demonstrated a value (to the shareholders) of board commitment that: (a) was strictly positive for exogenous β provided b was not too high, but (b) evaporated for endogenous board bias. A carefully calibrated friendly board was shown to offset the inherent communication disadvantage of cheap talk. We now revisit the value of commitment in the full-fledged setting.

In general, of course, commitment power is an asset—for the committing party, all else equal. Our setting however entails three players: we are mainly interested in the expected payoff to the *shareholders*, whereas it is the *board* (an intermediary) that may have commitment power. Given that the shareholders can *choose* a key preference parameter of the board—namely β—in their own interest, one might nonetheless conjecture that board commitment should ultimately benefit the shareholders. On the other hand, Corollaries 3 and 3′ have shown that noncommitment often, but not always, elicits greater board effort. Combined with the fact that the board earns rents (Lemma 6), and therefore requires no additional compensation at the margin for its efforts, such higher board effort should benefit the shareholders. To formally address these trade-

[21] The only case in which the effects of b on α^j and β^j are countervailing is under commitment for $b \in [\tilde{b}, \Delta]$. Then, $\beta^c(b) = b - \Delta < 0$, to keep the board indifferent between Cases (ii) and (iii); at the same time, $d\beta^c/db > 0$ and $d\alpha^c/db > 0$. We can show, however, that the function $\bar{\beta}^c(b) \equiv \alpha^c(b)(b - \Delta)$ is convex in b on $[\tilde{b}, \Delta]$ and hence can have only one minimum.

offs, define

$$VoC(b) \equiv EV^c(\alpha^c(b), \beta^c(b) \mid b) - EV^{nc}(\alpha^{nc}(b), \beta^{nc}(b) \mid b)$$

as the *value of board commitment* (to the shareholders). Our last result presents sufficient conditions for predicting the sign of VoC:

Proposition 3 (Value to Shareholders of Board Commitment)

(a) ***High*** q***:*** *If* $q \geq \hat{q}$*, then* $VoC(b) > 0$ *for any* $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$.

(b) ***Low*** q***:*** *If* $q < \hat{q}$*, then* $VoC(b) < 0$ *for any* $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$.

By improving communication, commitment power reduces the board's effort incentives, all else equal. How significant is this opportunity cost of commitment? The answer depends on the information advantage enjoyed by the CEO as captured by his signal precision. For high q the opportunity cost is limited because there is little need for advising. This results in $VoC(b) > 0$, at least for those intermediate CEO bias values for which the optimal board bias is antagonistic under commitment but friendly under noncommitment (part (a)). For low q, the opportunity cost is greater because board effort serves both an advising and monitoring role. As a result, $VoC(b) < 0$ for moderate levels of CEO bias (part (b)). The shareholders strategically utilize the communication handicap under cheap talk as an incentive instrument to elicit board effort (for free).[22]

[22] Both parts of Proposition 3 can be illustrated by simple revealed preference arguments. For the high-q case, suppose the shareholders nominate a (suboptimal) unbiased board, $\beta^c = 0$ for $b \in (\frac{\Delta}{2}, \hat{b}(q))$. This would leave them better off than under noncommitment with $\beta^{nc} = b - \frac{\Delta}{2}$ because they: (i) avoid any loss conditional on successful information gathering; (ii) incur the same loss as under noncommitment conditional on unsuccessful information gathering (the replication result, Lemma 4); and (iii) benefit from greater board effort, holding fixed α at $\alpha^{nc}(b)$. (The proof of Proposition 3 employs a slightly different replication argument.)

On the other hand, for $q < \hat{q}$ and $b \in (\hat{b}(q), \frac{\Delta}{2})$, the shareholders under noncommitment could set $\beta = 0$ (rather than the optimal $\beta^{nc} < 0$) and $\alpha = \alpha^c(b)$ to replicate the commitment outcome. But given the board's greater cost of ignorance, as compared with commitment, the incremental effort to be garnered from an antagonistic board outweighs the associated investment bias cost. (In either case of Proposition 3, establishing *strict* preference for the respective commitment regime is a technical matter.)

This illustrates the importance of, jointly, endogenous board bias and incentive complementarities of equity incentives and communication efficiency.[23]

6 Conclusion

This paper studies the dual role of boards—as a monitor of the firm's top management and as a source of additional information—in a setting with strategic communication. We allow for the shareholders to control (at least partly) the board's preference over an impending decision; referred to as board bias. Communication is modeled rather generally, either with commitment or as cheap talk. All else equal, i.e., for given CEO and board bias, commitment fosters communication between CEO and board. This comes at an opportunity cost of reduced information gathering effort on the part of the board. Factoring in the endogenous nature of board bias changes the picture significantly.

Our analysis has generated a number of surprising result, which often can be traced back to the endogenous nature of the board bias. If the CEO's information is sufficiently precise, then the board may receive a larger equity stake and exert more effort under commitment (Corollaries 2 and 3) because of the complementarity of equity incentives and the (high) cost of ignorance incurred by an antagonistic board in inducing board effort. On the other hand, if the CEO's signal is rather noisy, then the shareholders may benefit from lack of board commitment. By appointing an antagonistic board, the shareholders use the inherent communication disadvantage under cheap talk and the attendant strong information gathering incentives to their advantage (Proposition 3).

The model makes a number of restrictive assumptions. Relaxing some of

[23]Proposition 3 derives clear-cut predictions on $VoC(b)$ for a subset of the parameter space. In the remaining cases, i.e., if either (a) $q \geq \hat{q}$ and $b \in (\hat{b}(q), \Delta)$ or (b) $q < \hat{q}$ and $b \in (\frac{\Delta}{2}, \Delta)$, the shareholders face a trade-off that may favor either of the two commitment regimes. In cases (a) or (b), our earlier results imply for the determinants on the shareholder value as per (7): $\alpha^c < \alpha^{nc}$ (by Corollaries 2 and 2′); $e^c < e^{nc}$ (by Corollaries 3 and 3′); $\bar{L}_S^c < \bar{L}_S^{nc}$; and $L_S^c > L_S^{nc} = 0$. Simulations suggest that this tradeoff can go either way, depending on the CEO's signal precision q and the board's cost of effort, c.

those may be fruitful in future work. The most immediate one relates to the role of the shareholders. In this paper, the shareholders are assumed to be passive in that they only assemble and contract with the board. In many cases, shareholders such as funds are represented on the board of companies the funds invests in. It would be interesting to study the effects of such investor activism on the optimal board bias. For instance, if an activist investor has access to information generated by the board, and the authority to make or influence the investment decision, this will affect the board's information gathering effort as well as the communication subgame with the CEO. Studying this and related issues might require decomposing the board into individual directors, each with their own objective, as in Harris and Raviv (2008) or Malenko (2014).

Appendix A: Proofs

Preliminaries. The proofs below make frequent use of the following constructs:

- Unconditional probability of the CEO observing s: $Pr(s) = \frac{1}{2},\ s = 0, 1$.
- Conditional expectation of the state ω given signal s: $\mathbb{E}[\omega \mid s = 1] = Pr(\omega = 1 \mid s = 1) = q$ and $\mathbb{E}[\omega \mid s = 0] = Pr(\omega = 1 \mid s = 0) = 1 - q$.
- Expected conditional variance of ω given s, or expected posterior information loss: $\Lambda_s = \mathbb{E}_s[Var(\omega \mid s)] = q(1-q) \equiv Q$.
- Ex-ante variance of ω, or prior information loss: $\Lambda_\emptyset = Var(\omega) = \frac{1}{4}$.

Proof of Lemma 1. Without commitment, the communication between the CEO and the board takes the form of cheap talk. Let σ_s represents the probability of the CEO reporting $r = 1$ if he observes signal s. The precision of the CEO 's signal is q, i.e. with probability q, the signal is the same with the state. Anticipating the CEO's optimal reporting strategy, the board's optimal decision based on the CEO report is $\bar{y}^{nc}(r) = \Gamma(r) + \beta$, where:

$$\Gamma(r=1) = Pr(\omega = 1 \mid r = 1) = \frac{q\sigma_1 + (1-q)\sigma_0}{\sigma_0 + \sigma_1}, \tag{9}$$

$$\Gamma(r=0) = Pr(\omega = 1 \mid r = 0) = \frac{q(1-\sigma_1) + (1-q)(1-\sigma_0)}{2 - \sigma_0 - \sigma_1}. \tag{10}$$

Anticipating the board's decision, the CEO upon observing ω chooses his reporting strategy so as to maximize his objective in (4). Without loss of generality, we assume that in any informative equilibrium, $\bar{y}^{nc}(1) > \bar{y}^{nc}(0)$. Then, the first derivative of the CEO's objective in (4) with respect to σ is proportional to:

$$\begin{aligned} \phi(s) &\equiv 2b + 2Pr(\omega = 1 \mid s) - \bar{y}^{nc}(1) - \bar{y}^{nc}(0) \\ &= 2(b - \beta) + 2Pr(\omega = 1 \mid s) - \Gamma(1) - \Gamma(0). \end{aligned}$$

If $\phi(s) > 0$, then $\sigma_s^* = 1$; if $\phi(s) < 0$, then $\sigma_s^* = 0$; if $\phi(s) = 0$, then $\sigma_s^* \in [0, 1]$.

It is easy to show that $\frac{\partial \Gamma}{\partial \sigma_s} > 0$ if $r = s$, and $\frac{\partial \Gamma}{\partial \sigma_s} < 0$ if $r \neq s$. Therefore, $Pr(\omega = 1 \mid s = 0) \leq \Gamma(0) \leq \Gamma(1) \leq Pr(\omega = 1 \mid s = 1)]$, with at least one inequality strict. Then:

$$\begin{aligned} \phi(s=1) &= 2(b-\beta) + 2Pr(\omega = 1 \mid s = 1) - \Gamma(1) - \Gamma(0) > 2(b-\beta), \\ \phi(s=0) &= 2(b-\beta) + 2Pr(\omega = 1 \mid s = 0) - \Gamma(1) - \Gamma(0) < 2(b-\beta). \end{aligned}$$

We consider two cases. First, suppose $b > \beta$. In this case, $\phi(s = 1) > 0$. As a result, $\sigma_1^* = 1$, i.e., the CEO always truthfully reports a high signal. Then, $\Gamma(0) = Pr(\omega = 1 \mid s = 0)$, i.e., a low report perfectly reveals the CEO's private signal. At the same time, $\Gamma(1) \leq Pr(\omega = 1 \mid s = 1)$. Using this, we have

$$\phi(s=0) \geq 2(b-\beta) + Pr(\omega = 1 \mid s = 0) - Pr(\omega = 1 \mid s = 1) = 2\left[(b-\beta) - \frac{\Delta}{2}\right].$$

If $b - \beta > \frac{\Delta}{2}$, then $\phi(s = 0) > 0$, which implies that $\sigma_0^* = 1$. That is, the CEO reports 1 all the time. Thus communication has to be uninformative.

If $b - \beta \leq \frac{\Delta}{2}$, then to show that a truth-telling equilibrium always exists, suppose that $\sigma_0^* = 0$. Then, $\Gamma(1) = Pr(\omega = 1 \mid s = 1)$. Together with $\sigma_1^* = 1$, this implies that

$$\begin{aligned} \phi(s=0) &= 2(b-\beta) + Pr(\omega = 1 \mid s = 0) - Pr(\omega = 1 \mid s = 1) \\ &= 2\left[(b-\beta) - \frac{\Delta}{2}\right] \leq 0. \end{aligned}$$

Therefore the CEO's optimal reporting strategy indeed has $\sigma_0^* = 0$. That is, for $b - \beta \leq \frac{\Delta}{2}$, the truth-telling equilibrium exists. Similar arguments apply if $\beta \geq b$.

To summarize the cheap-talk equilibrium:

(a) For $|b - \beta| \leq \frac{\Delta}{2}$, a truth-telling equilibrium exists in which $\bar{y}^{nc}(r) = \beta + Pr(\omega = 1 \mid s = r)$.

(b) For $|b - \beta| > \frac{\Delta}{2}$, only the babbling equilibrium exists. The board chooses investment according to its prior: $\bar{y}^{nc}(1) = \bar{y}^{nc}(0) = \frac{1}{2} + \beta$. ■

Proof of Lemma 2. With commitment, the uninformed board minimizes its expected loss subject to the CEO's truth-telling constraints.

$$\begin{aligned}\mathcal{SP}^c: \min_{\{y(1),y(0)\}} \quad & \frac{1}{2}q\left(y(1)-1-\beta\right)^2+\frac{1}{2}(1-q)\left(y(1)-\beta\right)^2 \\ + \quad & \frac{1}{2}q\left(y(0)-\beta\right)^2+\frac{1}{2}(1-q)\left(y(0)-1-\beta\right)^2,\end{aligned}$$

subject to:

$$\begin{aligned} & Pr(\omega=1\mid s=1)\left(y(1)-1-b\right)^2+Pr(\omega=0\mid s=1)\left(y(1)-b\right)^2 \\ \leq\ & Pr(\omega=1\mid s=1)\left(y(0)-1-b\right)^2+Pr(\omega=0\mid s=1)\left(y(0)-b\right)^2, \qquad (TT_1) \\ & Pr(\omega=0\mid s=0)\left(y(0)-b\right)^2+Pr(\omega=1\mid s=0)\left(y(0)-1-b\right)^2 \\ \leq\ & Pr(\omega=0\mid s=0)\left(y(1)-b\right)^2+Pr(\omega=1\mid s=0)\left(y(1)-1-b\right)^2. \qquad (TT_0)\end{aligned}$$

We solve the optimization problem in three steps: First, we characterize the optimal separating solution where $y(1)\neq y(0)$; then, the optimal pooling solution where $y(1)=y(0)$; lastly, by comparing the two, we find the global optimum.

Optimal separating solution. Without loss of generality, assume $y(1)>y(0)$. Then (TT_1) and (TT_0) can be reduced to:

$$y(1)+y(0)-2b-2Pr(\omega=1\mid s=1)\ \leq\ 0, \qquad (TT_1')$$

$$y(1)+y(0)-2b-2Pr(\omega=1\mid s=0)\ \geq\ 0, \qquad (TT_o')$$

respectively. Clearly, it cannot be the case that (TT_1') and (TT_0') are both binding. The Lagrangian reads as follows:

$$\begin{aligned}\mathcal{L}= \quad & \frac{1}{2}q\left(y(1)-1-\beta\right)^2+\frac{1}{2}(1-q)\left(y(1)-\beta\right)^2 \\ + \quad & \frac{1}{2}q\left(y(0)-\beta\right)^2+\frac{1}{2}(1-q)\left(y(0)-1-\beta\right)^2 \\ + \quad & \lambda_1\left[y(1)+y(0)-2b-2Pr(\omega=1\mid s=1)\right] \\ + \quad & \lambda_0\left[2b-y(1)-y(0)+2Pr(\omega=1\mid s=0)\right].\end{aligned}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial y(1)} = q(y(1) - 1 - \beta) + (1-q)(y(1) - \beta) + \lambda_1 - \lambda_0 = 0, \tag{11}$$
$$\frac{\partial \mathcal{L}}{\partial y(0)} = q(y(0) - \beta) + (1-q)(y(0) - 1 - \beta) + \lambda_1 - \lambda_0 = 0. \tag{12}$$

As argued above, $\lambda_1\lambda_0 = 0$. By (11) and (12), we get $y(1) = y(0) + (2q - 1)$. To characterize the optimal separating solution, we prove three claims:

Claim 1: (TT_1') is always slack for $b \geq \beta$, and (TT_0') is always slack for $b < \beta$. Suppose (TT_1') is binding, then (TT_0') must be slack, which, by complementary slackness, implies that $\lambda_0 = 0$. Then by the binding (TT_1') constraint, (11), (12) and $\lambda_0 = 0$, we have:

$$\begin{cases} y(1) = b + E[\omega|s=1] + \frac{\Delta}{2}, \\ y(0) = b + E[\omega|s=1] - \frac{\Delta}{2}, \\ \lambda_1 = -(b - \beta + \frac{\Delta}{2}). \end{cases} \tag{13}$$

For $b \geq \beta$, $\lambda_1 = -(b - \beta + \frac{\Delta}{2}) < 0$, a contradiction. Therefore, for $b \geq \beta$, (TT_1') has to be slack.

Similarly, if (TT_0') is binding, then (TT_1') must be slack and $\lambda_1 = 0$. Then, by the binding (TT_0') constraint, (11), (12), and $\lambda_1 = 0$, we have:

$$\begin{cases} y(1) = b + E[\omega|s=0] + \frac{\Delta}{2}, \\ y(0) = b + E[\omega|s=0] - \frac{\Delta}{2}, \\ \lambda_0 = (b - \beta - \frac{\Delta}{2}). \end{cases} \tag{14}$$

Similar arguments prove that (TT_0') has to be slack for $b < \beta$.

Claim 2: If $|b - \beta| < \frac{\Delta}{2}$, then both (TT_1') and (TT_0') are slack. To prove this claim, it suffices to solve a relaxed program that has (TT_0) and (TT_1) removed from $\mathcal{SP}^c$. It easy to verify that the solution to the relaxed program satisfies both truth telling constraints for $|b - \beta| < \frac{\Delta}{2}$.

Claim 3: If $b - \beta \geq \frac{\Delta}{2}$, then (TT_0') is binding; if $b - \beta \leq -\frac{\Delta}{2}$, then (TT_1') is binding. Suppose that (TT_0') were slack for $b - \beta \geq \frac{\Delta}{2}$. Then, by complementary slackness, $\lambda_0 = 0$. At the same time, by Claim 1, for $b - \beta \geq \frac{\Delta}{2}$, (TT_1') is also slack, which implies $\lambda_1 = 0$. Then, by (11) and (12), we get $y(1) = \beta + E[\omega|s = 1]$ and $y(0) = \beta + E[\omega|s = 0]$. Therefore:

$$\begin{aligned} y(1) + y(0) &= 2\beta + E[\omega \mid s = 1] + E[\omega \mid s = 0] \\ &\leq 2b + 2E[\omega \mid s = 0], \end{aligned} \tag{15}$$

where the inequality uses the fact that $b - \beta \geq \frac{\Delta}{2}$. Inequality (15) however contradicts (TT_0') being slack. Hence, (TT_0') is binding for $b - \beta \geq \frac{\Delta}{2}$, calling for investment amounts as in (14). Similar arguments show that (TT_1') is binding for $b - \beta \leq -\frac{\Delta}{2}$, calling for investment amounts as in (13).

To summarize, the optimal separating solution is characterized as follows. Denote by L_B^{sep} the board's value function for $y(1) \neq y(0)$. For $|b - \beta| < \frac{\Delta}{2}$: $y(r) = \beta + E[\omega \mid s = r]$ and $L_B^{sep} = \frac{Q}{2}$. On the other hand, for $|b - \beta| \geq \frac{\Delta}{2}$, by (13) and (14): $y(1) = b + E[\omega \mid s = \mathbb{1}_{\beta > b}] + \frac{\Delta}{2}$, $y(0) = b + E[\omega \mid s = \mathbb{1}_{\beta > b}] - \frac{\Delta}{2}$, and $L_B^{sep} = \frac{Q}{2} + \frac{1}{2}\left(|b - \beta| - \frac{\Delta}{2}\right)^2$.

Optimal pooling solution. Under pooling the board will invest on its prior, i.e., choose $y = \frac{1}{2} + \beta$, resulting in a loss the board of $L_B^{pool} = \frac{1}{8}$.

Compare separating solution and pooling solution. For $|b - \beta| < \frac{\Delta}{2}$, clearly $L_B^{sep} < L_B^{pool}$. For $|b - \beta| \geq \frac{\Delta}{2}$, in contrast:

$$L_B^{sep} - L_B^{pool} = \frac{Q}{2} + \frac{1}{2}\left(|b - \beta| - \frac{\Delta}{2}\right)^2 - \frac{1}{8} \left\{ \begin{array}{c} \leq \\ > \end{array} \right\} 0, \text{ for } |b - \beta| \left\{ \begin{array}{c} \leq \\ > \end{array} \right\} \Delta.$$

We are now ready to characterize the optimal solution for Program $\mathcal{SP}^c$. Denote $\bar{y}^c(r)$ as the optimal solution and $\bar{L}_B^c$ as the board's value function for Program $\mathcal{SP}^c$ (scaled by $\frac{1}{2}$). Then:

Case (i): For $|b - \beta| < \frac{\Delta}{2}$: $\bar{y}^c(r) = \beta + E[\omega \mid s = r]$ and $\bar{L}_B^c = \frac{\Lambda_s}{2} = \frac{Q}{2}$.

Case (ii): For $|b - \beta| \in [\frac{\Delta}{2}, \Delta]$: $\bar{y}^c(1) = b + E[\omega \mid s = \mathbb{1}_{\beta > b}] + \frac{\Delta}{2}$, $\bar{y}^c(0) = b + E[\omega \mid s = \mathbb{1}_{\beta > b}] - \frac{\Delta}{2}$, and $\bar{L}_B^c = \frac{Q}{2} + \frac{1}{2}\left(|b - \beta| - \frac{\Delta}{2}\right)^2$.

Case (iii): For $|b-\beta| > \Delta$: $\quad \bar{y}^c(1) = \bar{y}^c(0) = \frac{1}{2} + \beta$ and $\bar{L}_B^c = \frac{1}{8}$. ■

Proof of Lemma 3. Board commitment affects the outcome only if $|b - \beta| \in [\frac{\Delta}{2}, \Delta]$: commitment then results in constrained communication (Case (ii)), whereas cheap talk results in babbling. Consider first the case that $\beta \leq b$ so that $\beta \in [b-\Delta, b-\frac{\Delta}{2}]$. The differential loss term (see Tables 1 and 2) then is:

$$\begin{aligned} \bar{L}_S^{nc,Case(iii)} - \bar{L}_S^{c,Case(ii)} &= \frac{1}{2}\left(\Lambda_\emptyset + \beta^2 - \left[\Lambda_s + \left(b - \frac{\Delta}{2}\right)^2\right]\right) \\ &\propto \left(\frac{\Delta}{2}\right)^2 + \beta^2 - \left(b - \frac{\Delta}{2}\right)^2 \\ &= \beta^2 + b(\Delta - b) \;\equiv\; d_{\beta\leq b}(\beta, b, \Delta), \end{aligned}$$

using that $\Lambda_\emptyset - \Lambda_s = \frac{1}{4} - Q = \left(\frac{\Delta}{2}\right)^2$. Now, $d_{\beta \leq b}(\cdot)$ is increasing β for $\beta > 0$, and decreasing β for $\beta < 0$. As postulated in the lemma, $b \leq \Delta$, leaving two possibilities. If $b \in [\frac{\Delta}{2}, \Delta]$, then $\beta = 0$ is feasible, bounding $d_{\beta\leq b}(\cdot)$ from below by $b(\Delta - b)$. If $b \in [0, \frac{\Delta}{2}]$, then $\beta < 0$ always, and hence the lower bound on $d_{\beta \leq b}(\cdot)$ is $\lim_{\beta \to b - \frac{\Delta}{2}} d(\cdot) = \left(\frac{\Delta}{2}\right)^2$. In either case, $d_{\beta\leq b}(\cdot)$ is always positive.

If $\beta > b$, then the corresponding differential loss term collapses to a term proportional to $d_{\beta > b}(\beta, b, \Delta) = \beta^2 - b^2$, which is positive. ■

Proof of Lemma 6. For $j \in \{c, nc\}$, the board's expected utility is:

$$EU^j(e^j) \;=\; F + \alpha\left[\frac{1}{2}E_\omega[(\omega+\beta)^2] - \left(1 - e^j\right)\bar{L}_B^j(\beta, b)\right] - \frac{c{e^j}^2}{2}.$$

Even choosing zero effort would allow the board to break even:

$$\begin{aligned} EU^j(e^j) &\geq EU^j(e = 0) \\ &= F + \alpha\left[\frac{1}{2}E_\omega[(\omega+\beta)^2] - \bar{L}_B^j(\beta, b)\right] \\ &= F + \alpha\left[\frac{1}{2}\left(\left(\frac{1}{2} + \beta\right)^2 + \frac{1}{4}\right) - \bar{L}_B^j(\beta, b)\right], \end{aligned}$$

which is positive by $\bar{L}_B^j(\beta, b) \leq \frac{1}{8}$; thus the IR constraint is slack at $F = 0$. ■

Proof of Proposition 1. The shareholders' value is given by (7) with $F = 0$. It is convenient to work with the value function

$$EV^c(\beta \mid b) \equiv EV^c(\alpha^c(\beta, b), \beta \mid b), \tag{16}$$

where $\alpha^c(\beta, b) \in \arg\max_\alpha EV^c(\alpha, \beta \mid b)$. The solution to Program $\mathcal{P}^c$ entails $(\alpha^c(b), \beta^c(b))$ where $\alpha^c(b) = \alpha^c(\beta^c(b), b)$. Define B^k as the set of β to induce communication case $k \in \{i, ii, iii\}$ as in Table 1:[24]

$$\begin{cases} B^i = (b - \frac{\Delta}{2}, b], \\ B^{ii} = [b - \Delta, b - \frac{\Delta}{2}], \\ B^{iii} = (-\infty, b - \Delta). \end{cases}$$

With slight abuse of notation, define $\beta^k(b) \in \arg\max_{\beta \in B^k} \; EV^c(\beta \mid b)$.

The proof for part (a) proceeds in two steps: First we show, in a new Lemma 8, that the shareholders never choose β so as to "jump" across communication cases, i.e., for any b, if case k occurs "naturally" (i.e., for $\beta = 0$), then it is never optimal to set β such that $|b - \beta|$ would induce case $l \neq k$. We then characterize the optimal solution.

Lemma 8 ***(No Jumping Cases)*** *With commitment on the part of the board, the shareholders never choose β so as to switch communication cases. That is:*

- $\beta^c(b < \frac{\Delta}{2}) = \beta_i(b)$,
- $\beta^c(\frac{\Delta}{2} \leq b \leq \Delta) = \beta_{ii}(b)$,
- $\beta^c(b > \Delta) = \beta_{iii}(b)$.

We prove Lemma 8 in the following steps: Step (1)-(4) show that if the shareholders were to choose β to "jump" communication cases, they would choose the adjacent boundary value of β that just suffices to induce such a jump. Formally, we show that if the shareholders want to jump from Case k to l, then the optimal

[24] To avoid clutter we suppress the functional argument b in $B^k(b)$.

way to do so is by setting $\beta = \sup B^l$ if $l > k$, or by setting $\beta = \inf B^l$ if $l < k$. In Steps (5)-(7) we argue that the shareholders never want to jump cases.

Taking derivative of (16), which is differentiable almost everywhere, and applying the Envelope Theorem:

$$\begin{aligned}\frac{dEV^c}{d\beta} &= \frac{\partial EV^c(\alpha^c(\beta,b),\beta \mid b)}{\partial\beta} \\ &= [1-\alpha^c(\beta,b)]\left[-e(\cdot)\frac{\partial L_S}{\partial\beta} - [1-e(\cdot)]\frac{\partial \bar{L}_S^c}{\partial\beta} + \frac{\partial e(\cdot)}{\partial\beta}[\bar{L}_S^c(\beta,b) - L_S(\beta)]\right].\end{aligned}$$

Step 1: If $b \geq \frac{\Delta}{2}$, then $\beta_i(b) = b - \frac{\Delta}{2} + \varepsilon$, where $\varepsilon \to 0$. To prove this claim, note that in Case (i) we have $\frac{\partial L_S}{\partial\beta} = \beta$, $\frac{\partial \bar{L}_S^c}{\partial\beta} = \beta$, and $\frac{\partial \bar{L}_B^c}{\partial\beta} = 0$. Hence:

$$\left.\frac{dEV^c}{d\beta}\right|_{\beta\in B^i} = -[1-\alpha^c(\beta,b)]\beta,$$

which implies $sign(\left.\frac{dEV^c}{d\beta}\right|_{\beta\in B^i}) = -sign(\beta)$. For any $b \geq \frac{\Delta}{2}$ and $\beta \in B^i$, we have $\beta > 0$. Therefore, $\beta_i(b \geq \frac{\Delta}{2}) = b - \frac{\Delta}{2} + \varepsilon$.

Step 2: If $b \leq \Delta$, then $\beta_{iii}(b) = b - \Delta - \varepsilon$, where $\varepsilon \to 0$. Similar arguments as in Step 1 show that $\left.\frac{dEV^c}{d\beta}\right|_{\beta\in B^{iii}} = -[1-\alpha^c(\beta,b)]\beta$. For any $b \leq \Delta$ and $\beta \in B^{iii}$, we have $\beta < 0$; hence, $\beta_{iii}(b \leq \Delta) = b - \Delta - \varepsilon$.

Step 3: If $b > \Delta$, then $\beta_{ii}(b) = b - \Delta$. To prove this claim, note that if the shareholders were to set β to induce Case (ii), then $\beta \in B^{ii} = [b-\Delta, b-\frac{\Delta}{2}]$. Also, $\frac{\partial L_S}{\partial\beta} = \beta$, $\frac{\partial \bar{L}_S^c}{\partial\beta} = 0$, and $\frac{\partial \bar{L}_B^c}{\partial\beta} = -(b-\beta-\frac{\Delta}{2}) \leq 0$. Hence:

$$\left.\frac{dEV^c}{d\beta}\right|_{\beta\in B^{ii}} = [1-\alpha^c(\beta,b)]\cdot[-e(\cdot)\beta + \underbrace{\frac{\partial e}{\partial\beta}}_{\leq 0}(\bar{L}_S^c - L_S)].$$

Note that in Case (ii), $\bar{L}_S^c - L_S = \frac{1}{2}\left[\Lambda_s + (b-\frac{\Delta}{2})^2 - \beta^2\right]$. For any $b > \Delta$ and $\beta \in B^{ii}$, we have $\beta \in (0, b-\frac{\Delta}{2}]$. Hence $\bar{L}_S^c - L_S > 0$, and consequently, $\left.\frac{dEV^c}{d\beta}\right|_{\beta\in B^{ii}} < 0$. As a result, $\beta_{ii}(b > \Delta) = b - \Delta$.

Step 4: If $b < \frac{\Delta}{2}$, then $\beta_{ii}(b) = b - \frac{\Delta}{2} < 0$. Proceeding as in Step 3 shows:

$$\begin{aligned} \left.\frac{dEV^c}{d\beta}\right|_{\beta\in B^{ii}} &= [1-\alpha^c(\beta,b)]\left[-e(\cdot)\beta + \frac{\partial e}{\partial \beta}[\bar{L}_S^c - L_S]\right] \qquad (17) \\ &= -\frac{\alpha^c(\beta,b)[1-\alpha^c(\beta,b)]}{2c}\left[\underbrace{(b-\beta-\frac{\Delta}{2})^2(b-\frac{\Delta}{2}+2\beta)+q(1-q)(b-\frac{\Delta}{2})}_{\equiv g(\beta|b)}\right]. \end{aligned}$$

For any $b < \frac{\Delta}{2}$ and $\beta \in B^{ii}$, we have $\beta \leq b - \frac{\Delta}{2} < 0$. Therefore, $\left.\frac{dEV^c}{d\beta}\right|_{\beta\in B^{ii}} > 0$. As a result, $\beta_{ii} = b - \frac{\Delta}{2}$. (We will use below the $g(\cdot)$ function defined here.)

Step 5: The shareholders will not jump between Cases (i) and (ii); that is, $\beta^c(\frac{\Delta}{2} \leq b \leq \Delta) \neq \beta_i(b)$ and $\beta^c(b < \frac{\Delta}{2}) \neq \beta_{ii}(b)$. To prove this claim, it is readily verified that $EV^c(\cdot)$ is continuous at $\beta = b - \frac{\Delta}{2}$, because both $\bar{L}_S^c$ and $\bar{L}_B^c$ are continuous at $\beta = b - \frac{\Delta}{2}$. Given the continuity of $EV^c(\cdot)$ at $\beta = b - \frac{\Delta}{2}$, it is straightforward that the shareholders will not switch between cases i and ii. As Steps 1 and 4 show, if the shareholders were to do so, they would choose $\beta = b - \frac{\Delta}{2}$, but then they can (at least) replicate such payoff by staying in the original communication case.

Step 6: The shareholders will not jump between Cases (ii) and (iii); that is, $\beta^c(\frac{\Delta}{2} \leq b \leq \Delta) \neq \beta_{iii}(b)$ and $\beta^c(b > \Delta) \neq \beta_{ii}(b)$. It is readily verified that $\bar{L}_B^c$ is continuous at $\beta = b - \Delta$. Denote by $\bar{L}_{S_k}^c$ the shareholders' loss given Case k:

$$\begin{aligned} \bar{L}_{S_{ii}}^c(\beta = b-\Delta, b) - \lim_{\varepsilon\to 0}\bar{L}_{S_{iii}}^c(\beta = b-\Delta-\varepsilon, b) &= \left(b - \frac{\Delta}{2} - \beta\right)\beta \\ &= \frac{\Delta}{2}(b-\Delta). \qquad (18) \end{aligned}$$

If $b > \Delta$, Case (iii) arises naturally, i.e., for $\beta = 0$. The shareholders could jump to Case (ii) by choosing $\beta = b - \Delta$ (Step 3). But doing so would be suboptimal because the term in (18) is positive for $b > \Delta$. Similar arguments show that if $\frac{\Delta}{2} \leq b \leq \Delta$, the shareholders will not jump from Case (ii) to (iii).

Step 7: The shareholders will not jump between Cases (i) and (iii); that is, $\beta^c(b > \Delta) \neq \beta_i(b)$ and $\beta^c(b < \frac{\Delta}{2}) \neq \beta_{iii}(b)$. By Step 2, if the shareholders were to jump from Case (i) to (iii), they would choose $\beta = b - \Delta - \varepsilon$. By Step 4, if $b < \frac{\Delta}{2}$, $\frac{dEV^c}{d\beta}\Big|_{\beta \in B^{ii}} > 0$, so that $EV^c(\beta = b - \Delta \mid b) < EV^c(\beta = b - \frac{\Delta}{2} \mid b)$. Combined with the fact that for $b \leq \frac{\Delta}{2}$, by (18), shows jumping from Case (ii) to (iii) is suboptimal. Reverse arguments show that the shareholders prefer not to jump from Case (iii) to (i), completing the proof of Lemma 8. ■

We now characterize the globally optimal solution. By Lemma 8, for $b < \frac{\Delta}{2}$, the shareholders will choose $\beta^c(b < \frac{\Delta}{2}) = \beta_i(b < \frac{\Delta}{2}) = 0$. The reason is that within Case (i) β does not affect $e^c(\cdot)$ but only introduces bias cost. Similarly, $\beta^c(b > \Delta) = \beta_{iii}(b > \Delta) = 0$.

If $b \in [\frac{\Delta}{2}, \Delta]$, communication Case (ii) arises "naturally" (for $\beta = 0$). By Lemma 8, $\beta^c(\frac{\Delta}{2} \leq b \leq \Delta) = \beta_{ii}(\frac{\Delta}{2} \leq b \leq \Delta)$. Denote by β^{int} the interior solution that satisfies the necessary first-order condition conditional on Case (ii):

$$\frac{dEV^c(\cdot)}{d\beta}\Bigg|_{\beta \in B^{ii}} = 0.$$

Using the $g(\cdot)$ function from (17), β^{int} is given by (ignoring irrelevant scalars):

$$g(\beta^{int} \mid b) \equiv \left(b - \frac{\Delta}{2} - \beta^{int}\right)^2 \left(b - \frac{\Delta}{2} + 2\beta^{int}\right) + q(1-q)\left(b - \frac{\Delta}{2}\right) = 0. \quad (19)$$

By (17), the shareholders' expected payoff is decreasing in β for any $\beta \in B^{ii}$ such that $\beta > 0$; hence, $\beta^{int} \leq 0$ must hold, with strict inequality for $b > \frac{\Delta}{2}$. The second derivative at this stationary point is:

$$\frac{d^2EV^c}{d\beta^2}\Bigg|_{\beta = \beta^{int}} = \frac{3\alpha^c(\cdot)[1-\alpha^c(\cdot)]}{c} \underbrace{\left(b - \beta^{int} - \frac{\Delta}{2}\right)}_{>0,\text{ for Case (ii)}} \beta^{int}, \quad (20)$$

making it impossible for there to be a local minimum for $\beta^{int} \leq 0$. This leaves one of two possibilities (see Figure 3 for illustration): either (a) the (unique) local maximum given by $\beta^{int}(b)$ falls in the interval $\left[b - \Delta, b - \frac{\Delta}{2}\right]$ and thus is feasible so that $\beta_{ii}(b) = b^{int}(b)$, or (b) $\beta^{int}(b) < b - \Delta$ in which case the corner solution $\beta_{ii}(b) = b - \Delta$ obtains. Plugging the corner solution $\beta = b - \Delta$ into the $g(\cdot)$ function in (19) and setting it equal to zero yields the unique CEO bias level, $\tilde{b}$, at which the interior solution becomes infeasible:

$$g\left(\tilde{b}-\Delta \mid \tilde{b}\right)=\frac{3}{4}\Delta^2\left(\tilde{b}-\frac{5}{6}\Delta\right)+\left(\tilde{b}-\frac{\Delta}{2}\right)Q=0 \iff \tilde{b}=\frac{\Delta}{2}\left(1+\frac{2-8Q}{3-8Q}\right).$$

Now note that, as $\lim_{b\downarrow\frac{\Delta}{2}}\beta^{int}(b)=0>\lim_{b\downarrow\frac{\Delta}{2}}b-\Delta$, so the interior solution is feasible and hence optimal at the lower bound of the b-interval $\left[\frac{\Delta}{2},\Delta\right]$. Together with uniqueness of $\tilde{b}$ this implies that $\beta_{ii}(b)=\beta^{int}$ (interior solution) for any $b\in\left[\frac{\Delta}{2},\tilde{b}\right]$, and $\beta_{ii}(b)=b-\Delta$ (corner solution) for any $b\in(\tilde{b},\Delta]$.

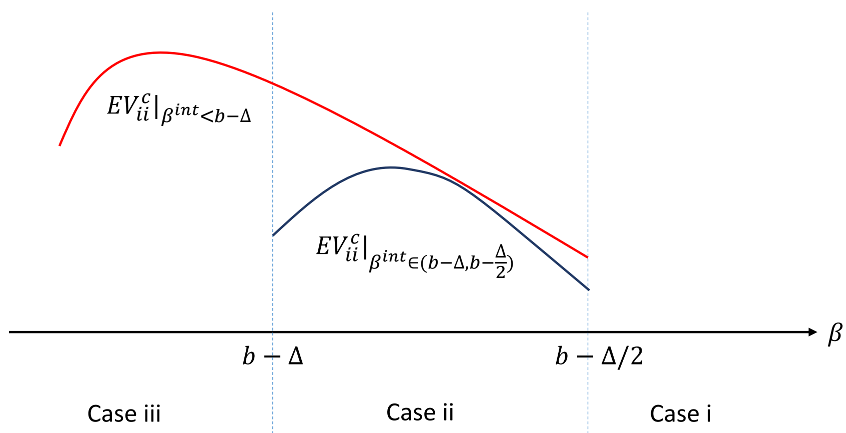

Fig.3: Interior and Corner Solution for β_{ii}

To summarize part (a): the optimal board bias with commitment is:

(1) For $b<\frac{\Delta}{2}$: $\beta^c(b)=0$, implementing Case (i).

(2) For $b\in[\frac{\Delta}{2},\tilde{b}]$: $\beta^c(b)=\beta^{int}\in[b-\Delta,b-\frac{\Delta}{2}]$, where $\tilde{b}=\frac{\Delta}{2}(1+\frac{2-8Q}{3-8Q})$ and β^{int} is determined by (19). This is the interior solution for Case (ii).

(3) For $b\in(\tilde{b},\Delta]$: $\beta^c(b)=b-\Delta$. This is the corner solution for Case (ii).

(4) For $b>\Delta$: $\beta^c(b)=0$, implementing Case (iii).

Continuity of $\beta^c(b)$ follows immediately from $\lim_{\beta\to\Delta/2}\beta^{int}(b)$ and $\lim_{\beta\to\Delta} b-\Delta$. We will prove single-troughedness of $\beta^c(b)$ after part (b), below.

Part (b): The optimal equity stake. Since in Cases (i) and (iii), $\beta^c = 0$ and α^c is constant in b, it remains to show that α^c is monotonically increasing in b in Case (ii). We first show that α^c is monotonically increasing in b for $b \in [\frac{\Delta}{2}, \tilde{b}]$. In this region, the optimal solution (α^c, β^{int}) is an interior one which satisfies the following first-order conditions:

$$\left.\frac{\partial EV^c(\alpha,\beta \mid b)}{\partial\beta}\right|_{\beta^{int}} = 0 \quad \text{and} \quad \left.\frac{\partial EV^c(\alpha,\beta \mid b)}{\partial\alpha}\right|_{\alpha^c} = 0,$$

which, when differentiated with respect to b, yield:

$$\begin{aligned} EV^c_{\alpha\alpha}\cdot\frac{d\alpha^c}{db} + EV^c_{\alpha\beta}\cdot\frac{d\beta^{int}}{db} + EV^c_{\alpha b} &= 0,\\ EV^c_{\beta\beta}\cdot\frac{d\beta^{int}}{db} + EV^c_{\beta\alpha}\cdot\frac{d\alpha^c}{db} + EV^c_{\beta b} &= 0. \end{aligned}$$

Using Cramer's rule,

$$\frac{d\alpha^c}{db} = \frac{EV^c_{\beta b}EV^c_{\alpha\beta} - EV^c_{\alpha b}EV^c_{\beta\beta}}{EV^c_{\alpha\alpha}EV^c_{\beta\beta} - (EV^c_{\alpha\beta})^2} \quad \text{and} \quad \frac{d\beta^{int}}{db} = \frac{-EV^c_{\alpha\alpha}EV^c_{\beta b} + EV^c_{\alpha b}EV^c_{\alpha\beta}}{EV^c_{\alpha\alpha}EV^c_{\beta\beta} - (EV^c_{\alpha\beta})^2}. \tag{21}$$

Clearly,

$$\begin{aligned} EV^c_{\alpha\alpha} &= -\frac{2\left[\bar{L}^c_S(\beta,b) - L_S(\beta)\right]\bar{L}^c_B(\beta,b)}{c} < 0,\\ EV^c_{\beta\beta} &= \frac{\alpha^c(1-\alpha^c)}{c}\left[\underbrace{\frac{b-\frac{\Delta}{2}}{\beta^{int}}}_{-}\underbrace{[\bar{L}^c_S(\beta,b) - L_S(\beta)]}_{+} + 2\underbrace{\left(b - \frac{\Delta}{2} - \beta^{int}\right)}_{+}\underbrace{\beta^{int}}_{-}\right] < 0,\\ EV^c_{\alpha\beta} &= \frac{(1-2\alpha^c)}{c}\cdot\underbrace{\frac{\partial\left[\bar{L}^c_B(\beta,b)[\bar{L}^c_S(\beta,b) - L_S(\beta)]\right]}{\partial\beta}}_{=0,\text{ from F.O.C. in (17)}} = 0. \end{aligned}$$

The derivatives in (21) then reduce to

$$\frac{d\alpha^c}{db} = -\frac{EV^c_{\alpha b}}{EV^c_{\alpha\alpha}} \quad \text{and} \quad \frac{d\beta^{int}}{db} = -\frac{EV^c_{\beta b}}{EV^c_{\beta\beta}}.$$

Now note:

$$
\begin{aligned}
EV^c_{\alpha b} &= (b-\frac{\Delta}{2}) + \frac{1-2\alpha^c}{c}\cdot\frac{\partial\left[\bar{L}^c_B(\beta,b)[\bar{L}^c_S(\beta,b)-L_S(\beta)]\right]}{\partial b} \\
&= (b-\frac{\Delta}{2}) + \frac{1-2\alpha^c}{c}\left\{(b-\frac{\Delta}{2}-\beta^{int})[\bar{L}^c_S(\beta,b)-L_S(\beta)] + (b-\frac{\Delta}{2})\bar{L}^c_B(\beta,b)\right\} \\
&> 0.
\end{aligned}
$$

Thus, $\frac{d\alpha^c}{db} > 0$ for any $b \in (\frac{\Delta}{2}, \tilde{b}]$.

Now consider the case of $b \in (\tilde{b}, \Delta]$, resulting in the corner solution $\beta^c = b - \Delta$:

$$
\begin{aligned}
\alpha^c &= \frac{1}{2} - \frac{\frac{1}{4} - \bar{L}^c_S(\beta,b)}{\frac{2}{c}[\bar{L}^c_S(\beta,b)-L_S(\beta)]\bar{L}^c_B(\beta,b)} \\
&= \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}[\Lambda_s + (b-\frac{\Delta}{2})^2]}{\frac{8}{c}(b\Delta - \frac{3}{4}\Delta^2)}.
\end{aligned}
\tag{22}
$$

Therefore, given $b \in (\tilde{b}, \Delta]$, $\frac{d\alpha^c}{db} > 0$.

It remains to verify the single-troughedness of $\beta^c(b)$. A sufficient condition for this is that the interior solution $\beta^{int}(b)$ is monotonically decreasing in b over the relevant range:

$$
\begin{aligned}
EV^c_{\beta b} &= -\frac{\alpha(1-\alpha)}{c}\left\{[\bar{L}^c_S(\beta,b)-L_S(\beta)] + (b-\frac{\Delta}{2}-\beta)(b-\frac{\Delta}{2}+\beta)\right\} \\
&= -\frac{\alpha(1-\alpha)}{2c}\left[\underbrace{3[(b-\frac{\Delta}{2})^2-\beta^2] + q(1-q)}_{H}\right].
\end{aligned}
$$

To show that $H > 0$, we plug in the first-order condition from (17):

$$
\begin{aligned}
H &= \frac{1}{b-\frac{\Delta}{2}}\left[(b-\frac{\Delta}{2})q(1-q) + 3((b-\frac{\Delta}{2})[(b-\frac{\Delta}{2})^2-\beta^2]\right] \\
&= \frac{1}{b-\frac{\Delta}{2}}\left[-(b-\frac{\Delta}{2}-\beta)(b-\frac{\Delta}{2}+2\beta) + 3((b-\frac{\Delta}{2})[(b-\frac{\Delta}{2})^2-\beta^2]\right] \\
&= \frac{2(b-\frac{\Delta}{2}-\beta)}{b-\frac{\Delta}{2}}\left[[\frac{1}{2}(b-\frac{\Delta}{2})+\beta]^2 + \frac{3}{4}(b-\frac{\Delta}{2})^2\right] \\
&> 0.
\end{aligned}
$$

Thus, $EV^c_{\beta b} < 0$. It follows that $\frac{d\beta^{int}}{db} < 0$, and $\beta^c(b)$ is single-troughed. ∎

Proof of Lemma 7. The shareholders choose (α, β) to maximize their expected utility under noncommitment, which reads:

$$\begin{aligned}
EV^{nc}(\alpha, \beta \mid b) &= (1-\alpha)\left[\frac{1}{4} - e^{nc}(\alpha,\beta,b)L_S(\beta) - (1-e^{nc}(\alpha,\beta,b))\,\bar{L}^{nc}_S(\beta,b)\right] \\
&= (1-\alpha)\left[\frac{1}{4} - e^{nc}(\alpha,\beta,b)\frac{1}{2}\beta^2 - (1-e^{nc}(\alpha,\beta,b))\left(\frac{1}{2}\Lambda_{k(\beta)} + \frac{1}{2}\beta^2\right)\right] \\
&= (1-\alpha)\left[\frac{1}{4} - \frac{1}{2}\beta^2 - \left(1 - \frac{\alpha\bar{L}^{nc}_B(\beta,b)}{c}\right)\left(\frac{1}{2}\Lambda_{k(\beta)}\right)\right] \\
&= (1-\alpha)\left[\frac{1}{4} - \frac{1}{2}\beta^2 - \left(1 - \frac{\alpha\Lambda_{k(\beta)}}{2c}\right)\left(\frac{1}{2}\Lambda_{k(\beta)}\right)\right].
\end{aligned}$$

Here, $k(\beta) \in \{i, iii\}$ represents the communication case, either perfect communication (Case (i), for $|b-\beta| \leq \frac{\Delta}{2}$) or no communication (Case (iii), for $|b-\beta| > \frac{\Delta}{2}$); hence, for given b, $k(\cdot)$ is a function of β. Note that, our expression for $EV^{nc}(\alpha,\beta,b)$ utilizes the facts that: $L_S(\beta) = \frac{1}{2}\beta^2$, $\bar{L}^{nc}_S(\cdot) = \frac{1}{2}\Lambda_{k(\beta)} + \frac{1}{2}\beta^2$, and $\bar{L}^{nc}_B(\cdot) = \frac{1}{2}\Lambda_{k(\beta)}$. By our preliminaries, $\Lambda_i = \Lambda_s = Q$ and $\Lambda_{iii} = \Lambda_\emptyset = \frac{1}{4}$.

Given that $\Lambda_{k(\beta)}$ is discontinuous in β at $\beta = b - \frac{\Delta}{2}$, the global optimization problem boils down to the discrete comparison between the following two programs: communication Case (i), i.e.,

$$EV^{nc}_i(b) = \max_{\alpha\in[0,1],\beta\in[b-\frac{\Delta}{2},b]} (1-\alpha)\left[\frac{1}{4} - \frac{\beta^2}{2} - \left(1 - \frac{\alpha\Lambda_s}{2c}\right)\frac{\Lambda_s}{2}\right], \tag{23}$$

versus Case (iii), i.e.,

$$EV^{nc}_{iii}(b) = \max_{\alpha\in[0,1],\beta\in(-\infty,b-\frac{\Delta}{2})} (1-\alpha)\left[\frac{1}{4} - \frac{\beta^2}{2} - \left(1 - \frac{\alpha\Lambda_\emptyset}{2c}\right)\frac{\Lambda_\emptyset}{2}\right]. \tag{24}$$

To avoid clutter, we omit the parameters q and c in $EV^{nc}_k(\cdot)$. Define $(\alpha_k(b), \beta_k(b))$ as the optimal solution in case $k \in \{i, iii\}$ and define:

$$D(b) \equiv EV^{nc}_i(b) - EV^{nc}_{iii}(b).$$

If $b \leq \frac{\Delta}{2}$, the communication between the CEO and the board is "naturally" truthful (Case (i)), hence $\beta_i(b) = 0$. Also the shareholders would choose $\beta_{iii}(b) = b - \frac{\Delta}{2} - \varepsilon$ if they were to jump to the babbling case, i.e., to Case (iii). Then:

$$\begin{aligned} D\left(b \mid b \leq \frac{\Delta}{2}\right) &\equiv EV_i^{nc}(b) - EV_{iii}^{nc}(b) \\ &= (1-\alpha_i(b))\left[\frac{1}{4} - \left(1 - \frac{\alpha_i(b)\Lambda_s}{2c}\right)\frac{\Lambda_s}{2}\right] \\ &\quad - (1-\alpha_{iii}(b))\left[\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2} - \varepsilon)^2 - \left(1 - \frac{\alpha_{iii}(b)\Lambda_\emptyset}{2c}\right)\frac{\Lambda_\emptyset}{2}\right]. \end{aligned}$$

By the Envelope Theorem, it is easy to see that $D\left(b \mid b \leq \frac{\Delta}{2}\right)$ is monotonically decreasing in b, i.e.,

$$\frac{dD\left(b \mid b \leq \frac{\Delta}{2}\right)}{db} = \frac{\partial D\left(b \mid b \leq \frac{\Delta}{2}\right)}{\partial b} = (1-\alpha_{iii}(b))\left(b - \frac{\Delta}{2} - \varepsilon\right) < 0. \tag{25}$$

Similarly, if $b > \frac{\Delta}{2}$, the communication between the CEO and the board is "naturally" babbling (Case (iii)), hence $\beta_{iii}(b) = 0$. Moreover, the shareholders would choose $\beta_i(b) = b - \frac{\Delta}{2}$ if they wanted to jump to Case (i). Then:

$$\begin{aligned} D\left(b \mid b > \frac{\Delta}{2}\right) &\equiv EV_i^{nc}(b) - EV_{iii}^{nc}(b) \qquad (26) \\ &= (1-\alpha_i(b))\left[\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2})^2 - \left(1 - \frac{\alpha_i(b)\Lambda_s}{2c}\right)\frac{\Lambda_s}{2}\right] \\ &\quad - (1-\alpha_{iii}(b))\left[\frac{1}{4} - \left(1 - \frac{\alpha_{iii}(b)\Lambda_\emptyset}{2c}\right)\frac{\Lambda_\emptyset}{2}\right]. \end{aligned}$$

By the Envelope Theorem:

$$\frac{dD\left(b \mid b > \frac{\Delta}{2}\right)}{db} = \frac{\partial D\left(b \mid b > \frac{\Delta}{2}\right)}{\partial b} = -(1-\alpha_i(b))\left(b - \frac{\Delta}{2}\right) < 0. \tag{27}$$

That is, $D(b)$ is always monotonically decreasing in b. It is easy to see that $D(b)$ is continuous at $b = \frac{\Delta}{2}$. Therefore, there must exist a $\hat{b}(q)$ such that for $b < \hat{b}(q)$, $D(b) > 0$ and the shareholders will choose Case (i). For $b \geq \hat{b}(q)$, $D(b) \leq 0$ and the shareholders will choose Case (iii).

Finally, we need to show that $\hat{b}(q) < \Delta$. Given that $D(b)$ is monotonically decreasing in b, a sufficient condition for that is $D(b = \Delta) < 0$. For that purpose, we first prove that for any $b \leq \Delta$, $\alpha_i(b) < \alpha_{iii}(b) < \frac{1}{2}$, which directly follows from the following expressions:

$$\begin{aligned}
\alpha_i\left(b \mid b > \frac{\Delta}{2}\right) &= \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2})^2 - \frac{\Lambda_s}{2}}{\frac{2}{c}(\frac{\Lambda_s}{2})^2}, \\
\alpha_{iii}\left(b \mid b > \frac{\Delta}{2}\right) &= \frac{1}{2} - \frac{\frac{1}{4} - \frac{\Lambda_\emptyset}{2}}{\frac{2}{c}(\frac{\Lambda_\emptyset}{2})^2}, \\
\alpha_i\left(b \mid b < \frac{\Delta}{2}\right) &= \frac{1}{2} - \frac{\frac{1}{4} - \frac{\Lambda_s}{2}}{\frac{2}{c}(\frac{\Lambda_s}{2})^2}, \\
\alpha_{iii}\left(b \mid b < \frac{\Delta}{2}\right) &= \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2})^2 - \frac{\Lambda_\emptyset}{2}}{\frac{2}{c}(\frac{\Lambda_\emptyset}{2})^2}.
\end{aligned}$$

Therefore, by the Envelope Theorem:

$$\frac{dD(b \leq \Delta)}{dc} = \frac{\partial D(b \leq \Delta)}{\partial c} = -(1 - \alpha_i(\cdot))\alpha_i(\cdot)\frac{\Lambda_s^2}{4c^2} + (1 - \alpha_{iii}(\cdot))\alpha_{iii}(\cdot)\frac{\Lambda_\emptyset^2}{4c^2} > 0.$$

The inequality uses $\alpha_i(b \leq \Delta) < \alpha_{iii}(b \leq \Delta) < \frac{1}{2}$ and $\Lambda_s < \Lambda_\emptyset$. Moreover, note that, by (26), $\lim_{c\to\infty} D(b = \Delta) = 0$. Therefore for all c, $D(b = \Delta) < 0$. ■

Proof of Proposition 2. The proof for the properties of the optimal board bias $\beta^{nc}(b)$ will rely on the functional properties of $\hat{b}(\cdot)$, but it will be more convenient to express $\hat{b}(\cdot)$ as a function of the effort cost parameter c rather than of q. We therefore reintroduce c as a functional argument. Recall that $\hat{b}(c)$ is determined by $D(\hat{b}(c), c) = 0$. That is:

$$EV_i^{nc}(\hat{b}(c), c) - EV_{iii}^{nc}(\hat{b}(c), c) \equiv 0. \tag{28}$$

Taking derivatives with respect to c on (28), we get

$$\underbrace{\left(\frac{dEV_i^{nc}}{db} - \frac{dEV_{iii}^{nc}}{db}\right)}_{<0,\text{ by } \frac{dD(b)}{db}<0}\frac{d\hat{b}}{dc} + \underbrace{\left(\frac{\partial EV_i^{nc}}{\partial c} - \frac{\partial EV_{iii}^{nc}}{\partial c}\right)}_{>0,\text{ by } \frac{dD(b\leq\Delta)}{dc}>0} = 0. \tag{29}$$

Therefore, $\frac{d\hat{b}}{dc} > 0$. Define $\hat{c}$ such that $\hat{b}(\hat{c}) = \frac{\Delta}{2}$. Then for $c < \hat{c}$, $\hat{b}(c) < \frac{\Delta}{2}$ and for $c \geq \hat{c}$, $\hat{b}(c) \geq \frac{\Delta}{2}$.

To solve for $\hat{c}$, we refer back to (28), that is,

$$EV_i^{nc}(\hat{b}(\hat{c}), \hat{c}) - EV_{iii}^{nc}(\hat{b}(\hat{c}), \hat{c}) = 0.$$

Plug in $\hat{b}(\hat{c}) = \frac{\Delta}{2}$ and refer back to the definition of EV_K^{nc}as per (23) and (24):

$$\begin{aligned} EV_i^{nc}(\hat{b}(\hat{c}), \hat{c}) &= (1 - \alpha_i)\left[\frac{1}{4} - \left(1 - \frac{\alpha_i \Lambda_s}{2\hat{c}}\right)\frac{\Lambda_s}{2}\right], \\ EV_{iii}^{nc}(\hat{b}(\hat{c}), \hat{c}) &= (1 - \alpha_{iii})\left[\frac{1}{4} - \left(1 - \frac{\alpha_{iii} \Lambda_\emptyset}{2\hat{c}}\right)\frac{\Lambda_\emptyset}{2}\right]. \end{aligned}$$

Equating $EV_i^{nc}(\hat{b}(\hat{c}), \hat{c})$ with $EV_{iii}^{nc}(\hat{b}(\hat{c}), \hat{c})$, we get $\hat{c} = \frac{q(1-q)}{4}$. It is readily verified that $\hat{c}$ is within the admissible region of c. Finally, by the duality of (q, c) (see Figure 4), there exists a unique $\hat{q}(c) = \frac{1}{2} + \frac{\sqrt{1-16c}}{2}$, such that $\hat{b}(\cdot) < \frac{\Delta}{2}$ for $q < \hat{q}(c)$, and $\hat{b}(\cdot) > \frac{\Delta}{2}$ for $q > \hat{q}(c)$.

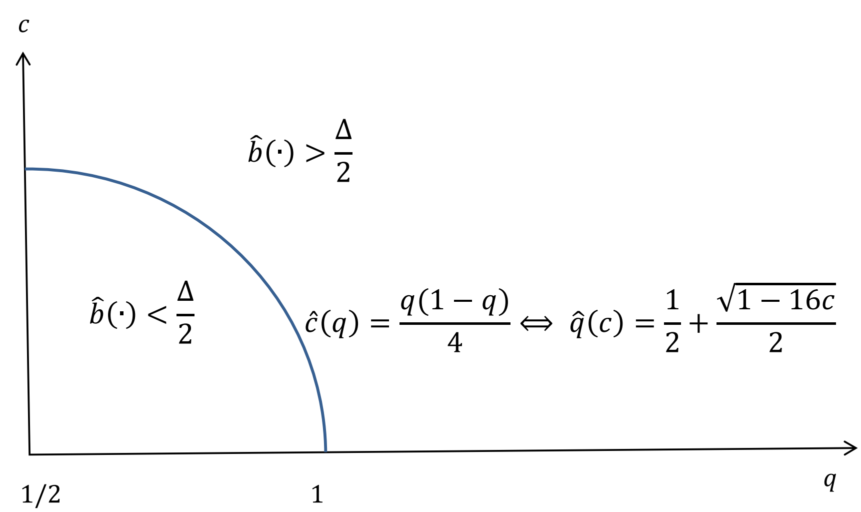

Figure 4: Duality of (q, c)

To prove the properties of $\alpha^{nc}(b)$, we look at the high and low -q cases separately. In the high-q case $(q > \hat{q})$, α^{nc} is constant in b if $b \in [0, \frac{\Delta}{2})$ or $b > \hat{b}(q)$.

We now examine the case where $b \in [\frac{\Delta}{2}, \hat{b}(q))$. In this region, $\beta^{nc} = b - \frac{\Delta}{2} > 0$ and Case (i) is implemented. Hence,

$$\alpha^{nc} = \alpha_i \left(b \mid b > \frac{\Delta}{2} \right) \;=\; \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2})^2 - \frac{\Lambda_s}{2}}{\frac{2}{c}(\frac{\Lambda_s}{2})^2}.$$

Thus, $\frac{d\alpha^{nc}}{db} > 0$ for $b \in [\frac{\Delta}{2}, \hat{b}(q))$. At $\hat{b}(q)$, α^{nc} jumps up because:

$$\lim_{b \to \hat{b}(q)^-} \alpha^{nc}(\cdot) = \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2})^2 - \frac{\Lambda_s}{2}}{\frac{2}{c}(\frac{\Lambda_s}{2})^2} \;<\; \frac{1}{2} - \frac{\frac{1}{4} - \frac{\Lambda_\emptyset}{2}}{\frac{2}{c}(\frac{\Lambda_\emptyset}{2})^2} = \lim_{b \to \hat{b}(q)^+} \alpha^{nc}(\cdot).$$

Similarly, in the low-q case ($q < \hat{q}$), α^{nc} is constant in b if $b \in [0, \hat{b}(q))$ or $b > \frac{\Delta}{2}$. We now examine the case of $b \in [\hat{b}(q), \frac{\Delta}{2})$. In this region, $\beta^{nc} = b - \frac{\Delta}{2} < 0$ and Case (iii) is implemented, so that

$$\alpha^{nc} = \alpha_{iii} \left(b \mid b < \frac{\Delta}{2} \right) \;=\; \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2})^2 - \frac{\Lambda_\emptyset}{2}}{\frac{2}{c}(\frac{\Lambda_\emptyset}{2})^2}.$$

Hence, $\frac{d\alpha^{nc}}{db} < 0$ for $b \in [\hat{b}(q), \frac{\Delta}{2})$. At $\hat{b}(q)$, α^{nc} jumps up because:

$$\lim_{b \to \hat{b}(q)^-} \alpha^{nc}(\cdot) = \frac{1}{2} - \frac{\frac{1}{4} - \frac{\Lambda_s}{2}}{\frac{2}{c}(\frac{\Lambda_s}{2})^2} < \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}(b - \frac{\Delta}{2})^2 - \frac{\Lambda_\emptyset}{2}}{\frac{2}{c}(\frac{\Lambda_\emptyset}{2})^2} = \lim_{b \to \hat{b}(q)^+} \alpha^{nc}(\cdot).$$

■

Proof of Corollary 2. *Part (a)* is proved in Proposition (1) and (2).

For *Part (b)*, to rank α^j across commitment settings, we plug in the optimal board bias levels to get the optimal equity stake. For $b \in (\frac{\Delta}{2}, \hat{b}(q))$, by Lemma 8 and Proposition 2, $\beta^c = \beta_{ii}(b)$ and $\beta^{nc} = b - \frac{\Delta}{2}$. Furthermore, communication case (i) is implemented under non-commitment. Therefore:

$$\begin{aligned}
\alpha^c &= \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}\left[\Lambda_s + (b - \frac{\Delta}{2})^2\right]}{\frac{2}{c}\left[\frac{1}{2}(\Lambda_s + (b - \frac{\Delta}{2} - \beta_{ii})^2)\right] \cdot \left[\frac{1}{2}(\Lambda_s + (b - \frac{\Delta}{2})^2 - \beta_{ii}{}^2)\right]}, \\
\alpha^{nc} &= \frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}\left[\Lambda_s + (b - \frac{\Delta}{2})^2\right]}{\frac{2}{c}\left(\frac{\Lambda_s}{2}\right)^2}.
\end{aligned}$$

Note that:

$$\begin{aligned}
\alpha^c > \alpha^{nc} &\Leftrightarrow \left[\frac{1}{2}(\Lambda_s + (b-\frac{\Delta}{2}-\beta_{ii})^2)\right]\cdot\left[\frac{1}{2}(\Lambda_s + (b-\frac{\Delta}{2})^2 - \beta_{ii}{}^2)\right] > \left(\frac{\Lambda_s}{2}\right)^2 \\
&\Leftrightarrow \underbrace{(b-\frac{\Delta}{2}-\beta_{ii})^2(b-\frac{\Delta}{2}+\beta_{ii}) + 2(b-\frac{\Delta}{2})q(1-q)}_{M(\beta_{ii})} > 0.
\end{aligned}$$

Hence to prove $\alpha^c > \alpha^{nc}$ it is sufficient to show that $M(\beta_{ii}) > 0$. Depending on the ranking of $\hat{b}(q)$ and $\tilde{b}(q)$, β_{ii} may take different values. If $\hat{b}(q) \leq \tilde{b}(q)$, then $\beta_{ii} = \beta^{int}$ for $b \in (\frac{\Delta}{2}, \hat{b}(q))$. Plugging in equation (19), we get:

$$\begin{aligned}
M(\beta_{ii} = \beta^{int}) &= (b-\frac{\Delta}{2}-\beta^{int})^2(b-\frac{\Delta}{2}+\beta^{int}) - 2(b-\frac{\Delta}{2}-\beta^{int})^2(b-\frac{\Delta}{2}+2\beta^{int}) \\
&= -(b-\frac{\Delta}{2}-\beta^{int})^2(b-\frac{\Delta}{2}+3\beta^{int}) \\
&= (b-\frac{\Delta}{2})q(1-q) - (b-\frac{\Delta}{2}-\beta^{int})^2 \underbrace{\beta^{int}}_{-} \\
&> 0.
\end{aligned}$$

If $\hat{b}(q) > \tilde{b}(q)$, then $\beta_{ii} = \beta^{int}$ for $b \in (\frac{\Delta}{2}, \tilde{b}(q))$ and $\beta_{ii} = b - \Delta$ for $b \in [\tilde{b}(q), \hat{b}(q))$. In the case of $\beta_{ii} = \beta^{int}$, same argument as above applies so that $M(\beta_{ii} = \beta^{int}) > 0$. If $\beta_{ii} = b - \Delta$, then

$$M(\beta_{ii} = b-\Delta) = \frac{\Delta^2}{4}(2b - \frac{3}{2}\Delta) + 2(b-\frac{\Delta}{2})q(1-q),$$

which is monotonically increasing in b. Therefore for $b \in [\tilde{b}(q), \hat{b}(q))$, $M(\beta_{ii} = b - \Delta) \geq M(\beta_{ii} = b-\Delta \mid b = \tilde{b}(q)) = M(\beta_{ii} = \beta^{int}) > 0$.

For $b \in (\hat{b}(q), \Delta)$, by Lemma 8 and Proposition 2, $\beta^c = \beta_{ii}(b)$ and $\beta^{nc} = 0$, so that Case (iii) obtains under noncommitment. Therefore, $\alpha^{nc}\left(b \in (\hat{b}(q), \Delta)\right) = \alpha^c(b > \Delta) > \alpha^c\left(b \in (\hat{b}(q), \Delta)\right)$. The last inequality arises from the monotonicity of $\alpha^c(b)$ shown above. ■

Proof of Corollary 3. By Corollary 2, $\alpha^c(b) > \alpha^{nc}(b)$ for $b \in (\frac{\Delta}{2}, \hat{b}(q))$. Combining this with the fact that $\bar{L}_B^c(b, \beta^c) = \frac{1}{2}\left[(b - \frac{\Delta}{2} - \beta_{ii})^2 + \Lambda_s\right] > \frac{1}{2}\Lambda_s = \bar{L}_B^{nc}(b, \beta^{nc})$ verifies that $e^c(\cdot) > e^{nc}(\cdot)$ for $b \in (\frac{\Delta}{2}, \hat{b})$.

For $b \in (\hat{b}(q), \Delta)$, $\alpha^{nc}(b) > \alpha^c(b)$, and $\bar{L}_B^{nc}(b, \beta^{nc}) = \frac{1}{8} \geq \bar{L}_B^c(b, \beta^c)$. Therefore $e^{nc}(\cdot) > e^c(\cdot)$ for $b \in (\hat{b}(q), \Delta)$. ■

Proof of Corollary 2′. *Part (a)* is proved in Propositions 1 and 2. For *Part (b)*, to rank α^j across commitment settings, first note that for $b \notin \left(\hat{b}(q), \Delta\right)$, $\beta^j = 0$ under both commitment settings, and the same communication case is implemented across commitment settings. Hence $\alpha^c(b) = \alpha^{nc}(b)$ for $b \notin \left(\hat{b}(q), \Delta\right)$. For $b \in \left(\hat{b}(q), \Delta\right)$, $\beta^{nc} = 0$ and Case (iii) obtains under noncommitment. Therefore, $\alpha^{nc}\left(b \in (\hat{b}(q), \Delta)\right) = \alpha^c(b > \Delta) > \alpha^c\left(b \in (\hat{b}(q), \Delta)\right)$. The last inequality holds by monotonicity of $\alpha^c(b)$ as per Proposition 1. ■

Proof of Corollary 3′. By Corollary 2′, $\alpha^c(b) \leq \alpha^{nc}(b)$ for any b. Combined with the fact that $\bar{L}_B^c(b, \beta^c) \leq \frac{1}{2}\Lambda_s = \bar{L}_B^{nc}(b, \beta^{nc})$, we have $e^c(\cdot) \leq e^{nc}(\cdot)$. ■

Proof of Corollary 1″. Part (a) is obvious. With noncommitment (part (b)), by Proposition 2, for high-q, both $\beta^{nc}(b)$ and $\alpha^{nc}(b)$ are positive and strictly increasing for any $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$. Therefore, $\bar{\beta}^{nc}(b) = \alpha^{nc}(b) \cdot \beta^{nc}(b)$ is positive and strictly increasing. In the low-q case, for $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$, $\beta^{nc}(b)$ is negative and strictly increasing but $\alpha^{nc}(b)$ is positive and strictly decreasing. Therefore, $\bar{\beta}^{nc}(b)$ is negative and strictly increasing.

With commitment (part (c)), by Proposition 1, for $b \in (\frac{\Delta}{2}, \Delta)$, $\beta^c(b)$ is negative and continuous and $\alpha^c(b)$ is positive and continuous, therefore $\bar{\beta}^c(b) = \alpha^c(b) \cdot \beta^c(b)$ is negative and continuous. Now we prove the single-troughedness property of $\bar{\beta}^c(b)$. Note that, for $b \in (\frac{\Delta}{2}, \tilde{b})$, $\beta^c(b) = \beta^{int}$ is negative and decreasing (the proof of Proposition 1), whereas $\alpha^c(b)$ is positive and increasing, hence $\bar{\beta}^c(b)$ is negative and decreasing for $b \in (\frac{\Delta}{2}, \tilde{b})$. For $b \in (\tilde{b}, \Delta)$, $\beta^c(b) = b - \Delta$ and $\alpha^c(b)$ is as in (22), therefore

$$\bar{\beta}^c(b) = \alpha^c(b) \cdot \beta^c(b) = \alpha^c(b)(b - \Delta) = \left(\frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}\left[\Lambda_s + (b - \frac{\Delta}{2})^2\right]}{\frac{8}{c}(b\Delta - \frac{3}{4}\Delta^2)}\right)(b - \Delta).$$

Take the third derivative of $\bar{\beta}^c$ with respect to b,

$$\frac{d^3\bar{\beta}^c}{db^3} = -\frac{96c(7-32Q)}{(3-4b\Delta-16Q)^4}.$$

That is, $\frac{d^2\bar{\beta}^c}{db^2}$ is monotonic in b for $b \in (\tilde{b}, \Delta)$. Moreover, it is readily verified that

$$\begin{aligned} \left.\frac{d^2\bar{\beta}^c}{db^2}\right|_{b=\Delta} &= 64c\Delta > 0, \\ \left.\frac{d^2\bar{\beta}^c}{db^2}\right|_{b=\tilde{b}} &= 16c\Delta(32\Delta^6 + 44\Delta^4 + 18\Delta^2 + 1) > 0. \end{aligned}$$

Therefore $\frac{d^2\bar{\beta}^c}{db^2} > 0$ for $b \in (\tilde{b}, \Delta)$. Combining with the fact that $\bar{\beta}^c(b)$ is continuous for the entire b region and decreasing for $b \in (\frac{\Delta}{2}, \tilde{b})$, it is then verified that $\bar{\beta}^c$ is single-troughed in b. ■

Proof of Proposition 3.

(a) High q: If $q \geq \hat{q}$, then $\hat{b}(q) \geq \frac{\Delta}{2}$. Also, $\hat{b}(q) < \Delta$. Hence, for any $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$, by Lemma 8, $\beta^c(b) = \beta_{ii}(b)$. That is, with commitment the shareholders prefer to stick with case (ii), choosing the conditionally optimal $\beta_{ii}(b) < 0$, rather than "jump" to case (i) by setting $\beta = \beta^{nc}(b)$. (Note that by Lemma 8, Step 1, the optimal board bias conditional on jumping to case (i) is indeed the corner solution $\beta = b - \frac{\Delta}{2} = \beta^{nc}(b)$.) That is, for any $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$, $EV^c(\alpha^c(b), \beta^c(b) \mid b) \geq EV^c(\alpha^{nc}(b), \beta^{nc}(b) \mid b)$. It remains to show that this inequality holds in a strict sense. For that purpose, note that $\beta^c(b) = \beta_{ii}(b) < b - \frac{\Delta}{2} = \beta^{nc}(b)$, and for the value function $EV^c(\beta \mid b) \equiv \max EV^c(\alpha(\beta, b), \beta \mid b)$, by (17),

$$\left.\frac{dEV^c(\beta \in B^{ii})}{d\beta}\right|_{\beta=b-\frac{\Delta}{2}} = -\frac{\alpha^c(\cdot)[1-\alpha^c(\cdot)]}{2c}\left[q(1-q)(b-\frac{\Delta}{2})\right] < 0.$$

Finally, for $\beta = \beta^{nc}(b)$, the communication between the CEO and the board is Case (i), where the commitment power does not make a difference. That is, $EV^c(\alpha^{nc}(b), \beta^{nc}(b) \mid b) = EV^{nc}(\alpha^{nc}(b), \beta^{nc}(b) \mid b)$. Therefore, *VoC* > 0 for any $b \in \left(\frac{\Delta}{2}, \hat{b}(q)\right)$, given $q \geq \hat{q}$.

(b) Low q: If $q < \hat{q}$, then $\hat{b}(q) < \frac{\Delta}{2}$. Hence, for any $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$, revealed preference argument leads to $EV^{nc}(\alpha^{nc}(b), \beta^{nc}(b) \mid b) \geq EV^{nc}(\alpha^{c}(b), \beta^{c}(b) \mid b)$. It remains to show that this inequality holds in a strict sense. For that purpose, note that for $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$, $EV^{nc}(\alpha^{nc}(b), \beta^{nc}(b) \mid b) = EV^{nc}_{iii}(b)$ and $EV^{nc}(\alpha^{c}(b), \beta^{c}(b) \mid b) = EV^{nc}_{i}(b)$. By Lemma 7, for $b > \hat{b}(q)$, $EV^{nc}_{iii}(b) > EV^{nc}_{i}(b)$. Finally, for any $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$, if $\beta = \beta^c(b) = 0$, the communication between the CEO and the board is Case (i), where the commitment power does not make a difference. That is, $EV^{c}(\alpha^{c}(b), \beta^{c}(b) \mid b) = EV^{nc}(\alpha^{c}(b), \beta^{c}(b) \mid b)$. Therefore, $VoC < 0$ for any $b \in \left(\hat{b}(q), \frac{\Delta}{2}\right)$, given $q < \hat{q}$. ■

Appendix B: Feasible Parameter Range for (c, q)

To ensure the interior optimal α and e, we need to impose the joint parameter restrictions on c and q: We first bound c from above, i.e., $c \leq \bar{c}(q)$, to ensure $\alpha^j(\cdot) \geq 0$. Since dilution concerns are most severe in Case (i), $\alpha^j(\beta) \geq \alpha_i(b) = \frac{1}{2} - \frac{\frac{1}{4} - \frac{Q}{2}}{\frac{2}{c}(\frac{Q}{2})^2} \geq 0$, using (22). Therefore $c \leq \frac{Q^2}{1-2Q} \equiv \bar{c}(q)$.

We now bound c from below, i.e., $c \geq \underline{c}(q)$, to ensure $e^j(\cdot) \leq 1$. For $q \geq \hat{q}$:

$$e^j(\cdot) \leq e^{nc}\left(b \geq \hat{b}\left(q \mid q > \hat{q}\right)\right) = \left(\frac{1}{2} - \frac{\frac{1}{4} - \frac{\Lambda_\emptyset}{2}}{\frac{2}{c}(\frac{\Lambda_\emptyset}{2})^2}\right)\frac{\Lambda_\emptyset}{2c} \leq 1.$$

Therefore $c \geq \frac{1}{24}$. For $q < \hat{q}$, on the other hand:

$$e^j(\cdot) \leq e^{nc}\left(b = \hat{b}\left(q \mid q < \hat{q}\right)\right) = \left(\frac{1}{2} - \frac{\frac{1}{4} - \frac{1}{2}(\hat{b}\left(q \mid q < \hat{q}\right) - \frac{\Delta}{2})^2 - \frac{\Lambda_\emptyset}{2}}{\frac{2}{c}(\frac{\Lambda_\emptyset}{2})^2}\right)\frac{\Lambda_\emptyset}{2c} \leq 1.$$

Plugging in $\hat{b}\left(q \mid q < \hat{q}\right) = \frac{\Delta}{2} - \frac{\Delta\sqrt{2c(Q-4c)}}{4c}$, and the identity $(\frac{\Delta}{2})^2 e \equiv \frac{1}{4} - Q$, we can then derive the lower bound $c \geq \frac{Q-2Q^2}{2+4Q} \equiv \underline{c}(q)$.

Lastly, we bound q from above, i.e., $q < \bar{q}$, to the parameter range of c thus derived is nonempty:

$$\underline{c}(q) < \bar{c}(q) \Leftrightarrow \frac{Q-2Q^2}{2+4Q} < \frac{Q^2}{1-2Q} \Leftrightarrow Q > \frac{1}{6} \Leftrightarrow q < \frac{1}{2} + \frac{\sqrt{3}}{6} \equiv \bar{q} \Leftrightarrow \underline{c}(q) > \frac{1}{24}.$$

Hence the joint parameter restrictions are $\underline{c}(q) \leq c \leq \bar{c}(q)$ and $q < \bar{q}$. ■

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