A Parametric Approach to Counterparty and Credit Risk

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Abstract

In this paper, we present the results of a business solution on how to measure credit and counterparty risk with the main focus on OTC derivatives. Moreover, we use this approach to include the measurement of liquidity risk exposure.

We explain how we measure the exposure for each counterparty with netting arrangements and collaterals. Further we introduce the concept of PFE (potential future exposure) and explain why we opted for a parametric approach. We then develop the concepts of credit loss and default probability as a result of a Poisson process. Further we use the concept of unexpected loss in order to derive the economic capital as the difference between the unexpected loss and the credit loss. Finally we show how this approach can be applied as a refinement of liquidity risk measurement by considering collateral requirements, so as to enhance the monitoring of liquidity congruence between funds' asset and liability, especially under stressed market conditions.

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1 Introduction

In the course of the financial crisis starting in 2008 counterparty risk has become very important for participants on the global financial markets and in particular over-the-counter

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(OTC) derivatives have become instruments of interest since risks cannot be controlled by an exchange. Especially given the huge amount traded in the OTC derivatives market - there were notional amounts of $633 trillion and gross market values of $24.7 trillion outstanding at the end of 2012 (cf. [7]) - governments were driven to a higher regulation of these financial instruments. Apart from the size, regulator's concerns are justified by the fact that from a systemic risk perspective, counterparty risk is quite concentrated and post-Lehman data from the Office of the Comptroller of the Currency (OCC) highlighted that five U.S. banks (JPMorgan, Citi, Bank of America, Goldman Sachs and Morgan Stanley) were the major players in the derivatives market (cf. [30]). Some of these initiatives for a better regulation and monitoring are described in [3], but the most important regulatory requirement is that all OTC derivative contracts have to be reported to trade repositories (cf. [15]). Besides regulatory changes, there has been heavy impetus to trade all OTC derivatives with the help of collateralisation, higher margins, and central counterparties (CCP) as stated in [29].

This paper describes a pragmatic approach to credit and counterparty risk where the first is defined by the Credit Valuation Adjustment (CVA) risk and the second is defined as the risk to each party of a contract that the counterparty will not fulfil its contractual obligations (cf. [24]).[1]Non-fulfillment can be regarded as default or late delivery, or otherwise failure to complete/settle a transaction as previously agreed upon. In fact, as mentioned by the Basel Committee on Banking Supervision "during the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults" (cf. [5]). For this reason the Basel Committee on Banking Supervision has determined that "Banks will be subject to a capital charge for potential mark-to-market losses (i.e. credit valuation adjustment – CVA – risk) associated with a deterioration in the credit worthiness of a counterparty" (cf. [4]) that was not covered by the Basel II standard.

As mentioned by [28], with regards to OTC derivatives, notional amounts i.e. the gross nominal values "provide a measure of the size of the market, but do not provide a measure of risk. Risk in derivatives stems from various other variables including price changes, volatility, leverage and hedge ratios, duration, liquidity, and counterparty risk".

In this paper we will show a pragmatic approach to address most of the factors mentioned above by explaining in detail how to deal with data and risk mitigations as netting, collateral, diversification (through correlation) and CCPs. Starting with data, the availability of it determines the granularity of the calculated risk. In our framework, a position is specified by: counterparty, fund, netting group and instrument. This is because some mitigations as netting applies only within the same netting group at fund level as regulated by netting agreement such as ISDA (or by cross-margining agreement). In addition, risk is further reduced by collateral.

[1]Only contracts which were privately negotiated between counterparties are subject to counterparty risk, hence OTC derivatives constitute the main focus describing counterparty risk. Of course, cash accounts, bonds or other instrumenst that were privately arranged also bear the risk of a defaulting counterparty, but for all other instruments we can apply the upcoming formulas in a simplified way. Therefore we look only at OTC derivatives throughout this paper except for paragraphs where it is necessary to elaborate on the specific instruments, for instance in the investigation on the recovery rate.

The risk is defined for the current exposure as well as for the Potential Future Exposure (PFE) and depends on the probability of default for the counterparty as derived from the company's implied rating (which, in turn, is a measure of counterparty creditworthiness). It is important to note that "strong capital requirements are a necessary condition" "but by themselves are not sufficient" (cf. [4]), hence potential collateral requirement is introduced in order to take into account liquidity risk.[2]

What does a negative or a positive net exposure of a contract mean from a risk perspective for the investor? As described by [28] there is an asymmetry of in-the-money contracts in favor of the defaulting counterparty because clients terminate contracts in their favor but not those where the counterparty is in favor. As mentioned by [28] "Lehman's bankruptcy illustrates the asymmetry between the treatment of derivatives payables and derivatives receivables. Moody's (2008) mentions that when Lehman's default was being considered at the weekend session (September 13-14) at NY Fed, only a limited amount of outstanding derivatives payable trades were re-priced as participants expected significant price changes and operational difficulty in replacing a sizeable volume of trades." In addition it must be noted that the replacement cost for the residual derivatives payable obligations (i.e. the unsecured receivables for those who have lost their counterparty) can be very high (cf. Figure 1).

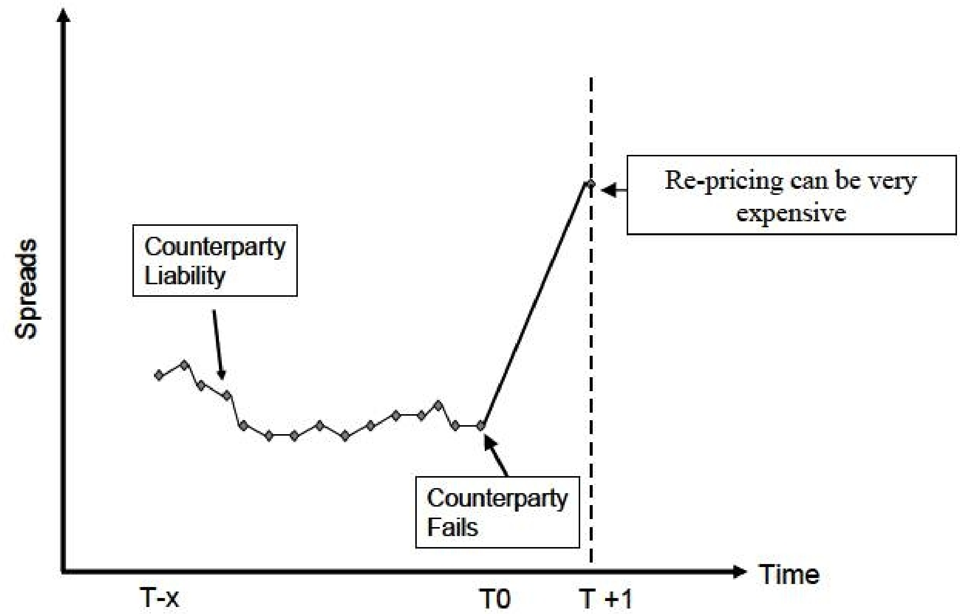

Figure 1: Spread-time combination (Figure 1 of [30]).

[2]During the early "liquidity phase" of the financial crisis, many banks – despite adequate capital levels – still experienced difficulties because they did not manage their liquidity in a prudent manner. The crisis again drove home the importance of liquidity to the proper functioning of financial markets and the banking sector. Prior to the crisis, asset markets were buoyant and funding was readily available at low cost. The rapid reversal in market conditions illustrated how quickly liquidity can evaporate and that illiquidity can last for an extended period of time. The banking system came under severe stress, which necessitated central bank action to support both the functioning of money markets and, in some cases, individual institutions (cf. [4]).

In order to understand the exposure at risk, it is required to measure first the net current exposure with each counterparty. Netting is not only between trades but is netting of collaterals. In this respect, good data is crucial as some credit support annex (CSA)[3] claim collateral on a fund level, whilst others on a position level. In addition to the above, net potential future exposure is calculated in an adverse market scenario. The credit loss is calculated for both current exposure as well as for the potential future exposure and we introduce the concept of unexpected loss, i.e. the deviation of the actual loss as well as we show the influence of CCPs on mitigating counterparty risk.

To measure the risk for an investor, and to avoid a cumbersome computation, a hybrid between Value-at-Risk (VaR), explained in the textbooks [20] and [17], and the add-on is introduced in order to get both the potential future exposure and its corresponding credit loss. This hybrid VaR is calculated for every position and depends on the risk of a specific asset type (e.g. futures/forwards on equity, options on equity, futures/forwards on commodities, swaptions, etc.) and it is used for determining the potential future exposure. As netting can only be done within the same netting group, we need to distinguish by netting group.

In the following table we summarize asset types and netting groups:[4]

Instrument Type	Netting Group
CDS	ISDA
Certificate	CERT
Forward/Future on loans, equity, commodity, swaptions	ISDA
Inflation linked forward/future	ISDA
Foreign exchange forward/future	ISDA
Forward/Future on bonds	GMRA
Forward/Future on mortgage	TBA
Swap	ISDA
Bond swap	ISDA
Not specified option	OTHER
Option on future	TBA
All other options	ISDA
Repurchase agreement	GMRA
Warrant	WARRANT

Table 1: Instrument types and corresponding netting groups.

For other instruments that bear counterparty risk besides OTC derivatives, netting also has to be considered, but unless there are different other agreements, instruments

[3] A CSA is a legal document that regulates the mitigation of credit risk for transactions of derivatives. It is explained in detail in Section 2.5.1 of [12]

[4] The netting groups are: CERT, GMRA, ISDA, TBA, OTHER, and WARRANT. Where CERT stands for "certificates", GMRA for "Global Master Repurchase Agreement", ISDA for "International Swaps and Derivatives Association", TBA for "to be announced"[5] , WARRANT for "warrants" and OTHER for investments that cannot be allocated to the netting groups.

of the same type can be netted against each other, e.g. a bond with any other bond.

The paper is structured as follows. The next section shows how to measure the current exposure. Section 3 describes the development of a potential future exposure and following this, a model is presented to calculate the expected (the credit loss) and unexpected loss for the default of a counterparty. In Section 5 we explain correlations of positions in the valuation of the different counterparty risk figures. The final section addresses the liquidity risk through potential collateral requirements.

2 Current Exposure Measurement

In this section we only focus on the quantification of the current exposure. The current exposure is defined as the amount at risk should the counterparty default now and is normally assumed to be the market value also called the mark-to-market (MtM) value (cf. [19]). Due to the changes of the contract values over time as the market moves, the current exposure independent from its observation level is the only exposure known with certainty at the observation date, whereas all future exposures are uncertain. To measure the current exposure, the granularity of data is crucial. First, we define current exposure at the contract level. Then we derive the exposure at the counterparty level without and with netting arrangements and lastly we show the impact of collateral as a tool for reducing the current exposure. In a general way the current exposure is defined as the net aggregate value[6] of all contract positions minus collateral, where collateral can be posted on contract or netting group or fund level for the respective counterparty depending on CSA.

If a counterparty defaults, we are interested in the Net Replacement Value (NRV) that the affected investor has to deposit. In our setting the NRV of a counterparty k is the current exposure of the investor with counterparty k, denoted by NRV_k. For calculating the NRV of a specific counterparty, we use the following notations[7]:

K	:=	Number of counterparties,
$\mathrm{NRV}_{i,g,f,k}$	:=	Net replacement value of position i in netting group g, fund f and with counterparty k,
$\mathrm{NRV}_{i,f,k}$	:=	Net replacement value of position i in fund f and with counterparty k (no consideration of netting groups),
NRV_k	:=	Aggregated net replacement value of counterparty k,
F_k	:=	Number of funds with positions with counterparty k,
$G_{f,k}$	:=	Number of netting groups in fund f and with counterparty k,
$N_{g,f,k}$	:=	Number of positions in netting group g, fund f and with counterparty k,

[6] The value of a position is the price of it used for calculating the NAV of the account (mandate/ mutual fund/ etc.) to which the position belongs to. In the whole methodology the term "value" is used for describing this value of a position.

[7] All notations are for the current date $t = 0$.

$N_{f,k}$	:=	Number of positions in fund f and with counterparty k (no consideration of netting groups),
$\mathrm{CM}_{i,g,f,k}$	:=	Collateral or margin amount of position i in netting group g, fund f and with counterparty k,[8]
$\mathrm{CM}_{g,f,k}$	:=	Collateral or margin amount of counterparty k for fund f and netting group g,
$\mathrm{CM}_{i,f,k}$	:=	Collateral or margin of position i in fund f and with counterparty k (no consideration of netting groups),
$V_{i,f,k}$	:=	Value of position i in fund f and with counterparty k (no consideration of netting groups),
$V_{i,g,f,k}$	:=	Value of position i in netting group g, fund f and with counterparty k.

As we have to specify every single position by counterparty, fund, netting group and instrument type, we need four indices. Therefore $V_{i,g,f,k}$ is the most granular description of the derivative position's value and $\mathrm{NRV}_{i,g,f,k}$ is the corresponding NRV if all information is available. If netting is not allowed or should not be considered the respective notations are $V_{i,f,k}$ and $\mathrm{NRV}_{i,f,k}$, whereas $\mathrm{Col}_{i,f,k}$ denotes the collateral in this case. Depending on the CSA collateral can be available on contract or netting group level for the counterparty, hence these amounts are denoted by $\mathrm{Col}_{i,g,f,k}$ and $\mathrm{Col}_{g,f,k}$ respectively. As described in Section 2.5 of [12] the bilateral collateral account has to be adjusted mark-to-market: if the mark-to-market NRV of the collateralised position is positive the investor receives collateral payments and if it is negative the investor has to post collateral onto the account. The collateral amount can also be positive or negative in the case of collateral only on netting group level, depending on the fact if the client collateral exceeds the broker collateral in the respective netting group or the other way round. In this document we will mainly consider broker collateral, i.e. collateral the investor receives, which is positive. In the section on potential collateral requirements (Section 6), we consider, also, the client collateral to be posted by the investor (which is negative).

2.1 Exposure at Contract Level

The value of contract i with a counterparty k is known only for the current date ($t = 0$). For any future date t, this value $V_{i,g,f,k}(t)$ is uncertain. Regarding the current date we write $V_{i,g,f,k}(0) := V_{i,g,f,k}$. If the counterparty defaults at time prior to the contract maturity, maximum economic loss equals the replacement cost of the contract:

- If the contract value is positive, the investor does not receive anything from the defaulted counterparty, but has to pay this amount to another counterparty to replace the contract.

[8] Collateral net of haircuts in case of OTC derivatives or margin amount in case of exchange traded and trades processed through central clearing.

- If the contract value is negative, the investor receives this amount from another counterparty, but has to forward it to the defaulted counterparty.

Thus the $\mathrm{NRV}_{i,g,f,k}$ on contract-level at time t is:

$$\mathrm{NRV}_{i,g,f,k}(t) = \max(V_{i,g,f,k}(t)\,, 0)\ .$$

Hence, today the $\mathrm{NRV}_{i,g,f,k}$ on contract-level is:

$$\mathrm{NRV}_{i,g,f,k} := \mathrm{NRV}_{i,g,f,k}(0) = \max(V_{i,g,f,k}, 0)\ . \tag{1}$$

2.2 Exposure at Counterparty Level

The NRV_k on counterparty-level at future time t can be defined as the loss if the counterparty k defaults at time t under assumption of no recovery. If counterparty risk is not mitigated in any way, i.e. there are no netting and collateral agreements, the counterparty-level NRV at time t equals the sum of contract-level exposures:

$$\mathrm{NRV}_k(t) = \sum_{f=1}^{F_k} \sum_{i=1}^{N_{f,k}} \mathrm{NRV}_{i,f,k}(t) = \sum_{f=1}^{F_k} \sum_{i=1}^{N_{f,k}} \max(V_{i,f,k}(t)\,, 0)\ . \tag{2}$$

Let us look at the current date $t = 0$, then the NRV of counterparty k is:

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \sum_{i=1}^{N_{f,k}} \mathrm{NRV}_{i,f,k} = \sum_{f=1}^{F_k} \sum_{i=1}^{N_{f,k}} \max(V_{i,f,k}, 0)\ . \tag{3}$$

If there are netting agreements between the considered counterparties, then the exposure at time t is given by the net positive value:

$$\mathrm{NRV}_k(t) = \sum_{f=1}^{F_k} \sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k}(t)\,, 0 \right)\ . \tag{4}$$

Then, for the current date, we get:

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k}, 0 \right)\ . \tag{5}$$

In case of several specific netting agreements with one counterparty, where not all trades have to be covered by netting agreements, we denote the m-th netting agreement NA_m. Then, the counterparty NRV with and without netting agreements at time t is given by (cf. equation (4) of [24]):

$$\mathrm{NRV}_k(t) = \sum_{f=1}^{F_k} \left[\sum_{m=1}^{G_{f,k}} \max\left(\sum_{i \in \mathrm{NA}_m} V_{i,m,f,k}(t)\,, 0 \right) + \sum_{i \notin \{\mathrm{NA}\}} \max\left(V_{i,f,k}(t)\,, 0\right) \right]\ , \tag{6}$$

where $\{\mathrm{NA}\}$ denotes the set of all netting agreements and $V_{i,m,f,k}$ is the value of position i in the netting group m of fund f with counterparty k. Regarding $t = 0$, it is:

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \left[\sum_{m=1}^{G_{f,k}} \max\left(\sum_{i \in \mathrm{NA}_m} V_{i,m,f,k}, 0 \right) + \sum_{i \notin \{\mathrm{NA}\}} \max\left(V_{i,f,k}, 0\right) \right] . \tag{7}$$

2.3 Exposure with Collateral Payments

Now, we want to measure the current exposure including collateral payments. In this subsection let us always consider $t = 0$ since the transfer to any other date t is possible without any problems. Supposed the counterparty posted collateral for every specific position and every position is encompassed by a netting agreement, the NRV of counterparty k is:

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k} - \mathrm{Col}_{i,g,f,k}, 0 \right) . \tag{8}$$

In case that not every position is covered by a netting agreement, we get the following formula for the NRV of counterparty k:

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \left[\sum_{m=1}^{G_{f,k}} \max\left(\sum_{i \in \mathrm{NA}_m} V_{i,m,f,k} - \mathrm{Col}_{i,m,f,k}, 0 \right) + \sum_{i \notin \{\mathrm{NA}\}} \max\left(V_{i,f,k} - \mathrm{Col}_{i,f,k}, 0\right) \right] , \tag{9}$$

where $\mathrm{Col}_{i,m,f,k}$ is the collateral amount corresponding to the contract of position i of netting agreement m in fund f with counterparty k. Now, we consider that collateral is posted on a netting group level. Hence, the NRV of counterparty k is:

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k}, 0 \right) - \mathrm{Col}_{g,f,k}, 0 \right] . \tag{10}$$

First, we add the positions which belong to the same netting group, are contained in the same fund, and belong to the same counterparty. If this sum is positive we continue with summing over all netting groups of the same fund belonging to the same counterparty and subtracting the respective collateral. If the figure, after subtraction, is still positive, this value represents the exposure of the fund with counterparty k. The sum of all funds is the NRV of counterparty k.

Now, let us suppose that not every position is covered by a netting agreement and the collateral is posted on a netting group level, then the NRV of counterparty k is:

$$\mathrm{NRV}_k = \sum_{f=1}^{F_k} \max\left[\sum_{m=1}^{G_{f,k}} \max\left(\sum_{i \in \mathrm{NA}_m} V_{i,m,f,k}, 0 \right) - \mathrm{Col}_{m,f,k} + \sum_{i \notin \{\mathrm{NA}\}} \max\left(V_{i,f,k}, 0\right), 0 \right] , \tag{11}$$

where $\mathrm{Col}_{m,f,k}$ denotes the collateral amount of the netting agreement m.

With netting agreements and collateral payments the current exposure is defined as the NRV over all counterparties k using equation (10):

$$\mathrm{NRV} = \sum_{k=1}^{K} \mathrm{NRV}_k \ .$$

Note, that in the whole document, every time collateral amounts are considered, the collateral amount after a certain haircut is meant. This haircut is regulated by the respective CSA and depends on the intsrument type, the instrument issuer and the maturity of the collateral.

3 Potential Future Exposure

The calculation of potential future exposure is much more complex compared to the current exposure. The following two methods are the most widely used for quantification of potential future exposure:

- Regulatory CEM/SM (Current Exposure Method/Standardised Method).
- Full revaluation/Monte-Carlo simulation.

The choice between these two methodologies is driven by the trade-off between accuracy, time, budget and level of calculation complexity. Therefore we will explain a third approach in detail since, in our opinion, it is an efficient trade-off. Figure 2 illustrates the idea of the potential future exposure that is described in the subsequent sections. The shaded area ilustrates that only positive outcomes of position values are counted to the relevant exposure.

3.1 Regulatory Methodology

The regulatory CEM (Current Exposure Method) was proposed by the Bank for International Settlements in 1988 (cf. [8]) and amended in 1996 (cf. [9]), whereas this amendment was updated in 2005, in order to assess the volatility of future exposure change. This was a big step forward with respect to the exposure measurement based on percentage of notional based on original maturity and it has been considered as a straightforward method to asses the potential future exposure in a meaningful way. The Basel Committee of Banking Supervision (BCBS) stated that "under the Current Exposure Method, banks must calculate the current replacement cost by marking contracts to market, thus capturing the current exposure without any need for estimation, and then adding a factor (the "add-on") to reflect the potential future exposure over the remaining life of the contract"; hence, "in order to calculate the credit equivalent amount of these instruments under this current exposure method, a bank would sum the replacement cost" ([10], p. 274), i.e. the "marking to market" of all its contracts with positive value, plus an add-on amount.

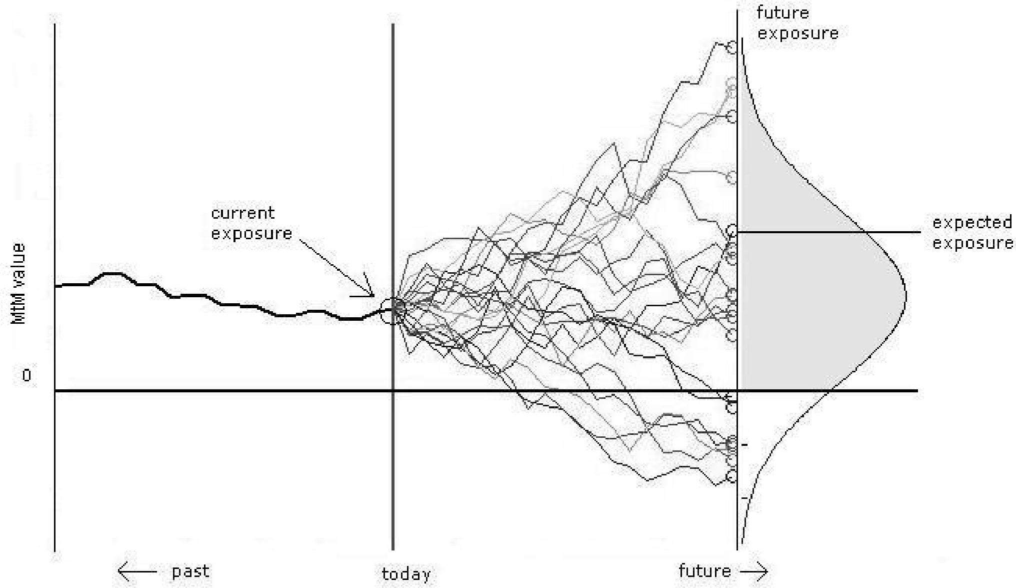

Figure 2: Simulated paths.

The table below shows the percentage, i.e. the Credit Conversion Factor (CCF), to be multiplied by the notional of a contract in order to estimate the potential future exposure as suggested by the BCBS (cf. [10]).

CCF/Add-on Matrix					
Residual Maturity	Interest Rate	Exchange Rate and Gold	Equity	Precious Metals (Except Gold)	Other Commoditites
<1Y	0.0%	1.0%	6.0%	7.0%	10.0%
1-5Y	0.5%	5.0%	8.0%	7.0%	12.0%
>5Y	1.5%	7.5%	10.0%	8.0%	15.0%

Table 2: Add-on matrix ([10], p. 274).

The add-on amount, which is supposed to capture the potential future exposure, is defined as:

$$\text{Add-onAmount} = \text{CCF} * \text{Notional}$$

In other terms the Exposure At Default (EAD) is defined as

$$\text{EAD} = \text{RC} * \text{Add-onAmount} - \text{C}_{adj},$$

where

RC	=	Replacement costs, i.e. MtM or variation margin to be paid by the Clearing Member in case of CCPs/Exchange derivatives,
C_{adj}	=	Collateral net of haircut adjusted by its volatility.

In case of netting the regulator allows for a discount (cf. p. 275 of [2]) on the add-on amount as follows: the add-on for netted transactions (Add-on$_{net}$) will equal the weighted average of the gross add-on (Add-on$_{gr}$) and the gross add-on adjusted by the ratio of net current replacement cost to gross current replacement cost (NGR). In formula:

$$\text{Add-on}_{net} = 0.4 * \text{Add-on}_{gr} + 0.6 * \text{NGR} * \text{Add-on}_{gr}$$

with

Add-on$_{gr}$	=	Sum of individual add-on amounts (calculated by multiplying the notional principal amount by the appropriate add-on factors) of all transactions subject to legally enforceable netting agreements with one counterparty,
NGR	=	Level of net replacement cost/level of gross replacement cost for transactions subject to legally enforceable netting agreements.

3.2 From the Add-on factors to the Parametric Approach

The CEM approach[9] has been widely criticised as (cf. [6]):

- It does not differentiate between margined and unmargined transactions;
- It does not sufficiently capture the level of volatilities as observed over the recent stress periods;
- The recognition of hedging and netting benefits through the Net-to-Gross-Ratio is too simplistic and does not reflect economically meaningful relationships between the derivative positions.

Our methodology, instead, is to interpret the add-on as the maximum credit exposure, i.e. the VaR with a certain confidence level (in our case we set it to 99.5%). For this purpose we assume arbitrary values for annual volatility (abbreviated as vola in the table) and delta for different underlyings as listed in Table 3[10].

For interest rate/fixed income derivatives the VaR factor increases with maturity. The BCBS proposes to distinguish between the following maturity buckets: up to 1 year, 1 to 5 years, and more than 5 years (cf. [10], p. 272). Table 4 states some assumptions for the level of increase. Under normal distribution assumption[11] for collateralised positions

[9]This method is under review by the regulator and a non-internal model method (NIMM) has been recently proposed in June 2013.

[10]Values based on historical estimates. Assumptions on volatility, correlations and other parameters can be set by the user who has to define a customized (potentially stressed) scenario.

[11]For sake of simplicity, we assume that the distribution is normal (a more refined approach, e.g. the Cornish–Fisher expansion, can be used in case the skewness and kurtosis are calculated).

we calculate the weekly[12] VaR:

$$\text{VaR}_{weekly} = \alpha \cdot \frac{\text{Vol}_{annual}}{\sqrt{52}} \cdot \text{Delta} \cdot T \ .$$

Assumptions for the extended VaR Matrix										
	Equity		Commodity		FI / IR		Credit		FX	
	Futures or Forwards	Options	Futures or Forwards	Options	Futures or Forwards or Swaps	Options or Swap-tions	Futures or Forwards or Swaps	Options or Swap-tions	Futures or Forwards	Options
Vola	30%	30%	30%	30%	5%	5%	40%	40%	10%	10%
Delta	1.00	0.50	1.00	0.50	1.00	0.50	1.00	0.50	1.00	0.50

Table 3: Arbitrary volatility and delta values.

Assumptions for the extended VaR Matrix			
Time to Maturity	$<$ 1 year	1 - 5 years	$>$ 5 years
Time Factor	1	3.5	10

Table 4: Time factor for interest rate and fixed income derivatives.

For uncollateralised positions we calculate the fortnightly VaR since we assume that it will take one week longer to liquidate the position in comparison to a collateralised position:

$$\text{VaR}_{fortnightly} = \alpha \cdot \frac{\text{Vol}_{annual}}{\sqrt{26}} \cdot \text{Delta} \cdot T \ .$$

where $\alpha := 2.3263$ (one sided 99.5% confidence level assuming a standard normal distributon) and $T :=$ Time Factor.

For example, the VaR factor for an uncollateralised interest rate swap with time to maturity in 4 years is 8% because of:

$$\text{VaR}_{fortnightly} = 2.3263 \cdot \frac{0.05}{\sqrt{26}} \cdot 1.00 \cdot 3.5 = 0.08 \ .$$

After calculating the fortnightly VaR for all different instrument types, underlyings and time to maturities, we have the following table for the VaR factors that are used in the calculation of the VaR of the different uncollateralised positions. The following two tables show the weekly and fortnightly VaR (for all different instrument types, underlyings

[12] The reporting frequency in many companies involved at capital markets is weekly as they would like to understand the potential future exposure in one week's time i.e. the time required for liquidating the positions. For uncollateralised positions the time required for liquidating a position is two weeks.

and time to maturities) depending on whether the positions are collateralised or uncollateralised. Of course these figures depend on the figures from the add-on matrix.

VaR Factors - Weekly						
Instrument Type	Instrument Subtype	Underlying	Netting Group	< 1Y	1Y - 5Y	> 5Y
Certificate		CTY	CERT	13.7%	13.7%	13.7%
Certificate		EQ	CERT	13.7%	13.7%	13.7%
Certificate		IR	CERT	2.3%	8.0%	22.8%
Fwd/Future		CR	ISDA	2.3%	8.0%	22.8%
Fwd/Future		CTY	ISDA	13.7%	13.7%	13.7%
Fwd/Future		EQ	ISDA	13.7%	13.7%	13.7%
Fwd/Future		FX	ISDA	4.6%	4.6%	4.6%
Fwd/Future	Bond Fwd	IR	GMRA	2.3%	8.0%	22.8%
Fwd/Future	TBA	IR	TBA	2.3%	8.0%	22.8%
Fwd/Future		IR	ISDA	2.3%	8.0%	22.8%
Fwd/Repo		IR	GMRA	2.3%	8.0%	22.8%
Option	Option	CR	ISDA	8.0%	28.2%	80.5%
Option	Warrant	CTY	WARRANT	13.7%	13.7%	13.7%
Option	Option	EQ	ISDA	6.8%	6.8%	6.8%
Option	Option	EQ	OTHER	6.8%	6.8%	6.8%
Option	Warrant	EQ	WARRANT	6.8%	6.8%	6.8%
Option	Option	FX	ISDA	2.3%	2.3%	2.3%
Option	Warrant	FX	WARRANT	2.3%	2.3%	2.3%
Option	Option	IR	ISDA	2.3%	8.0%	22.8%
Option	TBA	IR	TBA	2.3%	8.0%	22.8%
Option	Warrant	IR	WARRANT	2.3%	8.0%	22.8%
Option/Swap	Swaption	CR	ISDA	8.0%	28.2%	80.5%
Option/Swap	Swaption	IR	ISDA	2.3%	8.0%	22.8%
Swap		EQ	ISDA	13.7%	13.7%	13.7%
Swap		FX	ISDA	4.6%	4.6%	4.6%
Swap		IR	ISDA	2.3%	8.0%	22.8%
Swap/CDS		CR	ISDA	2.3%	8.0%	22.8%

Table 5: Weekly VaR factors.

VaR Factors - Fortnightly						
Instrument Type	Instrument Subtype	Underlying	Netting Group	< 1Y	1Y - 5Y	> 5Y
Certificate		CTY	CERT	9.7%	9.7%	9.7%
Certificate		EQ	CERT	9.7%	9.7%	9.7%
Certificate		IR	CERT	1.6%	5.6%	16.1%

Fwd/Future		CR	ISDA	1.6%	5.6%	16.1%
Fwd/Future		CTY	ISDA	9.7%	9.7%	9.7%
Fwd/Future		EQ	ISDA	9.7%	9.7%	9.7%
Fwd/Future		FX	ISDA	3.2%	3.2%	3.2%
Fwd/Future	Bond Fwd	IR	GMRA	1.6%	5.6%	16.1%
Fwd/Future	TBA	IR	TBA	1.6%	5.6%	16.1%
Fwd/Future		IR	ISDA	1.6%	5.6%	16.1%
Fwd/Repo		IR	GMRA	1.6%	5.6%	16.1%
Option	Option	CR	ISDA	8.0%	22.6%	64.5%
Option	Warrant	CTY	WARRANT	13.7%	13.7%	13.7%
Option	Option	EQ	ISDA	4.8%	4.8%	4.8%
Option	Option	EQ	OTHER	4.8%	4.8%	4.8%
Option	Warrant	EQ	WARRANT	4.8%	4.8%	4.8%
Option	Option	FX	ISDA	1.6%	1.6%	1.6%
Option	Warrant	FX	WARRANT	1.6%	1.6%	1.6%
Option	Option	IR	ISDA	1.6%	5.6%	16.1%
Option	TBA	IR	TBA	1.6%	5.6%	16.1%
Option	Warrant	IR	WARRANT	1.6%	5.6%	16.1%
Option/Swap	Swaption	CR	ISDA	8.0%	22.6%	64.5%
Option/Swap	Swaption	IR	ISDA	1.6%	5.6%	16.1%
Swap		EQ	ISDA	9.7%	9.7%	9.7%
Swap		FX	ISDA	3.2%	3.2%	3.2%
Swap		IR	ISDA	1.6%	5.6%	16.1%
Swap/CDS		CR	ISDA	1.6%	5.6%	16.1%

Table 6: Fortnightly VaR factors.

Let us take an example for calculating the weekly VaR factor for an uncollateralised interest rate swap with notional of € 100 million and a residual maturity of 4 years. The add-on amount is:

$$\text{VaR} = €100,000,000 \cdot 0.08 = €8,000,000$$

So, for this interest rate swap we have to add € 8,000,000.

3.3 Potential Future Exposure

The potential NRV of a counterparty k is the future exposure of the investor with counterparty k, denoted by NRV-VaR$_k$ where VaR stands for the amount that is added.
For calculating the potential future NRV of a specific counterparty, we use the notations stated at the beginning of Section 2. Additionally, if we take into consideration the VaR (i.e. the so-called add-on factor) of all positions, the following additional notations are

required:

$\text{VaRFactor}_{i,g}$	:=	VaR factor of position i in netting group g
$\text{VaR}_{i,g,f,k}$	:=	VaR of position i in netting group g, fund f and with counterparty k,
$\text{Not}_{i,g,f,k}$	:=	Notional of position i in netting group g, fund f and with counterparty k,
NRV-VaR_k	:=	Aggregated net replacement value including add-on of counterparty k.

Each add-on factor refers to the specific instrument in each netting group. This factor is independent of the fund and counterparty. As the add-on amount is calculated on position level, four indices in the formula of the VaR and the notional are needed. To calculate the add-on amount the following formula is used:

$$\text{VaR}_{i,g,f,k} = \text{Not}_{i,g,f,k} \cdot \text{VaRFactor}_{i,g} \ ,$$

where the VaR factor depends on the investment type. As above mentioned the BCBS suggested fixed add-on amounts for the different types of investments, depending on the tenor and type of investment (cf. [10]). As we want to distinguish the add-on factor by derivative type and underlying we require more information.

This add-on amount is used for computing the NRV with add-ons as shown in the formula below.

$$\text{NRV-VaR}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k} + \text{VaR}_{i,g,f,k}, 0\right) - \text{Col}_{g,f,k}, 0\right] \ . \tag{12}$$

4 Credit Loss

This chapter provides explanations for calculating the so-called credit loss (CL) in OTC counterparty risk reporting. There are two different concepts:

- The CL which is the expected loss for the current exposure with the respective counterparty. In this case the CL is based on the NRV_k of a counterparty k.
- The expected loss in a worst-case scenario based on the current exposure with the respective counterparty. In this case the CL is based on the NRV-VaR_k of a counterparty k.

In both cases we need the probability of default of the respective counterparty which we derive from its rating. To calculate this, we take the market implied rating (MIR) because it reflects more promptly the variation of the expected loss according to the markets than the stationary ratings. Furthermore, the loss given default (LGD)[13] is needed, defined as one minus the recovery rate.

[13] The LGD is often called "severity".

4.1 Probability of Default

In our model, the credit event is the first event of a Poisson counting process whose important characteristics we present in the following (a detailed analysis of the Poisson process can be found in Chapter I, Section 3 of [23]). A Poisson process is a stochastic process in which events occur continuously and independently of one another. Mathematically, the Poisson process is described by the so-called couning process $(N_t)_{t\geq 0}$ where N_t gives us the number of arrivals of the event that have occured in the interval $(0, t]$. t is a variable for the time, whereby 0 stands for the observation time, i.e. today. For mathematical reasons it is important to include the end points[14], so we take $(a, b] := \{t \geq 0 \mid a < t \leq b\}$. N_0 by definition is 0 with probability 1, which means that we are considering only arrivals at strictly positive times. This makes sense because we know if a counterparty is defaulted at the time of observation. If we want to know the number of arrivals between the times a and b, we have to calculate $N_b - N_a$. The Poisson process has three important characteristics as follows:

- *Poisson process is a pure birth process:* In an infinitesimal time interval dt only one arrival of the event may occur. This happens with the probability $\lambda\, dt$ independently of arrivals outside the interval. λ is called the intensity of the Poisson process.

- *Poisson process follows a Poisson distribution:* The number of arrivals N_t in a finite interval of length t follows the Poisson distribution, i.e.

$$\mathbb{P}(N_t = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \ .$$

  Moreover, the number of arrivals $N_{t_2} - N_{t_1}$ and $N_{t_4} - N_{t_3}$ in non-overlapping intervals $(t_1 \leq t_2 \leq t_3 \leq t_4)$ are independent.

- *Interarrival times follow an exponential distribution:* The interarrival times are independent and follow the exponential distribution with intensity λ, i.e.

$$\mathbb{P}(\text{interarrival time} > t) = e^{-\lambda t} \ .$$

For calculating the intensity of the Poisson process, we refer to Moody's[15] and display the historic default probabilities for companies of different ratings in Table 7. According to this matrix we have nine different intensities for every rating. For our purposes we need the intensity for the time scale of one week. To be conservative, the 1-year intensity is taken as basis and then it is scaled to one week. After calculating the intensity, we can compute the probability of default for a time horizon of one week. This is the time horizon we are interested in.

[14] As the Poisson process must be right-continuous.

[15] For the default rates of different time horizons the values of Exhibit 36 of [22] are used. The time horizons are from one year to ten years.

Rating\Year	1	2	3	4	5	6	7	8	9
Aaa	0%	0.016%	0.016%	0.048%	0.086%	0.132%	0.182%	0.186%	0.186%
Aa1	0%	0%	0%	0.094%	0.141%	0.158%	0.158%	0.158%	0.158%
Aa2	0%	0.014%	0.088%	0.192%	0.334%	0.431%	0.495%	0.561%	0.635%
Aa3	0.048%	0.127%	0.182%	0.254%	0.327%	0.38%	0.415%	0.435%	0.444%
A1	0.061%	0.217%	0.461%	0.672%	0.855%	0.992%	1.089%	1.18%	1.297%
A2	0.065%	0.174%	0.351%	0.578%	0.788%	1.073%	1.463%	1.918%	2.324%
A3	0.058%	0.214%	0.439%	0.627%	0.923%	1.235%	1.55%	1.907%	2.255%
Baa1	0.146%	0.38%	0.643%	0.897%	1.204%	1.501%	1.801%	2.016%	2.207%
Baa2	0.176%	0.486%	0.897%	1.519%	2.078%	2.686%	3.226%	3.723%	4.337%
Baa3	0.302%	0.876%	1.558%	2.219%	3.099%	3.993%	4.84%	5.847%	6.79%
Ba1	0.709%	1.986%	3.721%	5.546%	7.226%	9.012%	10.455%	11.506%	12.537%
Ba2	0.8%	2.286%	4.198%	6.249%	8.077%	9.538%	10.953%	12.522%	13.97%
Ba3	1.826%	5.291%	9.371%	13.66%	17.163%	20.379%	23.342%	26.385%	29.384%
B1	2.512%	6.969%	11.678%	15.866%	20.159%	24.509%	29.191%	33.167%	36.694%
B2	3.986%	9.863%	15.713%	21.076%	25.701%	29.903%	33.774%	37.281%	40.724%
B3	7.584%	16.097%	24.299%	31.262%	37.199%	42.764%	47.117%	51.039%	53.727%
Caa1	9.94%	21.715%	32.211%	40.782%	48.782%	54.343%	57.144%	60.441%	65.864%
Caa2	19.045%	30.446%	39.104%	46.371%	51.475%	55.336%	58.498%	61.973%	65.153%
Caa3	29.542%	45.41%	54.642%	61.612%	67.565%	69.136%	71.854%	75.593%	80.516%
Ca	38.739%	50.58%	59.678%	66.353%	71.652%	73.385%	75.92%	78.884%	78.884%
C	38.739%	50.58%	59.678%	66.353%	71.652%	73.385%	75.92%	78.884%	78.884%

Table 7: Probability of default matrix.

For further calculations we need a day-count convention. The day-count convention is for the time measurement between two dates T and t.[16] Let us take *Actual/365*, i.e. a year has 365 days, and the day-count convention for $T-t$ is given by

$$\frac{\text{actual number of days between } t \text{ and } T}{365} \; .$$

The formula for the intensity of the Poisson process is as follows:

$$\begin{aligned} p &:= \mathbb{P}(N_t > 0) = \mathbb{P}(N_t \geq 1) \\ &= 1 - \mathbb{P}(N_t = 0) = 1 - \frac{(\lambda_t t)^0}{0!} e^{-\lambda_t t} = 1 - e^{-\lambda_t t} \; . \end{aligned}$$

Therefore:

$$\lambda = -\frac{1}{t} \log(1-p) \; .$$

[16]For two different times $0 \leq s \leq t$ the difference $t-s$ is measured in years, cf. [21] for a complete discussion of day-count conventions.

Let us take an example. We want to calculate the probability of default for a B1 rated company and a time horizon of one week. The row of the matrix is as follows:

Rating\Year	1	2	3	4	5	6	7	8	9
B1	2.512%	6.969%	11.678%	15.866%	20.159%	24.509%	29.191%	33.167%	36.694%

Table 8: Probability of default matrix (for B1 rating).

Then we can calculate the intensity of the process taking the default probability of one year ($t = \frac{365}{365} = 1$):

$$\lambda(1 \text{ year}, B1) = -\frac{1}{1}\log(1 - 0.02512) = 0.02544.$$

After obtaining this intensity for the Poisson process, we can calculate the default probability of a B1 rated company in a week's time as follows:

$$p_{B1} \quad := \quad \mathbb{P}\Big(N_{\frac{7}{365}} > 0\Big) = 1 - e^{-\lambda(1 \text{ year}, B1)t} = 1 - e^{-0.02544 \cdot \frac{7}{365}} = 0.000488.$$

The whole calculation shown for a B1 rated company can be done for all other ratings resulting in the following:

- $\lambda(1 \text{ week}, Aaa) = 0.0$, $p_{Aaa} = 0.0$.
- $\lambda(1 \text{ week}, Aa1) = 0.0$, $p_{Aa1} = 0.0$.
- $\lambda(1 \text{ week}, Aa2) = 0.0$, $p_{Aa2} = 0.0$.
- $\lambda(1 \text{ week}, Aa3) = 0.000209$, $p_{Aa3} = 0.000004$.
- $\lambda(1 \text{ week}, A1) = 0.000265$, $p_{A1} = 0.0000051$.
- $\lambda(1 \text{ week}, A2) = 0.000282$, $p_{A2} = 0.0000054$.
- $\lambda(1 \text{ week}, A3) = 0.000252$, $p_{A3} = 0.0000048$.
- $\lambda(1 \text{ week}, Baa1) = 0.000635$, $p_{Baa1} = 0.0000122$.
- $\lambda(1 \text{ week}, Baa2) = 0.000765$, $p_{Baa2} = 0.0000147$.
- $\lambda(1 \text{ week}, Baa3) = 0.001314$, $p_{Baa3} = 0.0000252$.
- $\lambda(1 \text{ week}, Ba1) = 0.00309$, $p_{Ba1} = 0.0000593$.
- $\lambda(1 \text{ week}, Ba2) = 0.003488$, $p_{Ba2} = 0.0000669$.
- $\lambda(1 \text{ week}, Ba3) = 0.008004$, $p_{Ba3} = 0.0001535$.
- $\lambda(1 \text{ week}, B1) = 0.011049$, $p_{B1} = 0.0002119$.

- $\lambda(1 \text{ week}, B2) = 0.017666$, $p_{B2} = 0.0003387$.
- $\lambda(1 \text{ week}, B3) = 0.034252$, $p_{B3} = 0.00065669$.
- $\lambda(1 \text{ week}, Caa1) = 0.045468$, $p_{Caa1} = 0.0008716$.
- $\lambda(1 \text{ week}, Caa2) = 0.091756$, $p_{Caa2} = 0.0017582$.
- $\lambda(1 \text{ week}, Caa3) = 0.152069$, $p_{Caa3} = 0.0029122$.
- $\lambda(1 \text{ week}, Ca) = 0.212816$, $p_{Ca} = 0.0040731$.
- $\lambda(1 \text{ week}, C) = 0.212816$, $p_{C} = 0.0040731$.

To calculate the final probability of default for a certain rating class for a time horizon of one week, the transition probabilities from one rating class to another have to be incorporated. Table 1 of [26] shows the average one-year transition rates between 1975 and 2010 in percent:

	Aaa	Aa	A	Baa	Ba	B	Caa -C	NR
Aaa	94.2%	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	2.2%
Aa	1.7%	93.3%	2.1%	0.0%	0.0%	0.0%	0.0%	2.9%
A	0.0%	3.0%	91.7%	1.4%	0.0%	0.0%	0.0%	4.0%
Baa	0.0%	0.0%	3.8%	86.4%	3.2%	0.0%	0.0%	6.6%
Ba	0.0%	0.0%	0.0%	2.3%	88.9%	1.1%	1.5%	6.1%
B	0.0%	0.0%	0.0%	0.0%	12.8%	75.4%	3.8%	8.1%
Caa -C	0.0%	0.0%	0.0%	0.0%	0.0%	32.4%	62.1%	5.4%

Table 9: Rating transition matrix (for one year).

With similar calculations as above, we get a transition matrix for the time horizon of one week:

	Aaa	Aa	A	Baa	Ba	B	Caa -C	NR
Aaa	97.69%	1.48%	0.0%	0.0%	0.0%	0.0%	0.0%	0.83%
Aa	0.56%	97.78%	0.70%	0.0%	0.0%	0.0%	0.0%	0.96%
A	0.0%	0.73%	97.94%	0.35%	0.0%	0.0%	0.0%	0.98%
Baa	0.0%	0.0%	0.46%	98.35%	0.39%	0.0%	0.0%	0.80%
Ba	0.0%	0.0%	0.0%	0.38%	98.19%	0.18%	0.25%	1.0%
B	0.0%	0.0%	0.0%	0.0%	0.6%	98.84%	0.18%	0.38%
Caa -C	0.0%	0.0%	0.0%	0.0%	0.0%	17.32%	80.41%	2.27%

Table 10: Rating transition matrix (for one week).

Let $T_{i,j}$ denote the i,j-th entry of the transition matrix, e.g. $T_{Aaa,Aa} = 0.0148$. Then we calculate the probability of default for all different rating classes. For instance, we get for a B1 rated company:

$$\mathbb{P}_{B1} = p_{B1} + p_{Aaa}T_{B,Aaa} + \cdots + p_C T_{B,C}\,. \tag{13}$$

We still have to solve the problem that the transition probabilities are not on a very granular level. For example it is only known that the probability of switching from rating Aaa to Aa is about 1.5%, but the probability to switch from Aaa to Aa2 is not known. In order to calculate (13), we calculate p_B as expected value in a rudimentary way:

$$p_B = \frac{1}{3}p_{B1} + \frac{1}{3}p_{B2} + \frac{1}{3}p_{B3}\,. \tag{14}$$

Calculation (14) is done analogously for all other rating classes and this yields:

- $p_{Aaa} = 0.0$.
- $p_{Aa} = 0.000001$.
- $p_A = 0.000005$.
- $p_{Baa} = 0.000017$.
- $p_{Ba} = 0.000093$.
- $p_B = 0.00041$.
- $p_{Caa} = 0.00190$.
- $p_{Ca} = 0.00407$.
- $p_C = 0.00407$.

With these figures we can now calculate the final probabilites of default according to equation (13):

- $\mathbb{P}_{Aaa} = 0.0$, $\mathbb{P}_{Aa1} = 0.000001$, $\mathbb{P}_{Aa2} = 0.000001$, $\mathbb{P}_{Aa3} = 0.000005$.
- $\mathbb{P}_{A1} = 0.0000101$, $\mathbb{P}_{A2} = 0.0000104$, $\mathbb{P}_{A3} = 0.0000098$.[17]
- $\mathbb{P}_{Baa1} = 0.0000293$, $\mathbb{P}_{Baa2} = 0.0000318$, $\mathbb{P}_{Baa3} = 0.0000423$.
- $\mathbb{P}_{Ba1} = 0.0001616$, $\mathbb{P}_{Ba2} = 0.0001692$, $\mathbb{P}_{Ba3} = 0.0002558$.
- $\mathbb{P}_{B1} = 0.000625$, $\mathbb{P}_{B2} = 0.0007518$, $\mathbb{P}_{B3} = 0.0010699$.
- $\mathbb{P}_{Caa1} = 0.0042153$, $\mathbb{P}_{Caa2} = 0.0051019$, $\mathbb{P}_{Caa3} = 0.0062559$.
- $\mathbb{P}_{Ca} = 0.0074168$.

[17] One could argue that $\mathbb{P}_{A3}$ should be higher than $\mathbb{P}_{A2}$. This counterintuitive case is due to empirical data. Exhibit 36 of [22] is taken as basis and the empirical data yielded a lower p_{A3} than p_{A2}.

- $\mathbb{P}_C = 0.0074168$.

If a company is not rated we assumed the rating Baa2. For the purpose of OTC counterparty risk reporting, the values of derivatives are not affected by a change after the change in the rating class. A possible improvement could be to link a rating transition to price variation[18].

4.2 Credit Loss in Counterparty Reporting

In this section we use the calculations shown in the previous sections to obtain the credit loss in the OTC Counterparty Reporting. After calculating the probability of default, the LGD[19] is needed for the credit loss calculation. The recovery rate (RR) depends on the priority of claim. With the help of [14] we distinguish between the following priorities of investments[20]:

Instrument Type	RR
Cash	0.4
Covered Bonds	0.95
Covered Loans	0.95
Government Guarenteed Bonds	0.6
Loans	0.6
Senior Secured Bonds	0.6
Senior Unsecured Bonds	0.3
Subordinated Bonds (Senior & Junior)	0.2
Derivatives	0.2

Table 11: Recovery rates for different instrument types.

Therefore the LGD is 0.6, 0.05, 0.4, 0.7 or 0.8 respectively. The LGD is dependent on the specific instrument. For instance, if instrument i is a derivative, we get:

$$\mathrm{LGD}_i = 1 - \mathrm{RR}_i = 1 - 0.2 = 0.8 \ .$$

For purposes of OTC counterparty risk reporting, we are only interested in derivatives. All other possible instrument types listed in the table above are not relevant for this particular reporting but if the reporting is extended to other asset classes, this table can be consulted. Then, the credit loss (CL) is calculated as follows:

$$\mathrm{CL}_{i,g,f,k} = V_{i,g,f,k} \cdot \mathrm{LGD}_i \cdot p_k \ ,$$

[18] The change in price is discussed in [11].

[19] As the RR is normally seen as a random variable, the LGD is also a random variable (LGD = 1 - RR). When the expected loss is calculated, the expected value of the LGD is needed. That is a number between 0 and 1, not a random variable anymore, often called LD (loss on default). We take the LGD as a fixed number equal to the expected values.

[20] The recovery rate figures are average numbers derived from Exhibit 25 and 26 of [14].

where p_k denotes the probability of default for counterparty k with its specific rating, i.e. if counterparty k has market implied rating Aa1, we have: $p_k = \mathbb{P}_{Aa1}$. The credit loss is calculated for the current exposure (CL^{curr}) as well as for the future exposure in a worst case scenario (CL^{future}). That means we have the following two equations for the repective credit loss of a position i in netting group g in fund f with counterparty k:

- $$\mathrm{CL}^{curr}_{i,g,f,k} = V_{i,g,f,k} \cdot \mathrm{LGD}_i \cdot p_k \quad . \tag{15}$$

- $$\mathrm{CL}^{future}_{i,g,f,k} = (V_{i,g,f,k} + \mathrm{VaR}_{i,g,f,k}) \cdot \mathrm{LGD}_i \cdot p_k \quad . \tag{16}$$

The add-on amount is calculated as described in Chapter 3:

$$\mathrm{VaR}_{i,g,f,k} = \mathrm{Not}_{i,g,f,k} \cdot \mathrm{VaRFactor}_{i,g} \quad .$$

First the credit loss is calculated for every investment regardless of netting class, fund or counterparty as described above using formulas (15) and (16). Then it is aggregated by netting class, fund and counterparty. Afterwards, it is aggregated by counterparty. Finally, we have the following formulas for the credit loss based on the current exposure respectively based on the future exposure for a counterparty k. Logically these formulas are similar to equations (10) and (12).

- $$\mathrm{CL}^{curr}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{CL}^{curr}_{i,g,f,k}, 0\right) - \mathrm{Col}_{g,f,k}, 0\right] \quad .$$

- $$\mathrm{CL}^{future}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{CL}^{future}_{i,g,f,k}, 0\right) - \mathrm{Col}_{g,f,k}, 0\right] \quad .$$

Now it is possible to calculate the credit loss for the investor for OTC derivatives based on the current exposure as well as the credit loss for a future worst-case based on the potential future exposure:

- $$\mathrm{CL}^{curr} = \sum_{k=1}^{K} \mathrm{CL}^{curr}_k \quad , \tag{17}$$

- $$\mathrm{CL}^{future} = \sum_{k=1}^{K} \mathrm{CL}^{future}_k \quad , \tag{18}$$

where, as mentioned before, K is the number of different counterparties.

4.3 Central Counterparty

"Central Counterparties (CCPs) are designed to mitigate counterparty risk through multi-lateral netting, high levels of collateralisation as well as loss mutualisation" ([1], p. 1). Due to the financial crisis of 2008 there has been a concerted regulatory drive to substantially increase the proportion of derivatives that are centrally cleared. The primary motivation is to reduce bilateral counterparty risk, increase transparency and avoid contagion in the case of the default of a large financial institution. For derivatives cleared through a CCP, we use the LGD shown in the table below (c.f. [1] on page 15) that is dependent upon the underlying.

	Equity	Commodity[21]	Fixed Income / IR	Foreign Exchange	Credit
LGD	10.3%	10.3%	11.5%	17.4%	6.9%

Table 12: LGD of CCP-cleared derivatives according to their respective underlying.

Note that the used probability of default for a CCP-cleared position depends on the assumed rating of the CCP.

4.4 Unexpected Loss

Besides the credit loss, we are interested in the unexpected loss, i.e. the deviation of the actual loss. Chapter 5 of [27] provides a detailed analysis of the unexpected loss that we also used for this section, where we describe briefly the concept of the unexpected loss and its application in counterparty risk measurement. Figure 3 illustrates the difference between expected and unexpected loss.

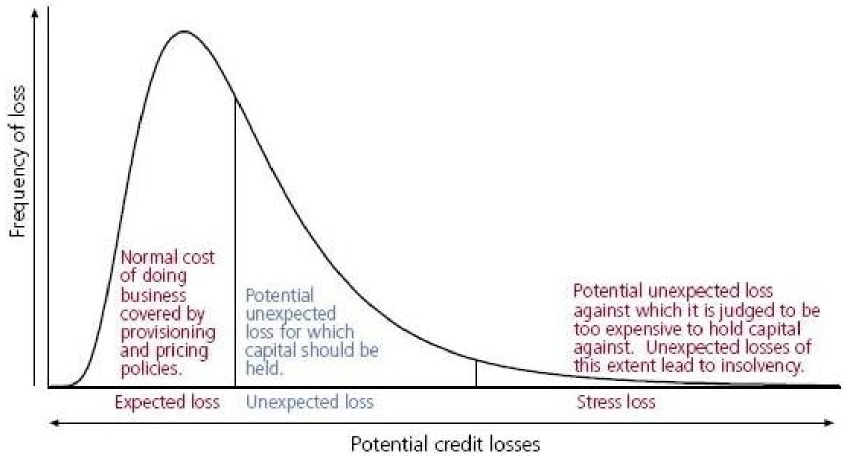

Figure 3: Potential credit losses (Figure of [31]).

[21] There is no figure for the LGD of derivatives with underlying commodity stated in [1], so we take the same as for underlying equity because of similarities in the volatility levels of these underlyings.

First, we have to define the actual loss of a position i, denoted by $\mathrm{L}_{i,g,f,k}$:

$$\mathrm{L}_{i,g,f,k} = V_{i,g,f,k} \cdot \mathrm{LGD}_i \cdot 1_{\{X_k=1\}} \tag{19}$$

with X_k denoting the default process for the counterparty k, i.e. starting with a probability space Ω. The space $\{X_k = 1\} = \{\omega \in \Omega : X_k(\omega) = 1\}$ denotes all default states and $\{X_k = 0\} = \{\omega \in \Omega : X_k(\omega) = 0\}$ is the set of all non-default states. At time $t = 0$ we know the state $\omega \in \Omega$ which is realised, i.e. the actual loss of a position i in $t = 0$ is either 0 or $V_{i,g,f,k} \cdot \mathrm{LGD}_i$.

The credit loss for the current exposure defined in Section 4.2 by equation (15) is the expected value of equation (19), whereas the potential deviation from the expected loss, i.e. from the credit loss that the investor can expect to incur, is quantified in terms of the standard deviation of the actual loss. This standard deviation is called unexpected loss and is calculated by taking the variance of $\mathrm{L}_{i,g,f,k}$. Therefore we need the following formula derived from the classical variance formula:

$$\mathbb{E}\left[1_{\{X_k=1\}}\right] = \mathbb{E}\left[1^2_{\{X_k=1\}}\right] = p_k^2 + \mathbb{V}(X_k) \ . \tag{20}$$

Let us denote the standard deviation of X_k by σ_{X_k}, i.e. $\mathbb{V}(X_k) = \sigma^2_{X_k}$.

$$\begin{aligned}
\mathbb{V}(\mathrm{L}_{i,g,f,k}) &= \mathbb{E}\left[\mathrm{L}^2_{i,g,f,k}\right] - \mathbb{E}[\mathrm{L}_{i,g,f,k}]^2 = \mathbb{E}\left[\mathrm{L}^2_{i,g,f,k}\right] - \left(\mathrm{CL}^{curr}_{i,g,f,k}\right)^2 \\
&\overset{(19)}{=} V^2_{i,g,f,k} \cdot \mathrm{LGD}_i^2 \cdot \mathbb{E}\left[1^2_{\{X_k=1\}}\right] - V_{i,g,f,k} \cdot \mathrm{LGD}_i \cdot p_k^2 \\
&\overset{(20)}{=} V^2_{i,g,f,k} \cdot \mathrm{LGD}_i^2 \cdot \sigma^2_{X_k} \ .
\end{aligned} \tag{21}$$

As X_k is a Bernoulli random variable, the variance is given by:

$$\sigma^2_{X_k} = p_k \cdot (1 - p_k) \ . \tag{22}$$

Finally, using equation (22) we can calculate the unexpected loss for the current exposure of a position i in netting group g in fund f of counterparty k, denoted by $\mathrm{UL}^{curr}_{i,g,f,k}$:

$$\mathrm{UL}^{curr}_{i,g,f,k} := \sqrt{\mathbb{V}(\mathrm{L}_{i,g,f,k})} = \sqrt{V^2_{i,g,f,k} \cdot \mathrm{LGD}_i^2 \cdot p_k \cdot (1 - p_k)} \ . \tag{23}$$

According to our calculations in Chapter 3 we calculate an add-on amount which is added to the current value of position i:

$$\mathrm{VaR}_{i,g,f,k} = \mathrm{Not}_{i,g,f,k} \cdot \mathrm{VaRFactor}_{i,g} \ .$$

This add-on amout is used for calculating the unexpected loss of the future exposure:

$$\mathrm{UL}^{future}_{i,g,f,k} = \sqrt{(V_{i,g,f,k} + \mathrm{VaR}_{i,g,f,k})^2 \cdot \mathrm{LGD}_i^2 \cdot p_k \cdot (1 - p_k)} \ . \tag{24}$$

First the unexpected loss is calculated using formulas (23) and (24) for every investment regardless of netting class, fund or counterparty. Then it is aggregated by netting class, fund and counterparty level. Afterwards, it is aggregated for the respective counterparties. Finally, we have the following formulas for the credit loss based on the current exposure based on the future exposure for a counterparty k respectively. Obviously these formulas are similar to equations (10) and (12).

- $$\mathrm{UL}_k^{curr} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{UL}_{i,g,f,k}^{curr}, 0\right) - \mathrm{Col}_{g,f,k}, 0\right] .$$

- $$\mathrm{UL}_k^{future} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{UL}_{i,g,f,k}^{future}, 0\right) - \mathrm{Col}_{g,f,k}, 0\right] .$$

Now it is possible to calculate the unexpected loss for the investor for OTC derivatives based on the current exposure, as well as the unexpected loss for a future worst case based on the potential future exposure:

- $$\mathrm{UL}^{curr} = \sum_{k=1}^{K} \mathrm{UL}_k^{curr} , \tag{25}$$

- $$\mathrm{UL}^{future} = \sum_{k=1}^{K} \mathrm{UL}_k^{future} , \tag{26}$$

where, as mentioned before, K is the number of different counterparties.

4.5 Economic Capital

In line of this paper and according to current regulation and market practices, we set the level of capital in a way that it equals the amount the company needs for absorbing unexpected losses over a certain time horizon at a given confidence level (cf. [18]). We follow the standard convention and measure the economic capital as the difference between the unexpected and expected losses (a detailed discussion of economic capital can be found in Chapter 5 of [27]). Therefore a current and a future economic capital can be distinguished as we distinguish current and future credit loss as well as current and future unexpected loss. Hence the following formula for the economic capital (which is denoted by EC) is as below:

- $$\mathrm{EC}_{i,g,f,k}^{curr} = \mathrm{UL}_{i,g,f,k}^{curr} - \mathrm{CL}_{i,g,f,k}^{curr} \tag{27}$$

- $$\mathrm{EC}_{i,g,f,k}^{future} = \mathrm{UL}_{i,g,f,k}^{future} - \mathrm{CL}_{i,g,f,k}^{future} \tag{28}$$

The calculations for the economic capital are similar to the calculations for the credit loss in Section 4.2 and in Section 4.4. The economic capital for each counterparty k based on formulas (27) and (28) is:

- $$\mathrm{EC}_k^{curr} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{EC}_{i,g,f,k}^{curr}, 0\right) - \mathrm{Col}_{g,f,k}, 0\right] .$$
- $$\mathrm{EC}_k^{future} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{EC}_{i,g,f,k}^{future}, 0\right) - \mathrm{Col}_{g,f,k}, 0\right] .$$

Then we calculate the economic capital based on the current exposure as well as the economic capital for a future worst case based on the potential future exposure:

- $$\mathrm{EC}^{curr} = \sum_{k=1}^{K} \mathrm{EC}_k^{curr} \quad . \tag{29}$$
- $$\mathrm{EC}^{future} = \sum_{k=1}^{K} \mathrm{EC}_k^{future} \quad . \tag{30}$$

Note that it is important to calculate first the economic capital for every investment i, then for each counterparty k. Finally the sum of the values for each counterparty gives us the economic capital for the investor. In fact if we had used equations (17) and (25) to calculate (29) (respectively equations (18) and (26) to calculate (30)), we would have subtracted the collateral twice.

5 Correlation

This chapter describes a method to include the correlation between different positions within a netting group first and, in a second stage, between netting groups. If there is a change in the value of a derivative this might impact the value of other derivatives of this counterparty. This effect can be properly captured on a netting group level as we cannot net positions from different netting groups. Therefore the next sections explain how to include the correlation into the future net replacement value and into the credit loss.

5.1 Correlation in Future Exposure

First, we need a correlation matrix with the dependences between the different positions of a netting group g. Let us denote this matrix by $\Sigma_{g,f,k}$. If the specific netting group consists of n positions, then it is a $n \times n$-matrix as below.

$$\Sigma_{g,f,k} := \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{pmatrix}$$

with c_{kl} denoting the correlation between position k and position l of netting group g. Next, a factor measuring the risk of a position in a netting group is needed for each position of the respective netting group. For this purpose, we take the respective VaR factor which can be found in the tables on page 11 and 12 in Section 3.2. For the n positions of the netting group, we get a n-variate vector as VaR factor vector of this netting group g which we denote by $\beta_{g,f,k}$:

$$\beta_{g,f,k} := (\text{VaRFactor}_{1,g,f,k}, \cdots, \text{VaRFactor}_{n,g,f,k})^\top .$$

Then we need the weight of each position within the netting group as a derivative with a very high notional has apparently a more significant effect on the future exposure than a derivative with a low notional. Therefore the notional of all positions of the netting group is needed:

$$\text{Not}_{g,f,k} := \sum_{i=1}^{N_{g,f,k}} \text{Not}_{i,g,f,k} .$$

The weight of a position i in the netting group is then

$$w_{i,g,f,k} := \frac{\text{Not}_{i,g,f,k}}{\text{Not}_{g,f,k}} .$$

Hence, the weight vector of the netting group is

$$w_{g,f,k} := (w_{1,g,f,k}, \cdots, w_{n,g,f,k})^\top .$$

Then, we define the vector $v_{g,f,k}$ as $w_{g,f,k}^\top \cdot \beta_{g,f,k}$. We obtain the VaR factor for the netting group g as

$$\text{VaRFactor}_{g,f,k} := \sqrt{v_{g,f,k}\ \Sigma_{g,f,k}\ v_{g,f,k}}$$

and hence the amount that has to be added to the values of the positions of the netting group g for calculating the future exposure is

$$\text{VaR}_{g,f,k} := \text{Not}_{g,f,k} \cdot \text{VaRFactor}_{g,f,k} .$$

In the end the future exposure of a counterparty k, i.e. the aggregated net replacement Value including add-on of counterparty k with integration of correlation is

$$\text{NRV-VaR}_k = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k}, 0\right) + \text{VaR}_{g,f,k} - \text{Col}_{g,f,k}, 0\right] . \tag{31}$$

5.2 Correlation in Credit Loss and Unexpected Loss

This section describes how to include the correlation within a netting group into the credit loss and unexpected loss. The current credit loss of a counterparty k remains unchanged as well as the unexpected loss:

$$\text{CL}_k^{curr} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \text{CL}_{i,g,f,k}^{curr}, 0\right) - \text{Col}_{g,f,k}, 0\right]$$

with

$$\mathrm{CL}_{i,g,f,k}^{curr} = V_{i,g,f,k} \cdot \mathrm{LGD}_i \cdot p_k$$

and

$$\mathrm{UL}_k^{curr} = \sum_{f=1}^{F_k} \max\left[\sum_{g=1}^{G_{f,k}} \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{UL}_{i,g,f,k}^{curr}, 0\right) - \mathrm{Col}_{g,f,k}, 0\right]$$

with

$$\mathrm{UL}_{i,g,f,k}^{curr} = \sqrt{V_{i,g,f,k}^2 \cdot \mathrm{LGD}_i^2 \cdot p_k \cdot (1-p_k)}\ .$$

To calculate the credit loss and unexpected loss for the future exposure we have to integrate the severity of position i of netting group g to the respective weight, i.e.

$$w_{i,g,f,k}^{\mathrm{CL}} := w_{i,g,f,k} \cdot \mathrm{LGD}_i \quad \text{or respectively} \quad w_{i,g,f,k}^{\mathrm{UL}} := w_{i,g,f,k} \cdot \mathrm{LGD}_i\ .$$

We again define the help variable

$$v_{i,g,f,k}^{\mathrm{CL}} := (w_{g,f,k}^{\mathrm{CL}})^\top \cdot \beta_{g,f,k}$$

or respectively

$$v_{i,g,f,k}^{\mathrm{UL}} := (w_{g,f,k}^{\mathrm{UL}})^\top \cdot \beta_{g,f,k}\ .$$

Then the VaR factor of the netting group is

$$\mathrm{VaRFactor}_{g,f,k}^{\mathrm{CL}} := \sqrt{v_{i,g,f,k}^{\mathrm{CL}}\ \Sigma_{g,f,k}\ v_{i,g,f,k}^{\mathrm{CL}}}$$

or respectively

$$\mathrm{VaRFactor}_{g,f,k}^{\mathrm{UL}} := \sqrt{v_{g,f,k}^{\mathrm{UL}}\ \Sigma_{g,f,k}\ v_{g,f,k}^{\mathrm{UL}}}\ .$$

Hence the amount that has to be added is

$$\mathrm{VaR}_{g,f,k}^{\mathrm{CL}} := \mathrm{Not}_{g,f,k} \cdot \mathrm{VaRFactor}_{g,f,k}^{\mathrm{CL}} \quad \text{or resp.} \quad \mathrm{VaR}_{g,f,k}^{\mathrm{UL}} := \mathrm{Not}_{g,f,k} \cdot \mathrm{VaRFactor}_{g,f,k}^{\mathrm{UL}}\ .$$

Next, we can calculate the future credit loss for the netting group g as

$$\mathrm{CL}_{g,f,k}^{future} := \mathrm{CL}_{g,f,k}^{curr} + \mathrm{VaR}_{g,f,k}^{\mathrm{CL}} \cdot p_k - \mathrm{Col}_{g,f,k}\ , \text{ whereas } \mathrm{CL}_{g,f,k}^{curr} := \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{CL}_{i,g,f,k}^{curr}, 0\right).$$

Then, the calculation of the future unexpected loss regarding netting group g is

$$\mathrm{UL}_{g,f,k}^{future} := \mathrm{UL}_{g,f,k}^{curr} + \mathrm{VaR}_{g,f,k}^{\mathrm{UL}} \cdot p_k - \mathrm{Col}_{g,f,k}\ , \text{ whereas } \mathrm{UL}_{g,f,k}^{curr} := \max\left(\sum_{i=1}^{N_{g,f,k}} \mathrm{UL}_{i,g,f,k}^{curr}, 0\right).$$

In the end the future credit loss of a counterparty k with correlation is

$$\mathrm{CL}_k^{future} = \sum_{f=1}^{F_k} \sum_{g=1}^{G_{f,k}} \max\left(\mathrm{CL}_{g,f,k}^{future}, 0\right) \tag{32}$$

and the future expected loss of a counterparty k with correlation can be calculated in the following way:

$$\mathrm{UL}_k^{future} = \sum_{f=1}^{F_k} \sum_{g=1}^{G_{f,k}} \max\Big(\mathrm{UL}_{g,f,k}^{future}, 0\Big) \ . \tag{33}$$

Using equations (32) and (33) the credit loss as well as the unexpected loss based on the potential future exposure with correlation between different positions within a netting group is as follows, (whereas K denotes the number of different counterparties):

- $$\mathrm{CL}^{future} = \sum_{k=1}^{K} \mathrm{CL}_k^{future} \ .$$

- $$\mathrm{UL}^{future} = \sum_{k=1}^{K} \mathrm{UL}_k^{future} \ .$$

5.3 Correlation Between Netting Groups

Besides the correlation of positions inherent of a specific netting group, there is also correlation between the different netting groups. Therefore a correlation matrix is needed displaying the dependences of the netting groups of a fund f and denoted by $\Sigma_{f,k}$. If the specific fund consists of n positions, then it is a $n \times n$-matrix as below.

$$\Sigma_{f,k} := \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{pmatrix}$$

with k_{lm} denoting the correlation between netting group l and netting group m of fund f. Then, the correlation of the netting groups is incorporated in the calculations as described for positions in a netting group in the previous section, whereas the weight of each netting group in fund f is calculated via the sum of all notionals of all positions of the specific netting group.

6 Potential Collateral Requirements

In this paragraph we develop the concept how collateral requirements might have an impact on a fund's/account's liquidity and risk. When the value of a position decreases it not only has an impact on the NAV but as well on the collateral. Three cases might happen:

1. The position changes from positive to negative i.e. the fund has to give back the collateral received and to post collateral.

2. The position remains positive, i.e. the fund has to give back part of the collateral received.

3. The position remains negative, i.e. the fund has to post additional collateral.

The above shows that a market/credit event might put stress on the liquidity of the fund which, if not carefully managed, could in turn lead to a sale of assets at unfavorable conditions.
In the following, we present the formula which describes the potential collateral requirements (PCR) of counterparty k, that has to be compared with the fund's liquidity, potential borrowing and eligible assets for collateral by using the known notations of Chapters 2 to 5:

$$\mathrm{PCR}_k = \sum_{f=1}^{F_k} \max\left[-\sum_{g=1}^{G_{f,k}} \min(V_{g,f,k} + \mathrm{VaR}_{g,f,k} - \mathrm{Col}_{g,f,k}, 0)\,, 0 \right]\,, \tag{34}$$

where the sum of the NRVs of all positions of a netting group g in a fund f with counterparty k is denoted by:

$$V_{g,f,k} := \sum_{i=1}^{N_{g,f,k}} V_{i,g,f,k}\,.$$

The difference between available assets (i.e. liquidity, borrowing and eligible assets for collateral) and PCRs can be used for managing liquidity risk as a shortfall measure.

Note that in equation (34) the collateral amount of a certain netting group $\mathrm{Col}_{g,f,k}$ can be positive or negative, i.e. broker and client collateral respectively have to be taken into consideration. As far as collateral received is booked off balance, a reduction of this collateral does not affect assets of the fund. Since often there is no sufficient information it's difficult to gauge, whether collateral received is on- or off-balance. For this reason the PCR is reported as a value independent from the way broker collateral is booked. In such instance the PCR will overstate the real requirements, that have to be covered by the fund's assets, by the amount of broker collateral booked off-balance. To correct that, in order to evaluate whether the eligible assets of a fund are sufficient to cover potential collateral requirements, the PCR has to be reduced by the amount of off-balance broker collateral before comparing it to the amount of assets available. Further, $\mathrm{VaR}_{g,f,k}$ denotes the diversificated add-on amount, described in Chapter 5.

7 Conclusion

In the paper, we have shown a pragmatic solution to the problem of measuring credit and counterparty risk and dealing with liquidity risk via collateral requirements. It is worth mentioning that the figures shown in the tables, for example in paragraph 3.2, should be estimated/calibrated in order to obtain a number of exceptions in the back-testing consistent with the level of confidence set for the VaR (e.g. if the VaR is set at 99.5%, the figures in the table should lead to a maximum number of exceptions of 5 over 1,000 runs).

Estimation and calibration can be carried out through Monte Carlo simulation (which presents some issues in terms of modeling the nonlinearities) or by historical simulation which accurately reflects the historical multivariate probability distribution but requires some corrections in order to incorporate volatility updating (cf. [16]). A possible extension of this paper could be to illustrate how to address the matter pragmatically. Finally, it goes without saying that, in the context of this paper, VaR is the Shortfall/CVaR for a lower confidence level.

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