Philosophy and Imprecise Probs

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Epistemology

"Fusion is a viable energy source"

hope, desire

believe, think it's quite likely that

representational attitudes

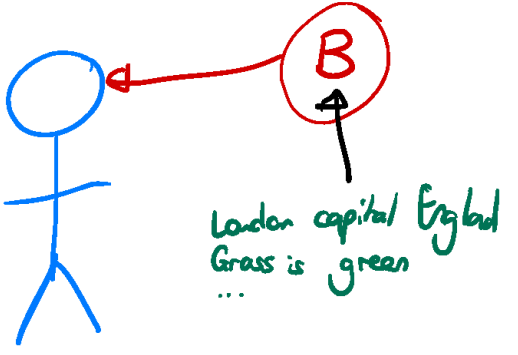
not values, or emotion

her belief state (doxastic state)

Model of belief

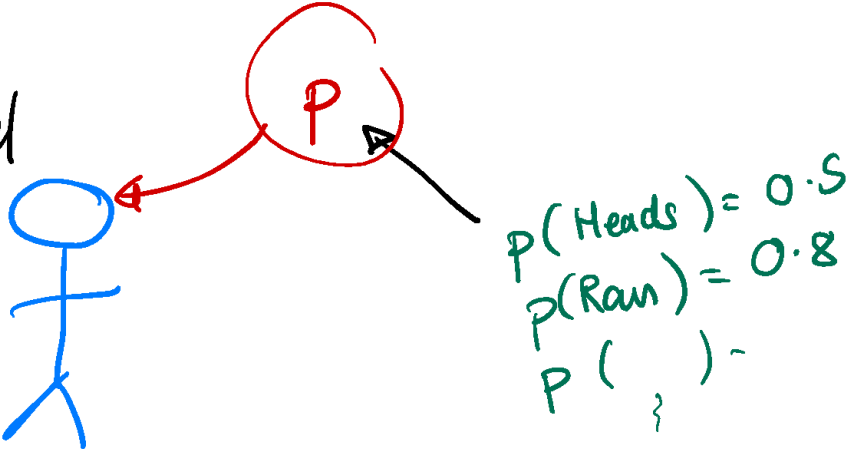
"representation of uncertainty"

Traditional: full belief



Alternative:

- Numerical degrees of bel
- Credence
- (subjective) probability

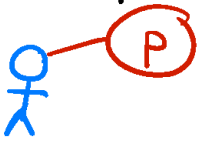


Bayesian Epistemology

Agents views of what the world is like

- belief state

best represented by numerical degrees of bel



$P: \wp(\Omega) \to \mathbb{R}$

Descriptive

- which should conform to the axs of prob

$P(E) = \sum_{\omega \in E} P(\omega)$ $\quad P(\Omega) = 1$ $\quad P(\omega) > 0$

Normative

Coherence

"hang together"

Roles of belief

- represent the world
  inference, reasoning

- guiding action
  decision making

Why should my degrees of belief be probs?
What's wrong with b(Rain) = 0.7 b(NoRain) = 0.7

- Pragmatic (practical) rationality

vs

- Epistemic (theoretical) rationality

Pascal's wager: you should believe in God
believing it yields vast benefits in afterlife

but it's not best reflection of evidence

Pragmatic justificahia of probabilism

Sure loss argumenb

Your bething prices should sahisfy axs

Epistemic justificahia of probabilism

"Accuracy"!

Credences aim to be close to tmth

&

measured by a strictly proper error loss finchia

Assume rather than justify precisia

Why imprecise

Descriptive

- psychological plausibility

What exactly is your credence Ramsey says?

Normative

- right response to incomplete, ambiguous evidence

A stranger approaches you and pulls objects out his bag

1) tube toothpaste
2) live jellyfish
3) travel toothpaste

What should your confidence be that next object is toothpaste? (Elga)

Imprecise Bayesian Epistemology

Agents views of what the world is like

- belief state

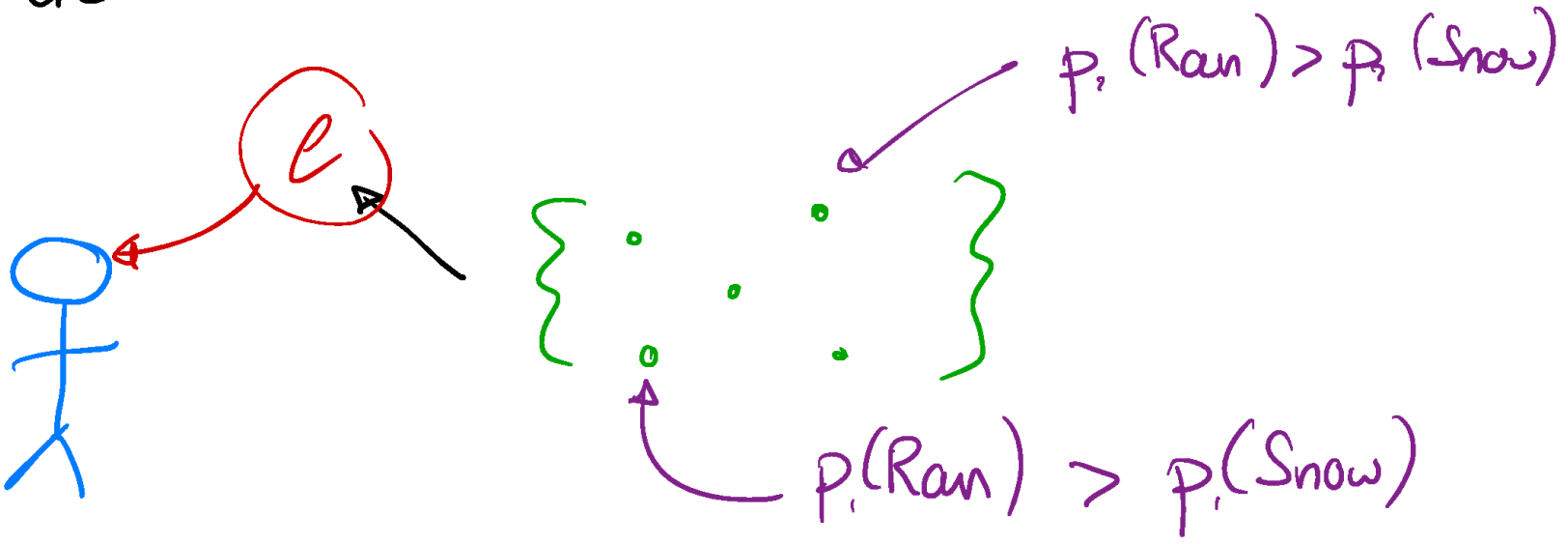
  best represented IMPRECISE PROBS

  Descriptive

- which should BE COHERENT

  Normative

Credal set model of belief (for coherent believers)



Encodes judgement:

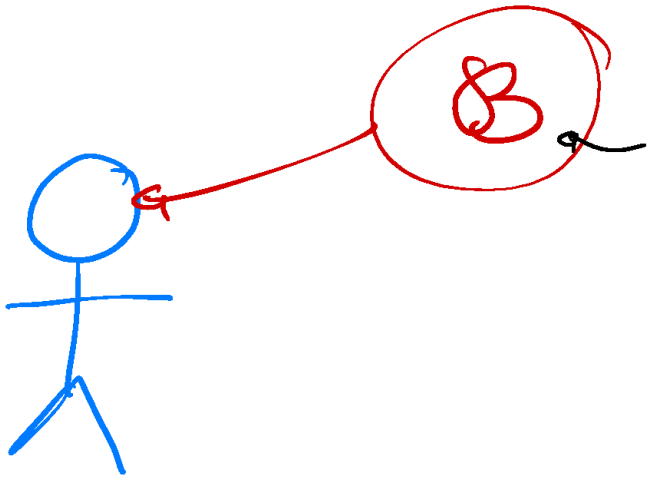
more likely to rain than snow

Descriptive:

rep by epistemic judgements

constraints

probability judgement



$\mathcal{B}$

$[\, pr(\text{Rain}) > pr(\text{Snow}) \,] \in \mathcal{B}$

$[\, pr(\text{AlicePasses}) > 0.9 \,] \in \mathcal{B}$

$[\, pr(H) < 1/4 \quad \text{or} \quad pr(H) > 3/4 \,] \in \mathcal{B}$

$[\, A \text{ and } B \text{ are prob independent} \,] \in \mathcal{B}$

$pr(A \wedge B) = pr(A) \times pr(B)$

probabilistic expectation

$[\, \mathbb{E}_{pr}[\text{mm of rain}] > 10 \,] \in \mathcal{B}$

Which kinds are legitimate?

Probability interval model

$$[\, 0.2 \leq \mathrm{pr}(A) \leq 0.7 \,]$$

Upper / Lower expectations

$$[\, 2 \leq \mathbb{E}(f) \leq 5 \,]$$

Desirable gambles

$$[\, \mathbb{E}(f) > 0 \,]$$

Choice f^s ??

$$[\, \exists f \in A \quad \mathbb{E}(f) > g \,]$$

- It seems to me more likely to rain than snow

$$pr(\text{Rain}) > pr(\text{Snow})$$

Imprecise:

- No judgement on particular likelihood of rain

$$pr(\text{Rain}) = ?$$

  - failure of elicitation
  - ☆ - no opinion

Coherent $\mathcal{B}$?

- $[pr(A) \geq 0.9] \in \mathcal{B}$

  Very likely Alice passes

- $[pr(B) \geq 0.9] \in \mathcal{B}$

  Very likely Bob passes

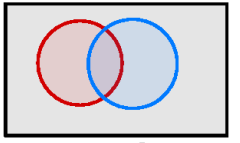
judgements I hold

because for probs p
$p(A) > 0.9 \;\&\; p(B) > 0.9$
$\Rightarrow p(A \& B) > 0.8$

Then committed to the judgement

$\Rightarrow [pr(A \wedge B) \geq 0.8] \in \mathcal{B}$

Quite likely they both pass



Coherence

Normative

natural extension: collecting all consequences

- If P_1 P_2 .. $\in$ 𝒮

  and every prob fⁿ satisfying P_1 & P_2 .. also satisfies Q

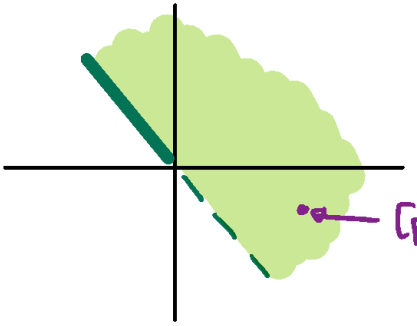
  then you are committed to Q

- Not committed to $\varnothing$ → Credal sets

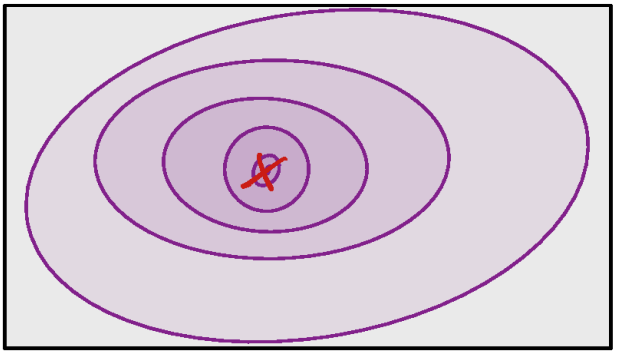
Me: Coherence as finitary consequences

Allow:

$$\begin{aligned} pr &< 1/2 \\ &< 1/4 \\ &< 1/8 \\ &\vdots \\ &> 0 \end{aligned}$$



probability filter



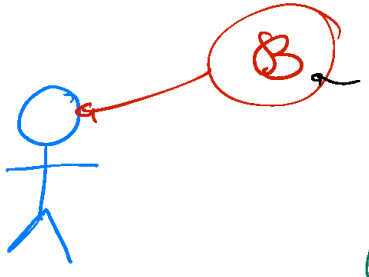
No representing (real valued) credal set

infinitesimals

Imprecise Bayesian Epistemology

- belief state ← Agents views of what the world is like

  best represented by collection of prob judgements



Descriptive

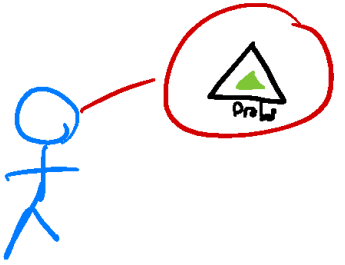
- which **should** be closed under entailment given the axioms of probability

Normative

To think about coherent believers,

credal sets

A set of prob f^ns



Fomal representation of ep judgements

Credal committee

Other norms?

- Regularity — If E is possible $cr(E) > 0$
- Principal Principle — credences match known chances
  $cr(A \mid ch(A) = r) = r$
- Principle of Indifference
  if no information, spread cred equally
  $cr(\omega_i) = cr(\omega_j)$
- Conditionalisation
  conditional credences should: $cr(A \mid E) = \frac{cr(A \wedge E)}{cr(E)}$
  when you learn you should follow your cond creds

§2

Challenges for IP

dec making

- sure loss args against imprecision
- paradoxes of sequential choice

learning

- dilation
  ↳ learning leads to more imprecision
- belief inertia
  ↳ pre-learning state is vacuous
  then you'll remain vacuous forever

Decision Making

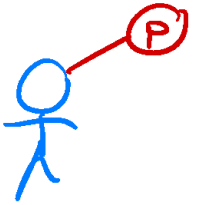
Beliefs + Values / Desires $\xrightarrow[\text{choice procedure}]{}$ Choice Fn

p + u $\xrightarrow[\text{expected utility}]{}$ EU_p

Which actions do your beliefs and desires rationalise?

You're a doctor.

Which drug?



	T	¬T
Pr	50%	50%
safe	4	4
gamble	10	0

$$EU_{.5}(g) = .5 \times 10 + .5 \times 0 = 5$$

$$EU_{.5}(s) = 4$$

$$g \succ_{.5} s$$

No information about T

	T	$\neg T$
Pr	?	?
safe	4	4
gamble	10	0

Expected value of s vs g?

No judgement about which has higher exp utility

What should you do?

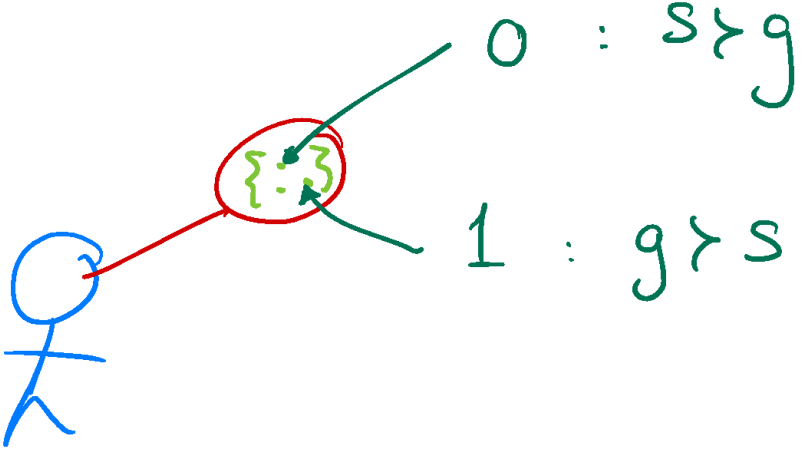
Beliefs + Values / Desires → (choice procedure) Choice F^{1}

Credal set + u →

- Γ-Maximin
- ϵ-Admissibility
- Maximality

Which should I choose?

	T	$\neg T$
pr	?	?
safe	4	4
gamble	10	0



Incomparability

Three decision theories

Γ - Maximin : ambiguity averse

maximise minimum expected utility

E - Admissibility : reject when:

judge something else is better

Maximality : reject when something else is judged better

} permissive choice rules

Γ-Minimax: ambiguity averse

select safe

	T	$\neg T$
pr	?	?
safe	4	4
gamble	10	0

$C\{s, g\} = \{s\}$

E-Admiss / Maximality: $C\{s, g\} = \{s, g\}$

Pattern not probabilistically rationalisable

D_1	T	¬T
Pr	?	?
★ safe	4	4
gamble	10	0

(like Ellsberg preferences)

D_2	T	¬T
Pr	?	?
★ safe	4	4
gamble	0	10

Challenge: Sure Loss arg

D_1	T	¬T
pr	?	?
safe	4	4
gamble	10	0

D_2	T	¬T
pr	?	?
safe	4	4
gamble	0	10

Don't take both!

	T	¬T	
s in both	8	8	← bad!
g in both	10	10	
$s_1 + g_2$	4	14	
$g_1 + s_2$	14	4	

Responses

- I don't expect to face such a nefarious bookie
- Reject package principle
  - evaluation of unit not combining individual

Note: undermines usual sure loss arg for coherence ... ?

Lecture 2

Three decision theories

Γ - Maximin : ambiguity averse

maximise minimum expected utility

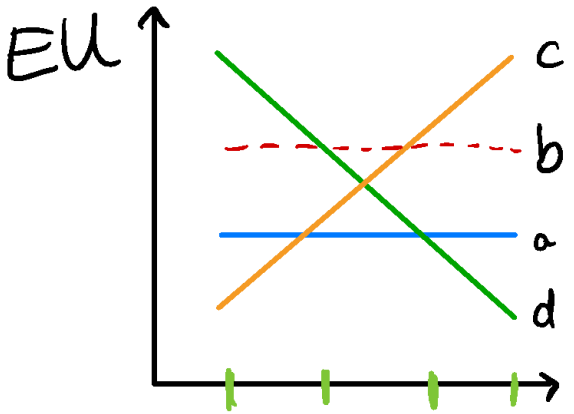
E - Admissibility : reject when:

judge something else is better

Maximality : reject when something else is judged better

} permissive choice rules

$x \in [0,1]$	$.5 \times x$	$.5 \times (1-x)$	$.5 \times x$	$.5 \times (1-x)$	
a	4	4	4	4	$EU = 4$
~~b~~	10	0	0	10	$EU = 5$
c	4	4	0	10	$EU = 2 + 5(1-x)$
d	10	0	4	4	$EU = 2 + 5x$



$C(\{a,b,c,d\}) =$

$C(\{a,c,d\}) =$

- Maximin: Maximise min EU

E-Admiss: do you judge something else better

Max: is there a you judge as better

A new kind of perspective:

Ask the dec theory itself
what dec theory it recommends

Are imprecise dec theories
self-undermining

Not sure which drug will be on offer

D_1	T	$\neg T$
s_1	4	4
g_1	10	0

D_2	T	$\neg T$
s_2	4	4
g_2	0	10

"Which should I choose in $D_1 \geq D_2$?"

"Whose advice should I follow?"

Yours or your colleague's?

"Which should I choose in D_1 & D_2 ?"

D_1	T	$\neg T$
s_1	4	4
g_1	10	0

D_2	T	$\neg T$
s_2	4	4
g_2	0	10

Γ-Maximin : s in both D_1 & D_2

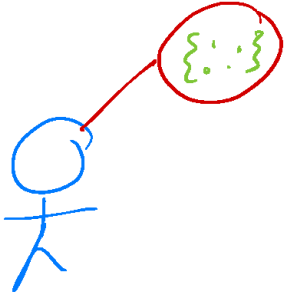
"Whose advice should I follow?"

Yours or your colleague's?

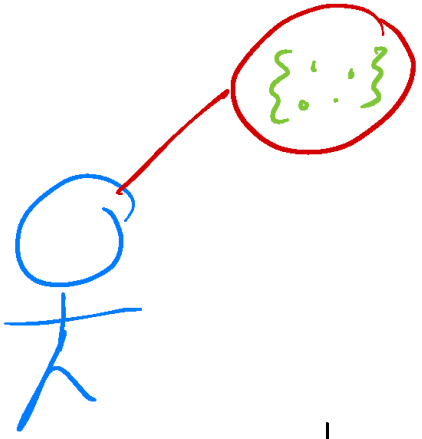
Apply the dec theory to this meta-decision

How good the advice is depends on

- which drug offered D_1 vs D_2
- whether I have trait T



probabilities over T and D_1 vs D_2

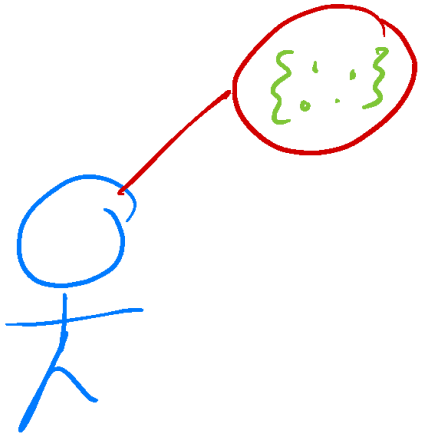


$pr(D_1) = pr(D_2) = .5$

D_i & T are prob indep

$pr(D_i \& T) = pr(D_i) \times pr(T)$

	Offered D_1		Offered D_2		
	T	$\neg T$	T	$\neg T$	
$x \in (0,1)$	$.5 \times x$	$.5 \times (1-x)$	$.5 \times x$	$.5 \times (1-x)$	
s in both	4	4	4	4	$EU = 4$
g in both	10	0	0	10	$EU = 5$
s_1, g_2	4	4	0	10	$EU = 2 + 5(1-x)$
g_1, s_2	10	0	4	4	$EU = 2 + 5x$



$pr(D_1) = pr(D_2) = .5$

D_i & T are prob indep

$pr(D_i \& T) = pr(D_i) \times pr(T)$

Γ-Maximin :

$MM\ EU\ (s \text{ in both}) = 4$

$MM\ \bar{E}U\ (g \text{ in both}) = 5$

$MM\ EU\ (s_1, g_2) = 2$

$MM\ \bar{E}U\ (g_1, s_2) = 2$

should choose g in both

Γ - Maximin :

"Which should I choose in $D_1 \gtrsim D_2$?"

s in both!

"Whose advice should I follow?"

Yours or your colleague's ?

his! ⇝ g in both

Me: Γ-Maximin, please advise me,
what should I do in decision D_1?
and what should I do in decision D_2?

Γ-Maximin: It depends how you're asking me the qu
Individually, for each you pick s
But the best advice I can give you is g in both

Me: You seem to be telling me not to trust you
If I take your strategy advice
you're telling me not to call you and check
once I learn which I'm facing
If I take your local advice
you're telling me it'd be better to
consult your colleague, Ell 0.5

Recall:

- bad performance in this one pair might be compensated elsewhere
  unlikely to face such a nefarious bookie

We've again just set up a case

It might be used for many more decisions than just $D_1^{50\%}/D_2^{50\%}$

Thm If

(1) You have precise probabilities over which decision you'll face μ

(2) μ spread over many decisions

$$\mu\{D \mid \text{Ell}_p(D) \text{ singleton}\} = 1 \quad \text{for all probs } p$$

Then

Γ-Maximin recommends following advice of a precise colleague

Undermining her ambiguity averse guidance

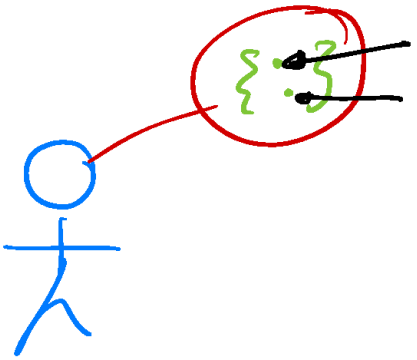
Decision theories which leave gaps

E-Admissibility & Maximality

E-Admissibility: Choiceworthy if some prob thinks it's best

reject when: you judge that: something else is better

Maximality: reject when there is something else which you judge as better



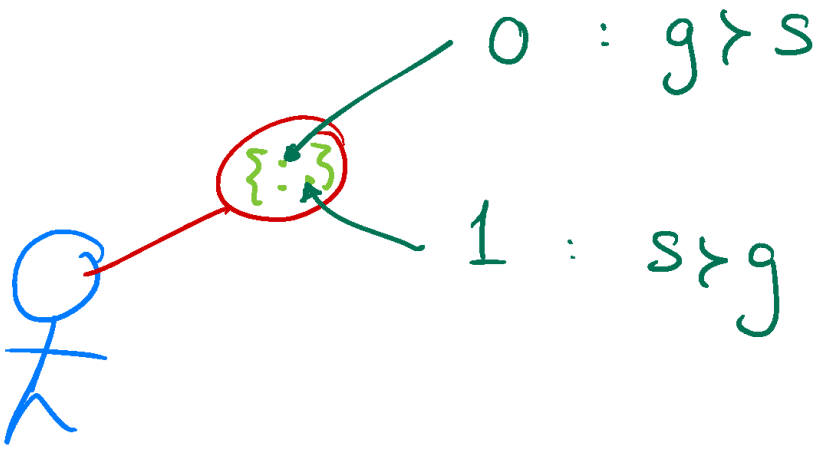
$a \succ b \succ c$

$c \succ b \succ a$

E-Ad: $C(\{a,b,c\}) = \{a, c\}$

Max: $C(\{a,b,c\}) = \{a,b,c\}$

Which should I choose in D_1



Neither is judged as rejected ...

$$C(\{s, g\}) = \{s, g\}$$

"Whose advice should I follow? "
Yours or your colleague's?

What to do when faced with this advice?

$$C(\{s,g\}) = \{s,g\}$$

What to do when faced with this advice?

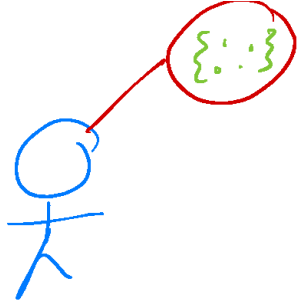
$$C(\{s, g\}) = \{s, g\}$$

Toss a coin? Unstable behaviour

Randomly pick

D_1 : 50% chance s_1
50% chance g_1

D_2 : 50% chance s_2
50% chance g_2



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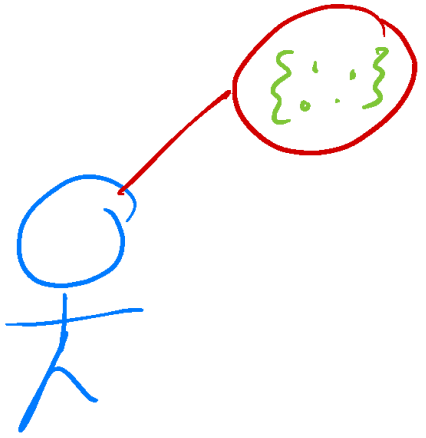
	Offered D_1		Offered D_2		
	T	$\neg T$	T	$\neg T$	
	$.5 \times x$	$.5 \times (1-x)$	$.5 \times x$	$.5 \times (1-x)$	
s in both	4	4	4	4	EU = 4
s_1, g_2	4	4	0	10	EU = $7 - 5x$
g_1, s_2	10	0	4	4	EU = $2 + 5x$
g in both	10	0	0	10	EU = 5
coin toss	7	2	2	7	EU = 4.5



$pr(D_1) = pr(D_2) = .5$

D_i & T are prob indep

$pr(D_i \& T) = pr(D_i) \times pr(T)$

Unanimously: g in both > coin toss

All dec theories for IP say don't toss coin

- "Which should I choose in D_1 & D_2 ?"

  D_1: either
  D_2: either

  toss a coin

- "Whose advice should I follow ?"

  Yours or your colleague's ?

  Not mine ! g-colleague gives better advice

- "Which should I choose in D_1 & D_2?"

  either... ish

- "Whose advice should I follow? Yours or your colleague's?"

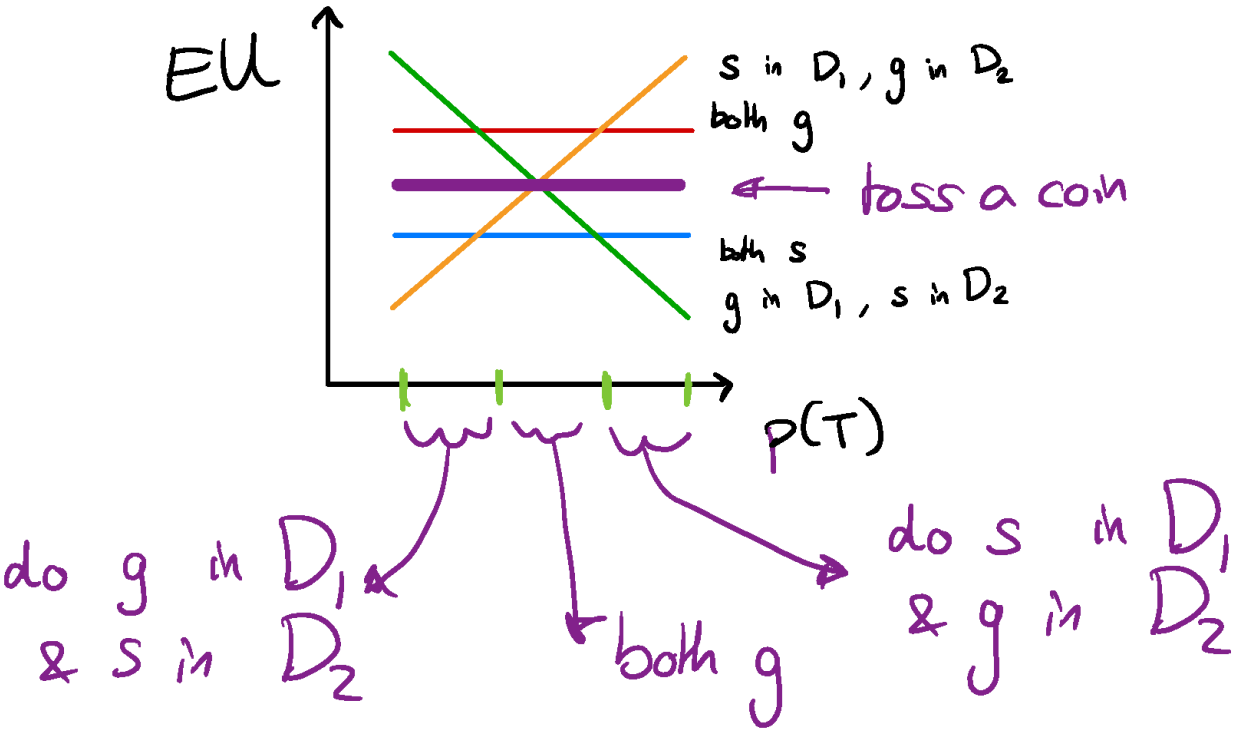
My advice is <u>not</u> to randomly pick in each D

You must either

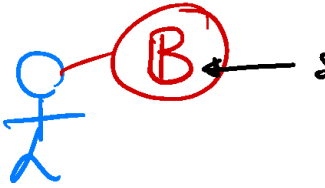
- S in D_1, g in D_2 ← follow advice of p^* ≤ 0.4
- both g ← follow advice of p^* $0.4 \leq p^* \leq .6$
- g in D_1, s in D_2 ← follow advice of p^* $\geq .6$

You must:

- s in D_1, g in D_2 ← follow advice of p^* ≤ 0.4
- both g ← follow advice of p^* $0.4 \leq p^* \leq .6$
- g in D_1, s in D_2 ← follow advice of p^* $\geq .6$



E-Admissibility



set of probs over
- states of the world
- which dec I'll face

Thm

whatever uncertainty over pass decs

E-Ad requires :

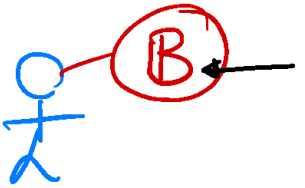
credal set over states of the world

pick $p^* \in \mathbb{P}$ and use it for everything

(at least μ-surely for $p^* \times \mu \in \mathbb{B}$)

or pick according to $b(\cdot|D)$ for some $b \in \mathbb{B}$

Maximality



set of probs over
- states of the world
- which dec I'll face

Thm If

(1) You have precise probabilities over which decision you'll face: μ

(2) μ spread over many decisions

$$\mu\{D \mid EU_p(D) \text{ singleton}\} = 1 \quad \text{for all probs } p$$

Then

Maximality requires acting like an EU reasoner

Question:

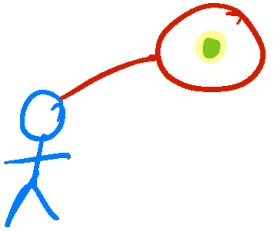
Are imprecise dec theories self-undermining

they recommend acting like a precise probabilist

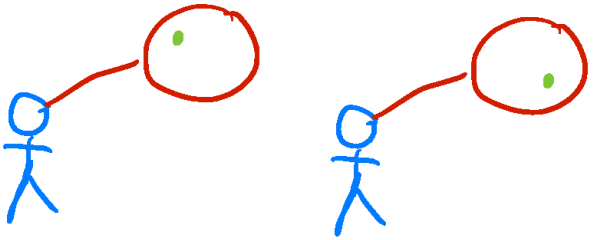
What's the value of imprecision?

Whats the diff

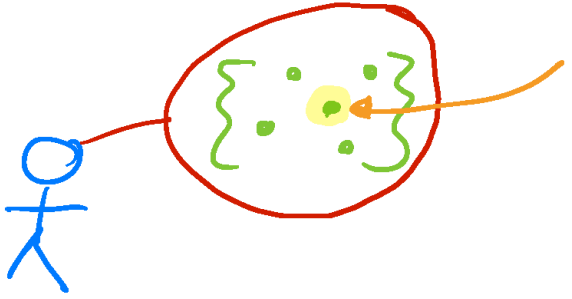
- Precise permissivism



other rationally permissible



- Imprecisia



use p^* for all decs!

Objective Bayesians

There's a single rational response to every body of evidence

Disagreement must be due to diff evidence

Subjective / Permissivism

There's many different rational responses

Obj Bayesians:

Principle of Indifference

start with least informative prior

spreading credence equally over possibilities

Response:

- still very informative
- language/partition dependent

Big question:

The status of choice set non-singleton

How to use the advice of imprecision?

Me: E-Ad, please advise me,
what should I do in decision D_1?

E-Ad: The choice set is $\{g, s\}$

Me: should I randomise (ie mix, toss a coin?)

E-Ad: No...! That's not an E-Ad action!

added after lectures

Me: So I'm allowed to do either? I should just pick?

E-Ad: Well, no, I don't recomend that

Me: What do you recommend

E-Ad: Do some more experimentation

Me: But I need to make the dec now

E-Ad: Sorry, I can't help
but at least you can trust advice I do give
those other advisers are going beyond
what your beliefs and values rationalise

Gamble Judgements

as primitive model for IP

My presentation

Beliefs + Values / Desires → Choice Fn

choice procedure

Credal set

utility function

E-Ad
Max
Γ-Maximin

Following Walley:

$\mathcal{D}$ as model of IP itself!

forget probabilities entirely

Motivations

- prob can't deal with "prob zero" challenges
- beliefs characterised by behavioural dispositions

Behaviourism Walley

beliefs are characterised by

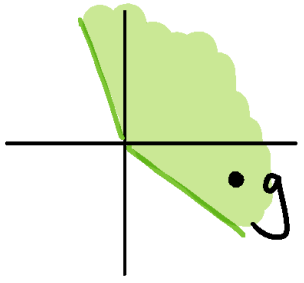
behavioural dispositions

in suitable conditions will produce actions

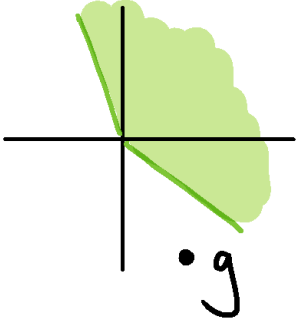
Glass has a disposition to shatter when struck

↳ a property of glass

Behaviourism Walley

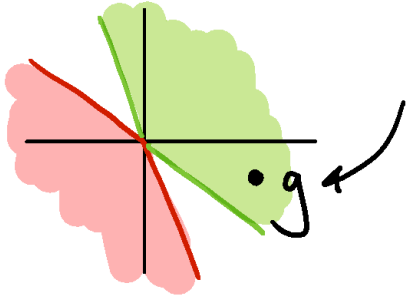


I have the behavioral dispositio
to accept g from {o, g}



?

Behaviourism Walley



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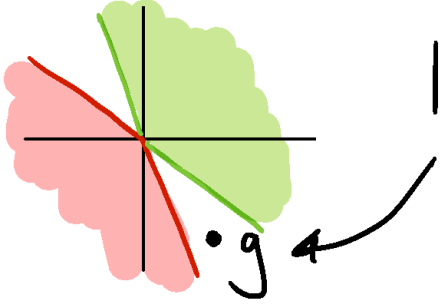
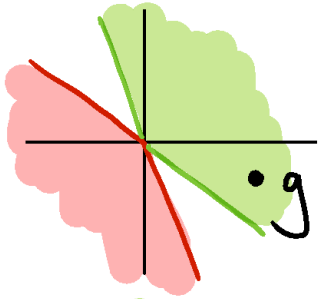


~~I have the behavioral disposition to reject g from {o, g}~~

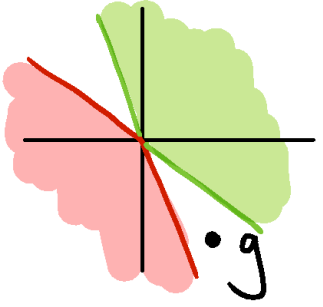
$\sim$

Γ-Maximin

Behaviourism Walley



I have the behavioral dispositic
to accept g from {0,g}



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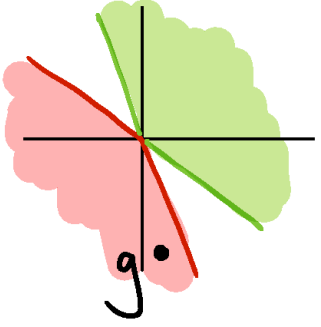

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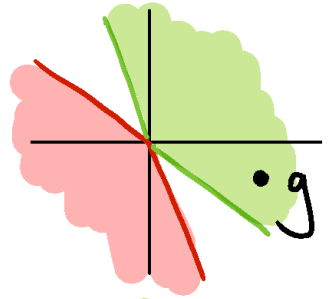
Behaviourism Walley



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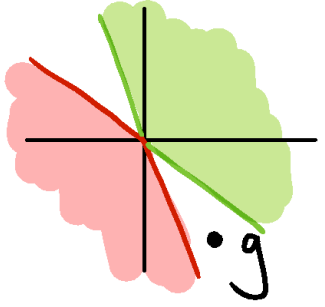
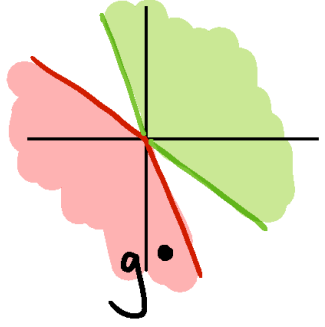


I have neithe dispositio

What does this look like?



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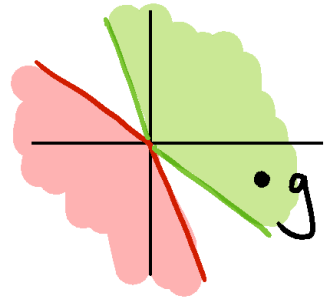
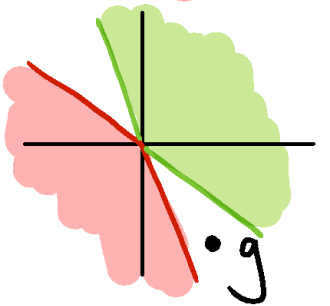
Behaviourism Walley



I have the behavioural disposition to accept g from {o,g}

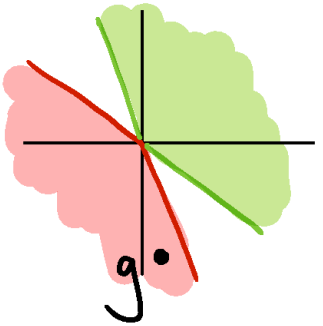


beh disp is unstable

??

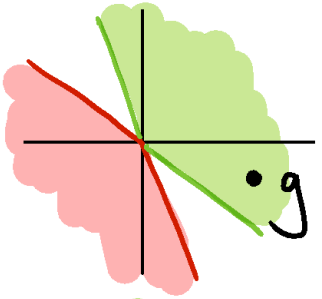
I'll sometimes accept sometimes reject

undermining

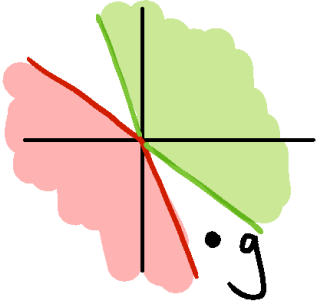


I have the behavioural disposition to reject g from {o,g}

Behaviourism Walley

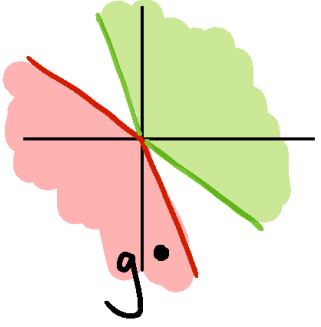


happy to accept



squeemish

now expressing mental state not behaviorism



happy to reject

Choice functions

What are they a model of?

Are they a model of

my belief state?

or

my approach to choice?

Credal set view

Bels + Values → Max → binary Choice functions

credal set

Bels + Values → E-Ad → mixing Choice functions

credal set

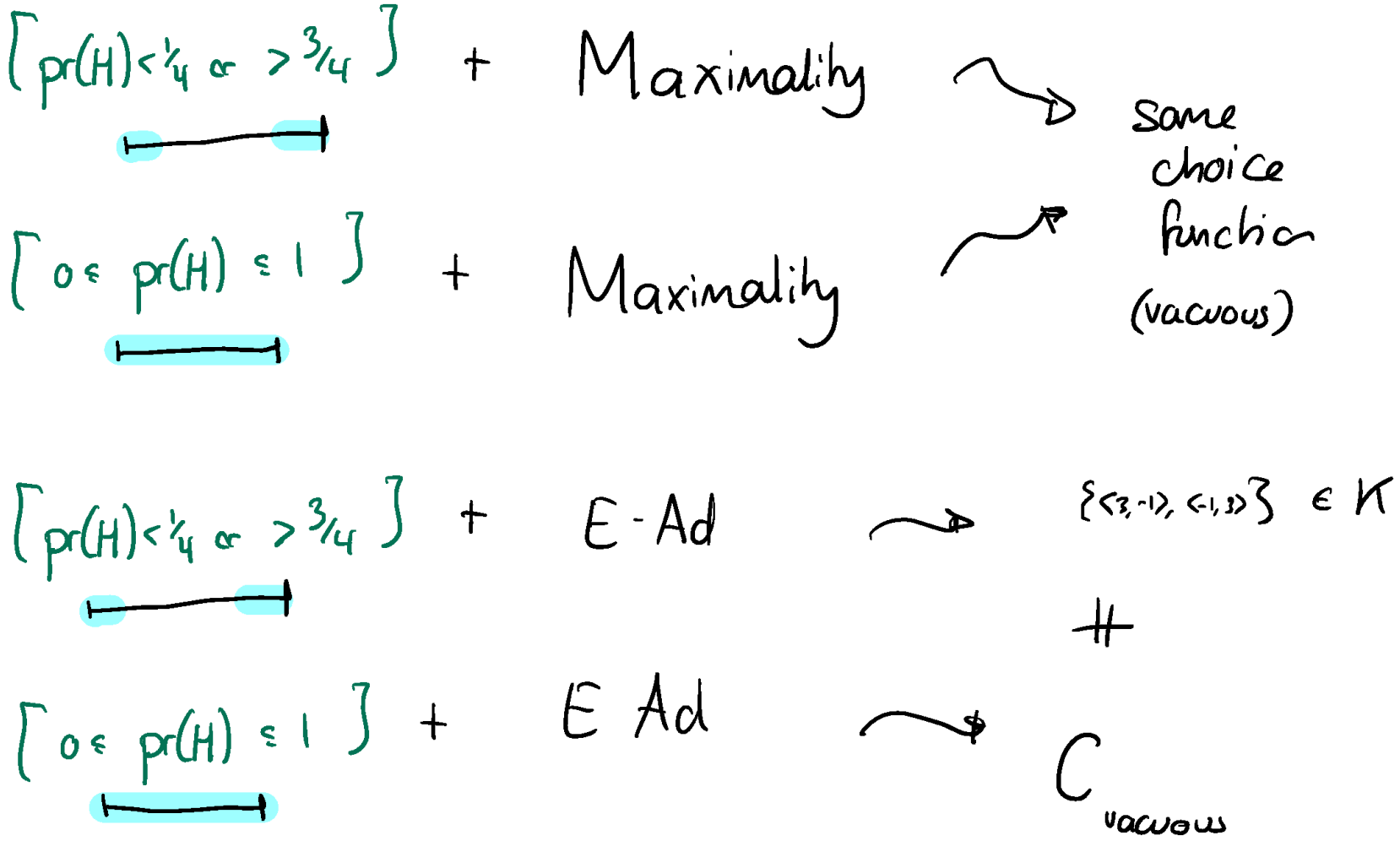
Bels + Values + Choice Approach → all coherent choice functions

credal set

Max / E-Ad / .. ?

Ignore components ?

Encompassed by choice f^{n} ?


[pr(H)<1/4 or >3/4] + Maximality
Same choice functio
[0≤ pr(H) ≤1] + Maximality
(vacuous)
[pr(H)<1/4 or >3/4] + E-Ad
{<3,-1>, <-1,3>} ∈ K
[0≤ pr(H) ≤1] + E Ad
C
vacuous

Further interpretation qns not covered in lecture

Are these (tryng to) model the same thing?

- precise probs
- credal set
- $\mathcal{D}$
- K
- R / C

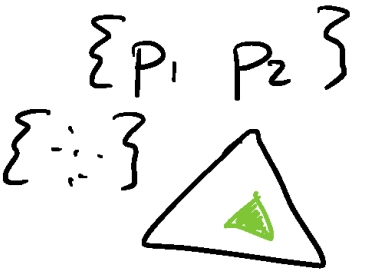
Are they a model of belief state?

Choice function represents $C(\{g, s\}) = \{g, s\}$

- actual behaviour
- behavioural dispositions
- what is rational to choose [given my evidence?]
- my judgements of what I should choose
- expressing which choices I'm comfortable with
- judgements of whether there's something else with higher expected value

What do imprecise prob models represent

→ credal set



(A) Failure of elicitation

(B) Incomplete epistemic judgements

(C) Views about range of rationally permissible probs

(D) Her behavioural dispositions

Differences?

Readings

General reference for lots of things!

- The Open Handbook of Formal Epistemology

Other references:

Bayesian Epistemology – credences and rational norms

- Hanti Lin - SEP - Bayesian Epistemology
- Mike Titlebaum 2022 - Fundamentals of Bayesian Epistemology 1: Introducing Credences. Chapter 1.

Arguments for rational norms

- Jim Joyce 'A Nonpragmatic Vindication of Probabilism' (*the original piece with good discussions, although the axioms he gives on accuracy aren't the right ones to use)*

Credal sets as probability constraints

- Catrin Campbell-Moore Probability filters and desirable gambles
    - Further refs*: "This is essentially the account by Van Fraassen (esp.: [14, §6-§7], [15, p.250–252] [7, p.345–347], [16, p.483–485]). And is close to some comments by Joyce [6], [17], [18]. For Van Fraassen, the agent's state of opinion is characterised by a collection of judgements and coherence is specified by entailment according to probability functions satisfying the judgements. A credal set is merely a "representor" of coherent collections of judgements. This model is also essentially proposed by Moss [13]."*

Imprecise probability discussions and responses to challenges

- Mahtani, "Imprecise Probabilities" in The Open Handbook of Formal Epistemology - nice overview of updating discussions and decision making (including responses to the the Elga challenge and references therein)
- Bradley, SEP – "Imprecise Probabilities"
- Joyce – Defense of Imprecise Credences in Inference and Decision Making

Decision making challenges for IP – self-undermining decision theories

- The sure-loss case was popularised by Elga presented as sequential problem.
    - See 3.2 of Mahtani for an overview.
    - Bradley and Steele should probabilities be sharp?
- Pettigrew, Campbell-Moore and Konek: Choosing How to Choose.