

1. **SIR model:** In a population of 100,000 individuals, 1,000 currently have a harmless cold as they are infected with certain virus, while 24,000 have already recovered from it and are now resistant. The number of infected people is currently increasing by approximately 150 per day, and it takes on average 10 days to recover after an infection.

- (a) Calculate the values of the parameters β and γ of a simple SIR model for this epidemic.
- (b) How many people will have been infected (including those who have already recovered) according to the predictions of this SIR model when the number of currently infected individuals reaches its maximum?

2. **Predictions of the SIR model**

In a population of 10,000,000 individuals, one individual is infected by a newly evolved virus and all others are susceptible at time $t = 0$

- (a) For what values of the basic reproduction rate $\mathcal{R}_0$ does the simple SIR model predict that in total 30%, 70%, 90% or 99% of the population will have been infected when the epidemic is finally over?
- (b) How many individuals will have been infected before the number of infected individuals reaches its maximum? What are the answers to this question according to the simple SIR model with the $\mathcal{R}_0$ values that you found in part (a)?

3. **Predator–prey interactions**

The interactions between the size N of a prey population and the size P of a predator population are modeled by the following differential equations:

$$\begin{aligned}\dot{N} &= N \cdot \left(1.2 - 10^{-6} \cdot N - \frac{80 \cdot P}{10^6 + N} \right) \\ \dot{P} &= \frac{0.2 \cdot N \cdot P}{10^6 + N} - m \cdot P\end{aligned}$$

- (a) To what values do $N(t)$ and $P(t)$ converge for $t \rightarrow \infty$ if $m = 0.1$?
- (b) What is the range of values for m for which $(N(t), P(t))$ converge to a non-trivial fixed point?
- (c) If you are ready to try out the R software, get the software from <https://www.r-project.org/> (and maybe also RStudio from <https://rstudio.com/>) and simulate the above system with various values of m to illustrate your prediction from (b). For this you may copy and modify code from

http://evol.bio.lmu.de/_statgen/maths/ws2021/multidimensional_modelling.R.

4. **Collecting data for the stats session**

Equip yourself with a drawing pin (also known as thumb tack in American English I guess) and toss it often enough to infer with reasonable precision how probable it is that the drawing pin lands on its head with the pin pointing upward when tossed. Send your results in an e-mail to me (Dirk). The reported data should include: In how many of your tosses did the pin point upward, in how many tosses did it point downward, what material is on the surface of the head of the drawing pin (e.g. plastic, brass, steel, aluminum,...) and your personal drawing-pin tossing style category: powerful, elegant, top spin, or unmotivated.