

1. Assume you toss your thumb tack 10 times. Let X_i denote the result of the i 'th throw. We set $X_i = 1$ if the pin pointed upwards and $X_i = 0$ otherwise.
 - (a) Define the random variable S that denotes the number of times a pin pointed upward as a mathematical term combining the X_i .
 - (b) Use notations with these random variables X_i to define the following events:
 - A: "The pin pointed downward in the 1st and 4th throw."
 - B: "The pin pointed upward in 5 out of 10 throws."
 - C: "The pin pointed upward in the first 3 throws, and 4 times in total."
 - D: "The pin pointed downward more than once."
 - (c) Assume the probability that the pin points upward after a throw is 0.4. What is the distribution of S ? Calculate the probabilities of the events A to D.
2. The manufacturer of the thumb tacks claims that the thumb tacks have been designed in a way such that they can also be used as a random generator, with the probability of the pin pointing upwards being 0.5.
 - (a) Perform experiments and calculations to check this. If you do not have a thumb tack, use data from one of the lines in http://evol.bio.lmu.de/_statgen/maths/ws2021/thumbtack_2020.csv (or http://evol.bio.lmu.de/_statgen/maths/ws2022/thumbtack_2022.csv if already available). You can read the data directly into R with the command

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read.csv("http://evol.bio.lmu.de/_statgen/maths/ws2022/thumbtack_2022.csv").
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As part of the test, answer the following questions:
 - i. Define your null hypothesis H_0 .
 - ii. What is the alternative hypothesis?
 - iii. What is the distribution of S under the null hypothesis?
 - iv. Which possible results are as extreme as or more extreme than your observation?
 - v. Calculate the p -value.
 - vi. Using a significance level of 5%, what does your test indicate? Formulate the results as a sentence.
 - (b) Discuss to what extent you can substantiate or even proof that the claim of the manufacturer is right or wrong.
 - (c) Do the same for the case that the manufacturer only claims that the probability is in the range between 0.45 and 0.55.
3. Test for the data on http://evol.bio.lmu.de/_statgen/maths/ws2021/thumbtack_2020.csv (or http://evol.bio.lmu.de/_statgen/maths/ws2022/thumbtack_2022.csv if already available) perform (separate) statistical tests for the following two null hypotheses:
 - (a) If you average over all thumb tacks the probability to point upwards, the result is 0.5.
 - (b) All thumb tacks have the same probability to point upwards.