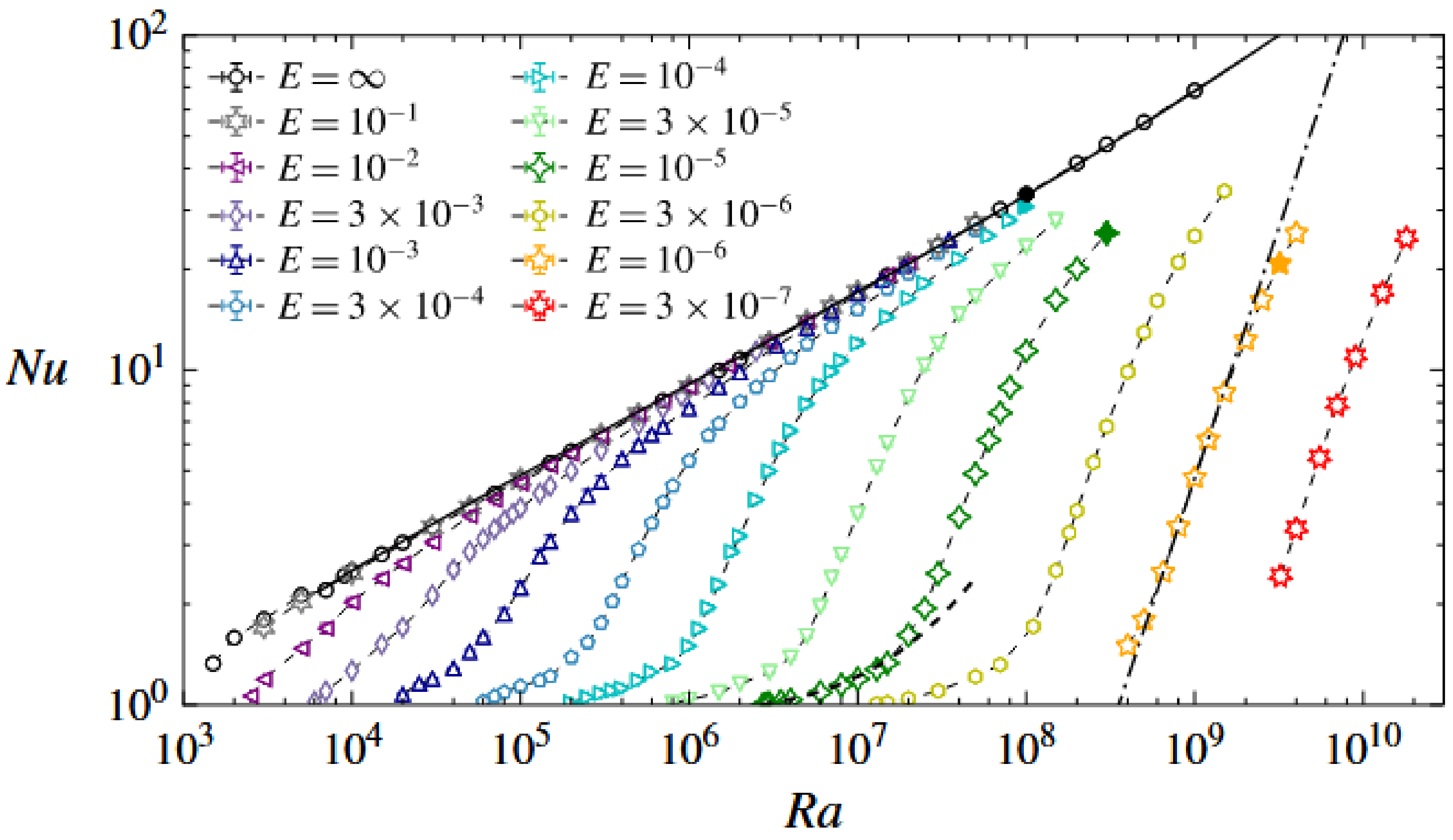
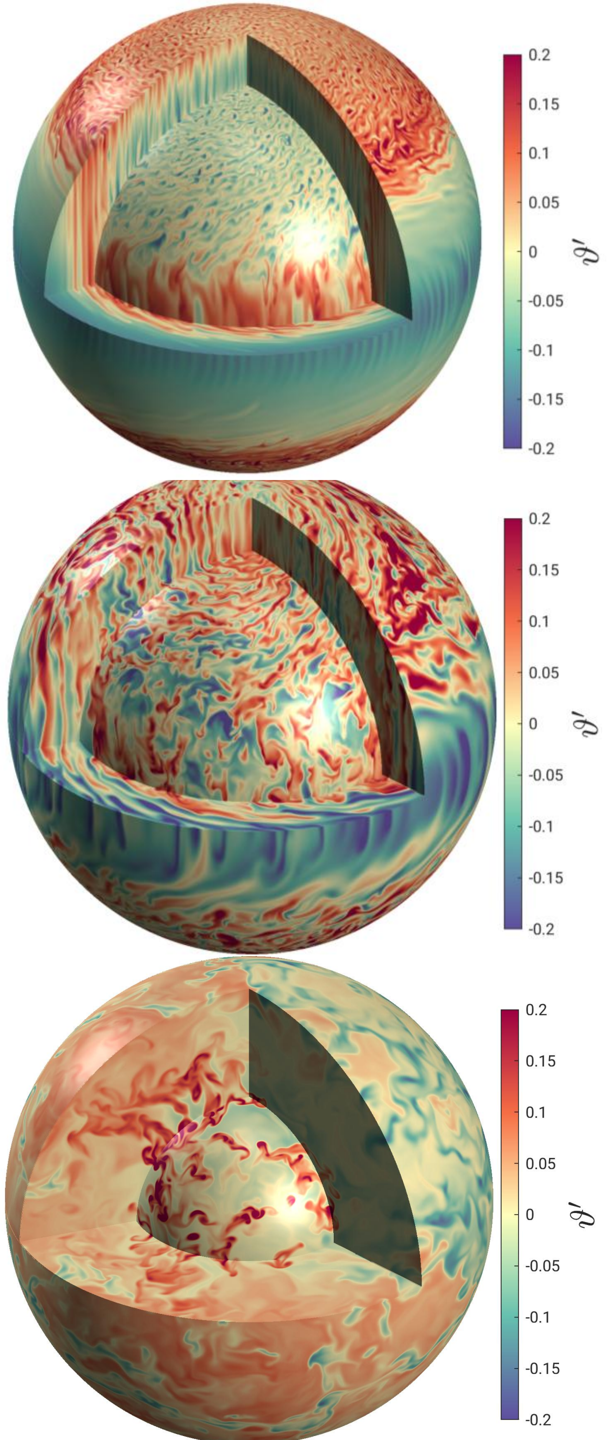
How does rotation influence heat transfer (Nu)



$Ro \approx 0.1$

$Ro \approx 0.5$

$Ro = \infty$



- Rotation-dominated regime

$$Ro = \frac{U}{2\Omega d} \ll 1$$

- Rotation-affected regime

$$Ro = \frac{U}{2\Omega d} \leq O(2)$$

- Buoyancy-dominated regime

$$Ro = \frac{U}{2\Omega d} > O(2)$$

How to choose the resolution?

- Key ideas

Grid size ≤ smallest dissipation scale

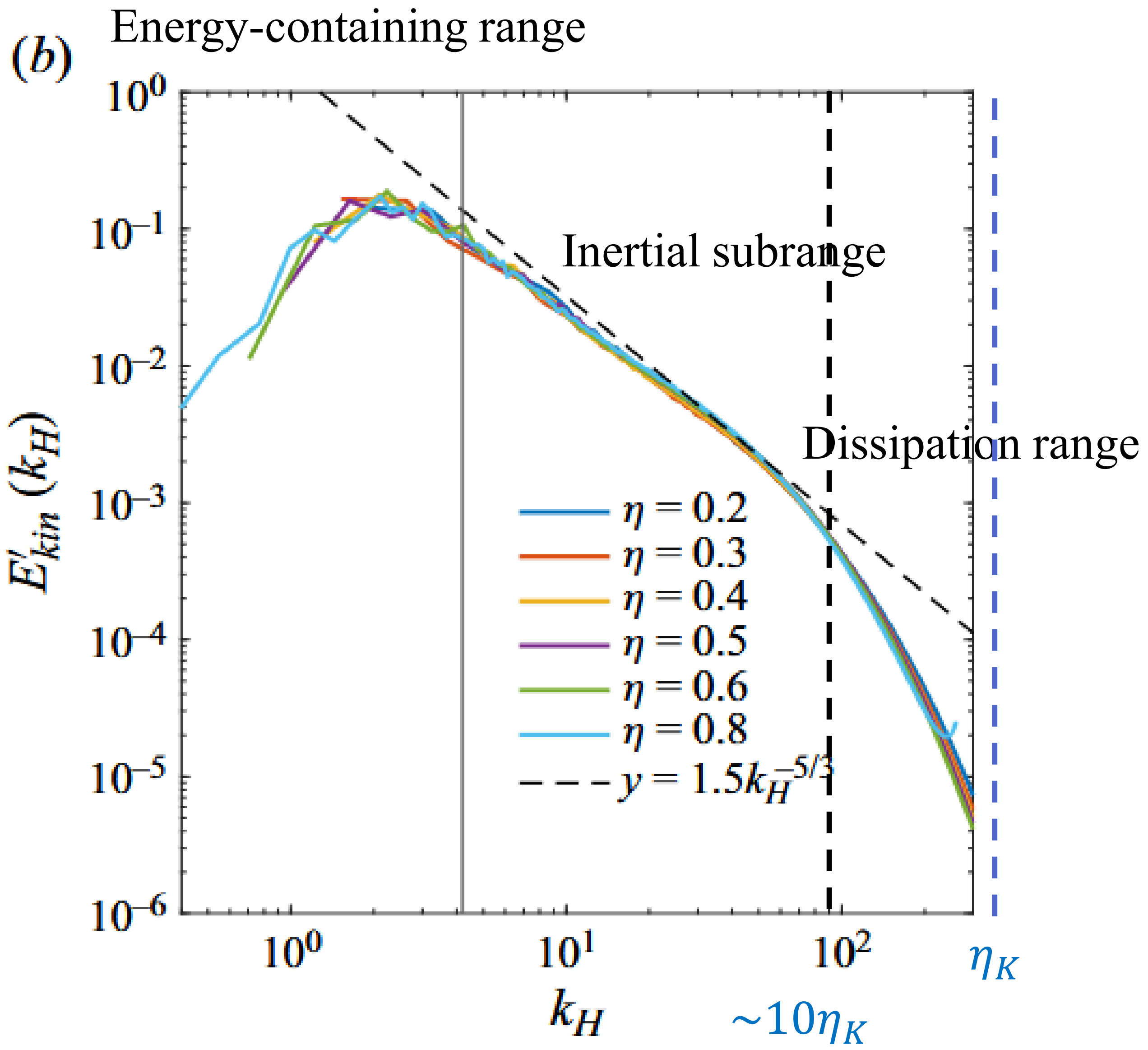
Kolmogorov scale: $\eta_K(\mathbf{x}, t) = \left(\nu^3/\epsilon_u(\mathbf{x}, t)\right)^{1/4}$

Batchelor scale: $\eta_B(\mathbf{x}, t) = \left(\nu\kappa^2/\epsilon_u(\mathbf{x}, t)\right)^{1/4}$

$$\eta_B = \eta_K Pr^{-\frac{1}{2}}$$

$Pr \leq 1$: $\eta_K < \eta_B$

$Pr > 1$: $\eta_B < \eta_K$



How to estimate η_K and η_B?

$$-\overline{\langle \boldsymbol{u} \cdot \nabla^2 \boldsymbol{u} \rangle} = \varepsilon_U = \frac{3}{1+\eta+\eta^2}\frac{Ra}{Pr^2}(Nu-1)$$

$$Pr \le 1: \quad \eta_K^{\mathrm{global}} \equiv \frac{\nu^{3/4}}{\langle \epsilon_u \rangle_{t,V}^{1/4}} = \frac{\mathcal{P}r^{1/2}}{\mathcal{R}a^{1/4}(\mathcal{N}u-1)^{1/4}} H.$$

$$Pr > 1: \quad \eta_B^{global} = \frac{\nu\kappa^2}{\langle \epsilon_u \rangle_{t,V}{}^{1/4}} = \frac{1}{Ra^{1/4}(Nu-1)^{1/4}} H$$

Let's choose $Pr = 1$ case as an example:

I wanna run simulations for $\eta = 0.4$ for $Ra = 10^6, 10^7, 10^8$.

How to choose resolution?

- Practical procedure

a) Start with a small case.

$$\eta = 0.4, Ra = 10^6$$

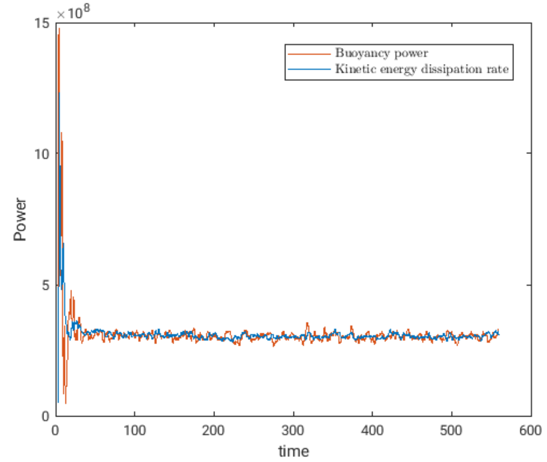
b) Estimate a resolution based on $\boldsymbol{\eta_K}$

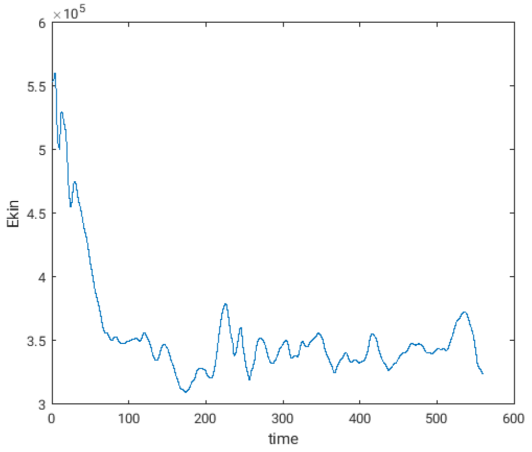
$$\frac{\eta_K^{globe}}{H} \approx 0.018 \qquad Nu \approx 10$$

$$\boldsymbol{N_r = 65}$$

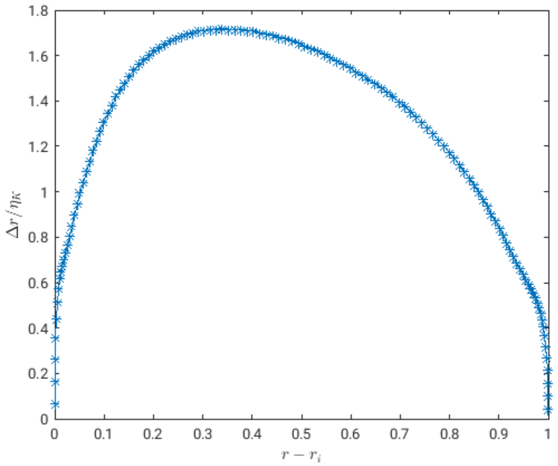
$$N_r \ge \frac{H}{\eta_K^{globe}} \approx 56 \qquad \boldsymbol{N_\phi = 384}$$

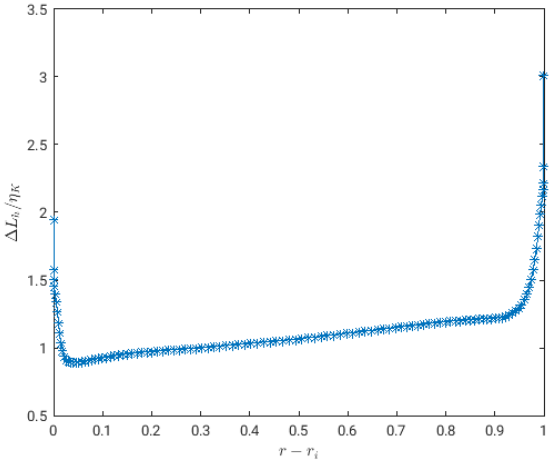
c) Perform the simulation until converged





d) Check the $\Delta r/\eta_K$ and $\Delta L_h/\eta_K$





$$Nu = \frac{\overline{\langle u_r T\rangle_s} - \frac{1}{Pr}\frac{\mathrm{d}\vartheta}{\mathrm{d}r}}{-\frac{1}{Pr}\frac{\mathrm{d}T_c}{\mathrm{d}r}} = -\eta\frac{\mathrm{d}\vartheta}{\mathrm{d}r}\bigg|_{r=r_i} = -\frac{1}{\eta}\frac{\mathrm{d}\vartheta}{\mathrm{d}r}\bigg|_{r=r_o},$$

$$\varepsilon_U = \frac{3}{1+\eta+\eta^2}\frac{Ra}{Pr^2}(Nu-1) \qquad \varepsilon_T = \frac{3\eta}{1+\eta+\eta^2}Nu$$

$$Nu_i = -\eta\frac{d\vartheta}{dr}\Big|_{r=r_i}, \quad Nu_o = -\frac{1}{\eta}\frac{d\vartheta}{dr}\Big|_{r=r_o},$$

$$Nu_{\epsilon_u} = \frac{1+\eta+\eta^2}{3}\frac{Pr^2}{Ra}\epsilon_u + 1, \quad Nu_{\epsilon_\vartheta} = \frac{1+\eta+\eta^2}{3\eta}\epsilon_\vartheta.$$

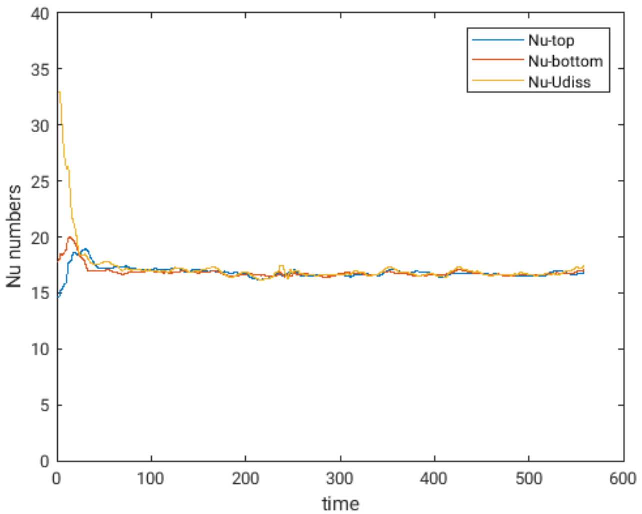
d) If the $\Delta r/\eta_K$ or $\Delta L_h/\eta_K > 2$ in the bulk

Say $\frac{\Delta r}{\eta_K} = 5$, try $N_r^{new} \approx 1.5 N_r^{old}$

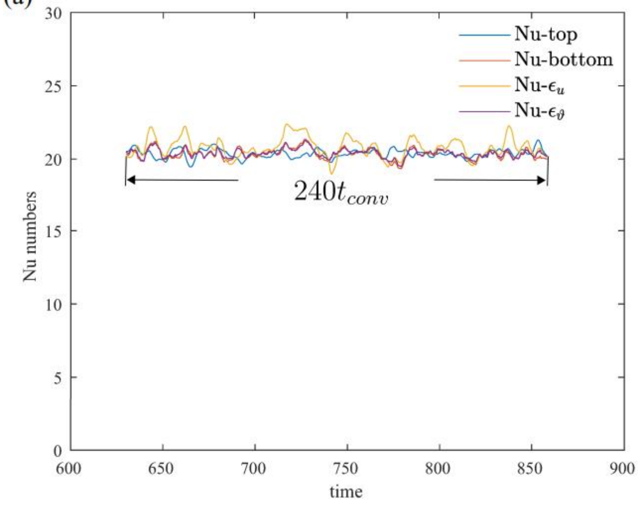
$$\eta_K^{global} = \frac{\nu^3}{\langle\epsilon_u\rangle_{t,V}}^{1/4}$$

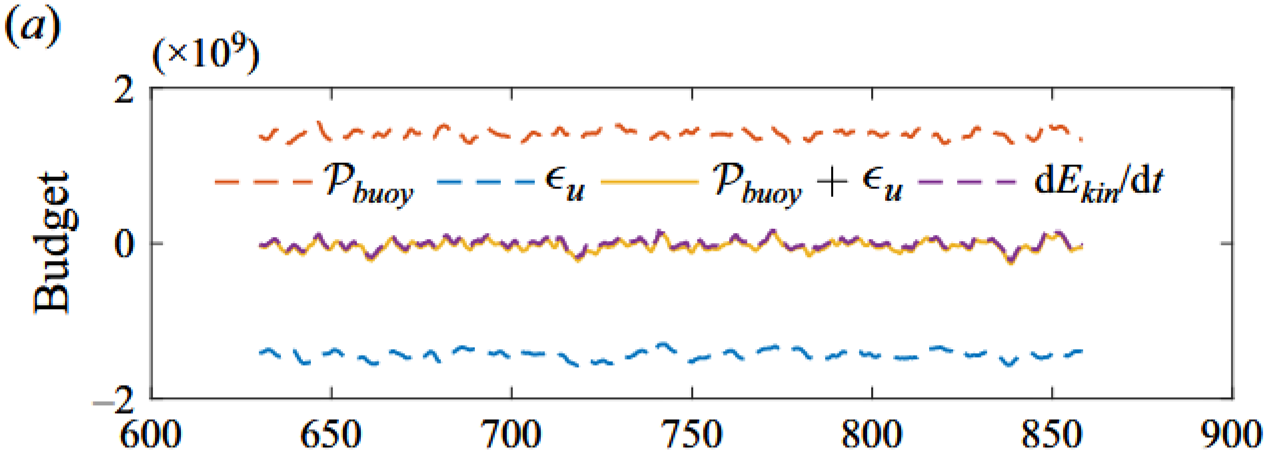
$N_r^{new} = 97$

e) Run it again and check the convergence

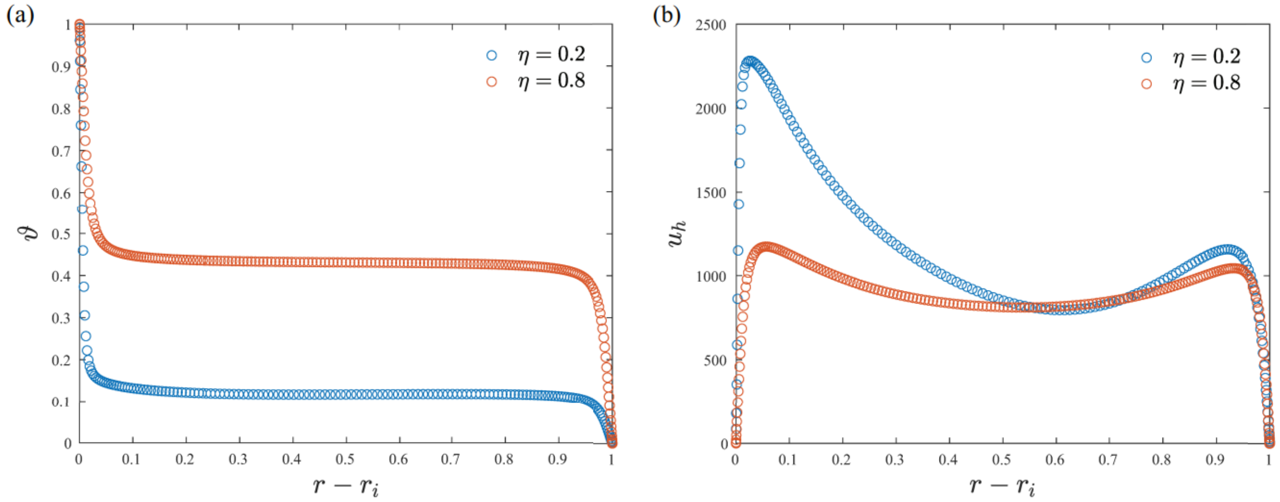


(a)





$$\mathcal{P}_{buoy} = \frac{Ra}{Pr}\langle gTu_r\rangle, \quad \epsilon_u = \langle \boldsymbol{u}\cdot\nabla^2\boldsymbol{u}\rangle.$$



At least 5 grid points inside BLs (10 is better)

d) Extroplate the resolution to other *Ra*

$$\eta_K^{\mathrm{global}} \equiv \frac{\nu^{3/4}}{\langle\epsilon_u\rangle_{t,V}^{1/4}} = \frac{\mathcal{P}r^{1/2}}{\mathcal{R}a^{1/4}(\mathcal{N}u-1)^{1/4}}H.$$

$$Nu \sim Ra^{1/3}$$

Ra	$N_r \times N_\phi \times N_\theta$
10^6	$65 \times 384 \times 192$
10^7	$121 \times 864 \times 432$
10^8	$201 \times 1792 \times 896$

We could obtain from boundary layer theories:

$$h^{\mathrm{BL}} \lesssim \begin{cases} 2^{-3/2}a^{-1}\mathcal{N}u^{-3/2}\mathcal{P}r^{3/4}A^{-3/2}\pi^{-3/4}H, & \mathcal{P}r < 3\times 10^{-4}, \\ 2^{-3/2}a^{-1}\mathcal{N}u^{-3/2}\mathcal{P}r^{0.5355-0.033\log\mathcal{P}r}H, & 3\times 10^{-4} \leqslant \mathcal{P}r \leqslant 1, \\ 2^{-3/2}a^{-1}\mathcal{N}u^{-3/2}\mathcal{P}r^{0.0355-0.033\log\mathcal{P}r}H, & 1 < \mathcal{P}r \leqslant 3, \\ 2^{-3/2}a^{-1}E^{-3/2}\mathcal{N}u^{-3/2}H, & \mathcal{P}r > 3, \end{cases}$$

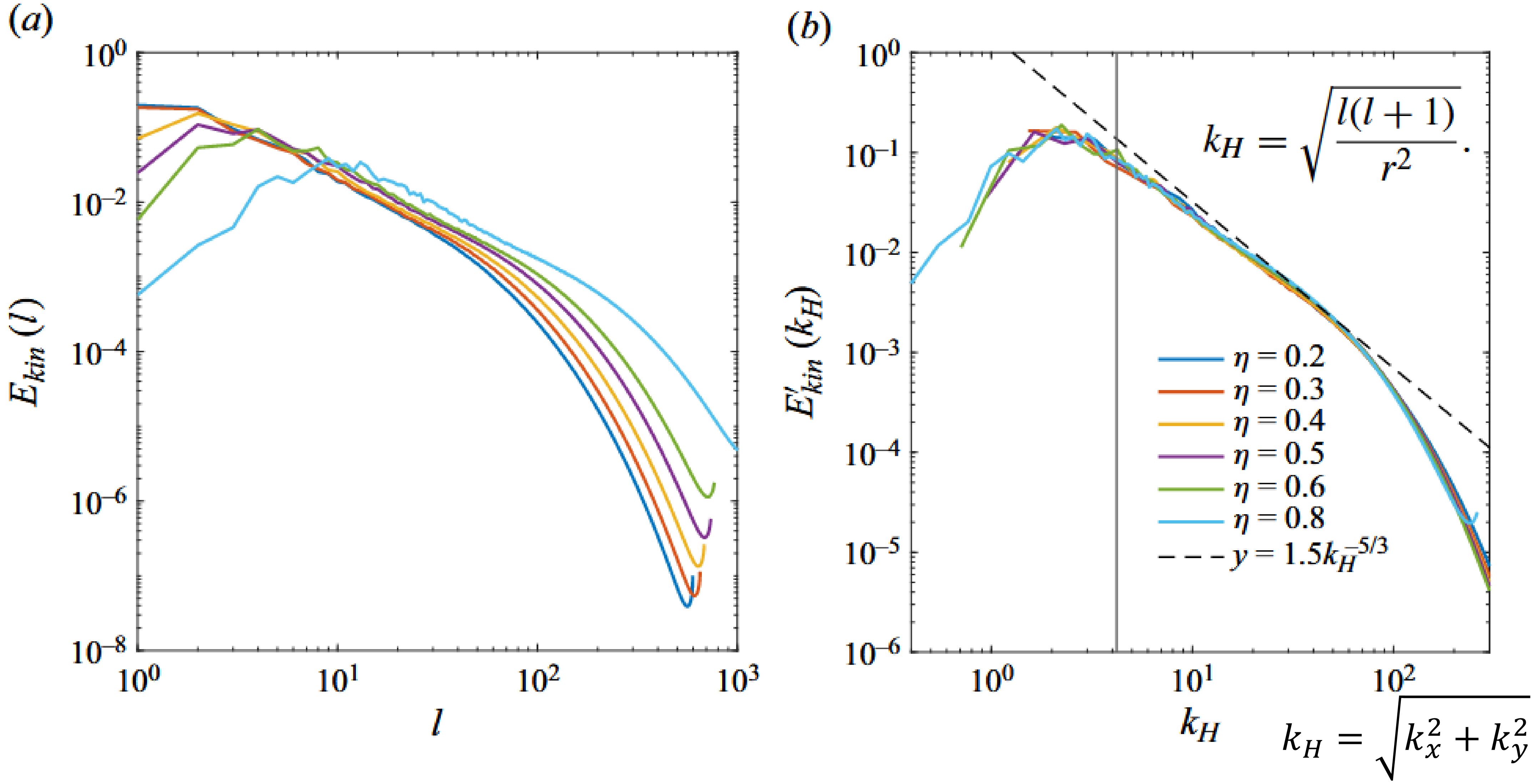


Figure 36. Kinetic energy spectra (*a*) $E_{kin}(l)$ with respect to harmonic degree l, and (*b*) $E'_{kin}(k_H)$ with respect to effective horizontal wavenumber k_H. The vertical black solid line in panel (*b*) is $k_H = 4.2$, which corresponds to $L = 2\pi/k_H = 1.5$. All the spectra are with the same $Ra = 3 \times 10^8$.

The smallest length scale

Dissipative spectrum: $D(k) = 2\nu k^2 E(k)$

Cumulative dissipation: $\varepsilon_{(0,k)} = \int_0^k D(k')dk'$

Wavelength: $l = 2\pi/k$, Kolmogorov scale: $\eta = \left(\frac{\nu^3}{\varepsilon_k}\right)^{\frac{1}{4}}$

$l = 2\pi/k \sim$ length scale $\Rightarrow \begin{cases} l_{min} \approx 4\eta \\ l_{0.8} \approx 10\eta \end{cases}$

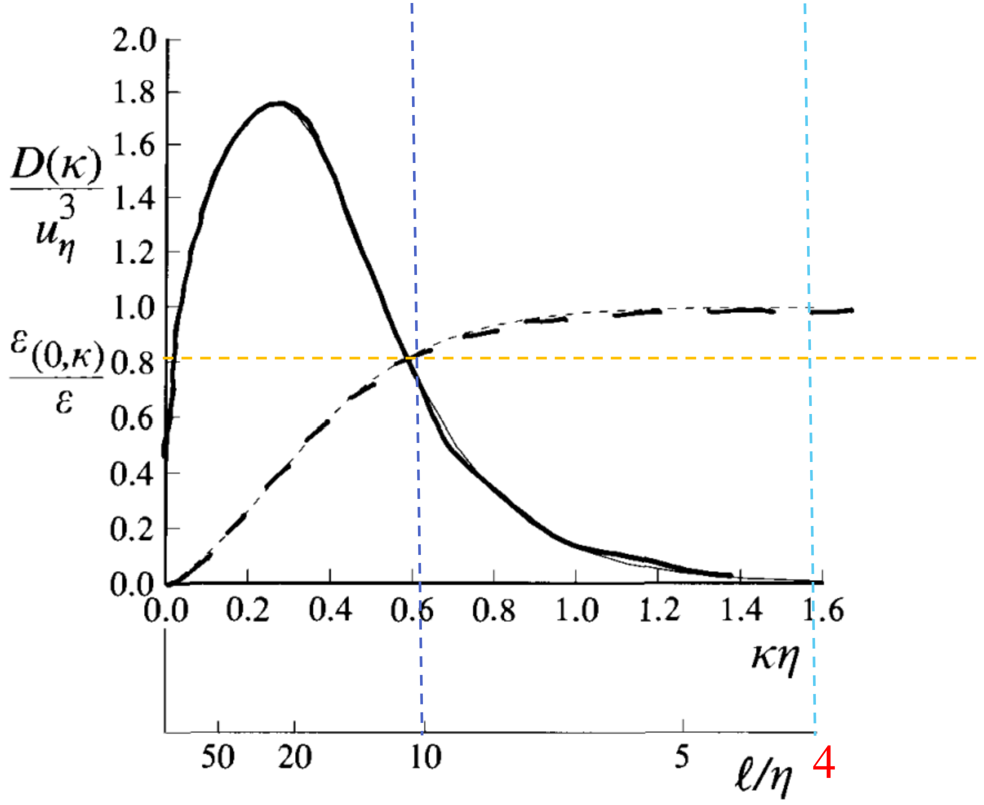


Fig. 6.16. The dissipation spectrum (solid line) and cumulative dissipation (dashed line) corresponding to the model spectrum Eq. (6.246) for $R_\lambda = 600$. $\ell = 2\pi/\kappa$ is the wavelength corresponding to wavenumber κ.

Max Planck Society

THANKS

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