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Workshop: Dynamos in a Nutshell

Double-Diffusive Convection

Mohammad Anas, Johannes Wicht, Xiaojue Zhu
Max Planck Institute for Solar System Research

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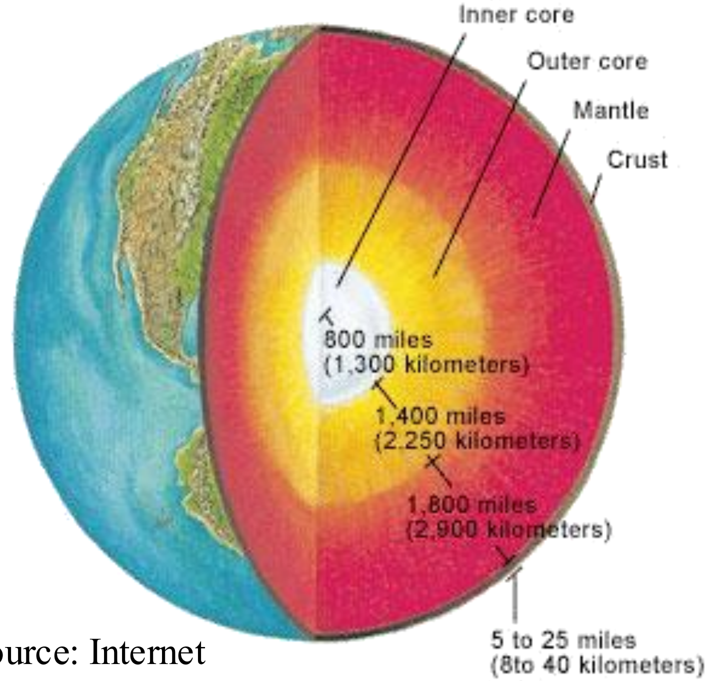
Motivation

- Convection (Thermal and Compositional) in Earth's outer core sustains Earth's magnetic field through **Dynamo** action.



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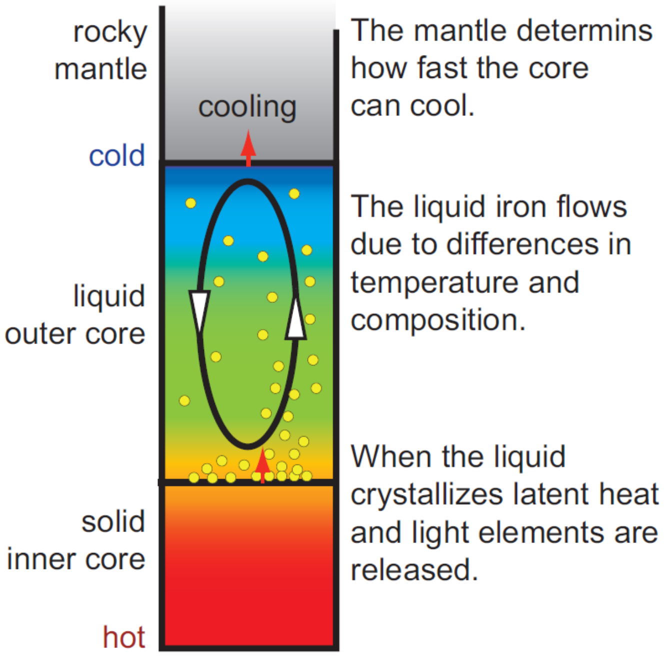
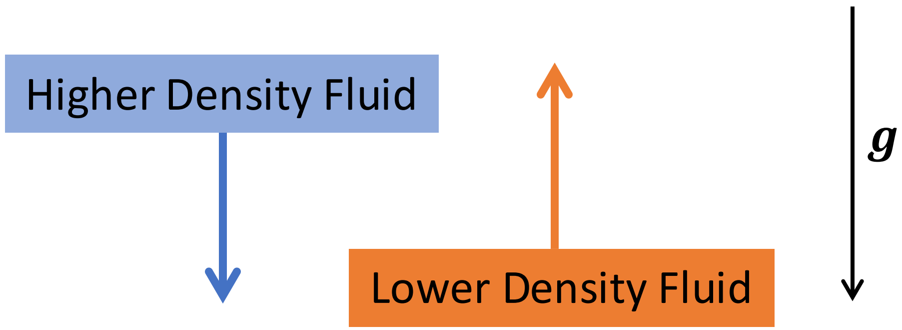
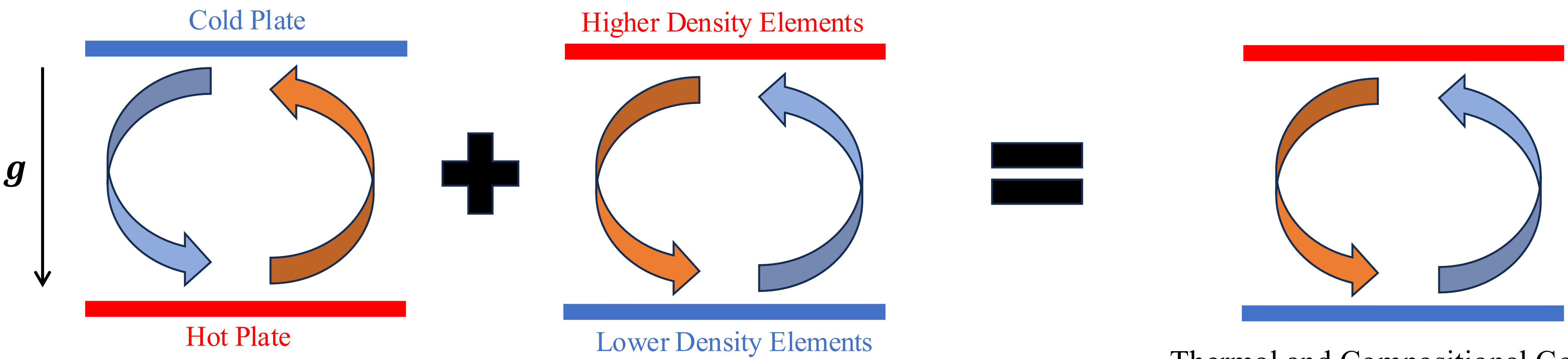


- By the crystallization of molten metal alloys (Fe, Ni) inner core grows and releases light elements (Si, S, O2)
- DDC also plays an important role on Gas giants like Jupiter, and in ocean where salt fingers form

Double-Diffusive Convection (DDC)

- Convection is a Buoyancy-Driven Flow
- Density gradient (parallel to gravity) causes convection
- Lower density fluid rises and higher density fluid sinks



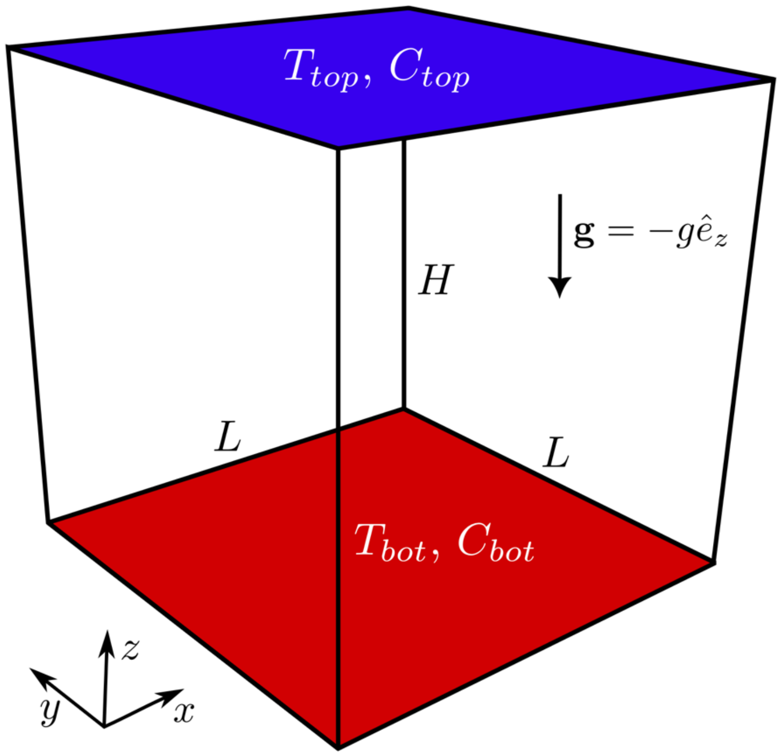


Thermal Convection
Rayleigh-Bénard Convection (RBC)

Compositional Convection

Thermal and Compositional Convection
(DDC)

Model for DDC



Governing Equations (under Boussinesq Approximation)

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nu \nabla^2 \boldsymbol{u} + (g\beta_T T - g\beta_C C)\hat{e}_z$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \kappa_T \nabla^2 T$$

$$\frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \nabla C = \kappa_C \nabla^2 C$$

$$\nabla \cdot \boldsymbol{u} = \mathbf{0}$$

$$\rho = \rho_0(1 - \beta_T(T - T_0) + \beta_C(C - C_0))$$

$\boldsymbol{u}$: velocity field, p: pressure field, T: temperature field, C: Compositional field

ν: kinematic viscosity, β_T: thermal expansion coefficient, β_C: compostional contraction coefficient

κ_T: thermal diffusivity, κ_C: compositional diffusivity, g: acceleration due to gravity

Non-Dimensional Equations

- Length scale (ℓ): H, Velocity scale (U_0): $\sqrt{g\beta_C \Delta C H}$, Time scale ($\tau_c$) = ℓ / U_0
- Temperature scale $T \rightarrow \frac{T-T_{bot}}{\Delta T} \in [0,1]$, Composition scale $C \rightarrow \frac{C-C_{bot}}{\Delta C} \in [0,1]$, $\Delta T = T_{top} - T_{bot}$, $\Delta C = C_{top} - C_{bot}$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \sqrt{\frac{Pr_C}{Ra_C}}\, \nabla^2 \boldsymbol{u} + \left(\frac{Pr_C}{Pr_T} \frac{Ra_T}{Ra_C} T - C \right) \hat{e}_z$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \frac{1}{Pr_T} \sqrt{\frac{Pr_C}{Ra_C}}\, \nabla^2 T$$

$$\frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \nabla C = \sqrt{\frac{1}{Ra_C Pr_C}}\, \nabla^2 C$$

$$\nabla \cdot \boldsymbol{u} = 0$$

Non-Dimensional Parameters

Input Parameters:

- Compositional Rayleigh number: $Ra_c = \frac{compositional\ buoyancy}{dissipative\ forces} = \frac{\beta_c g \Delta C H^3}{\nu \kappa_c}$
- Thermal Rayleigh number: $Ra_T = \frac{thermal\ buoyancy}{dissipative\ forces} = \frac{\beta_T g \Delta T H^3}{\nu \kappa_T}$
- Thermal Prandtl number: $Pr_T = \nu/\kappa_T$
- Compositional Prandtl number: $Pr_C = \nu/\kappa_C$
- Lewis number: $Le = \frac{\kappa_T}{\kappa_C} = \frac{Pr_C}{Pr_T}$
- Density ratio (Buoyancy ratio): $\Lambda = \frac{thermal\ buoyancy}{compositional\ buoyancy} = \frac{g\beta_T \Delta T}{g\beta_C \Delta C} = Le\frac{Ra_T}{Ra_C}$

Output Parameters:

- Heat and Composition Transfer: Nusselt number $Nu = \frac{transfer\ of\ heat\ or\ composition\ by\ convection}{transfer\ of\ heat\ or\ composition\ by\ diffusion} + 1$
- Flow strength: Reynolds number $Re = \frac{Inertial\ force}{viscous\ force} = \frac{UH}{\nu}$

Density Ratio (Λ) and Codensity Approach

$$\Lambda = \frac{g\beta_T \Delta T}{g\beta_C \Delta C} = \frac{Thermal\ Buoyancy}{Compositional\ Buoyancy}$$

$\Lambda \ll 1$:

- Compositional Buoyancy Controls Convection
- No-significant importance of Thermal Buoyancy

$\Lambda \gg 1$:

- Thermal Buoyancy Controls Convection
- No-significant importance of compositional Buoyancy

$\Lambda \sim 1$:

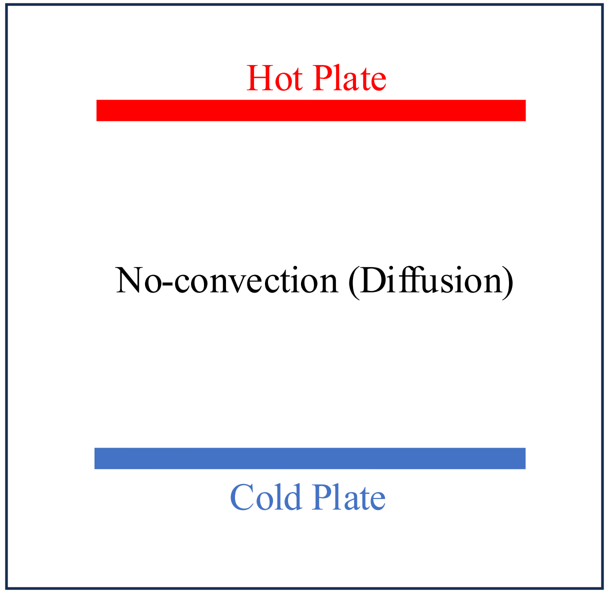
- Thermal and Compositional Buoyancy both Control Convection
- Both thermal and compositional buoyancy are important

- $Le \sim 1 \rightarrow \kappa_T \sim \kappa_C$: Equations for T and C are dynamically same

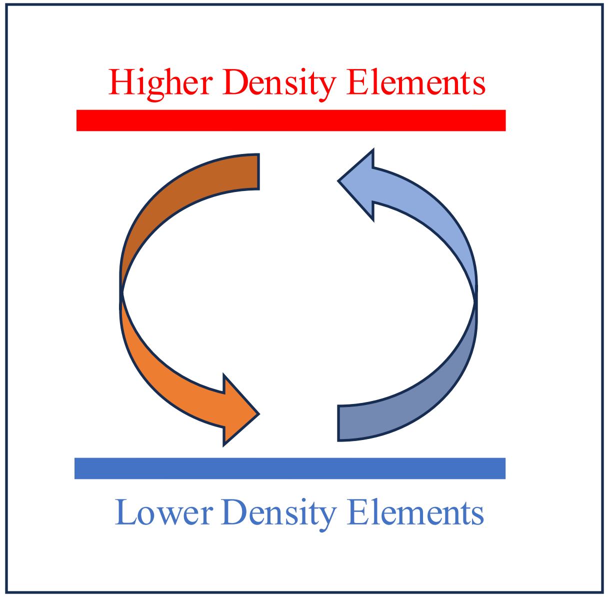
Codensity Approach:

- Solve one scalar equation when both T and C driving
- May not be a good approximation when:
- $Le \gg 1$ or $Le \ll 1$
- Both are not driving convection

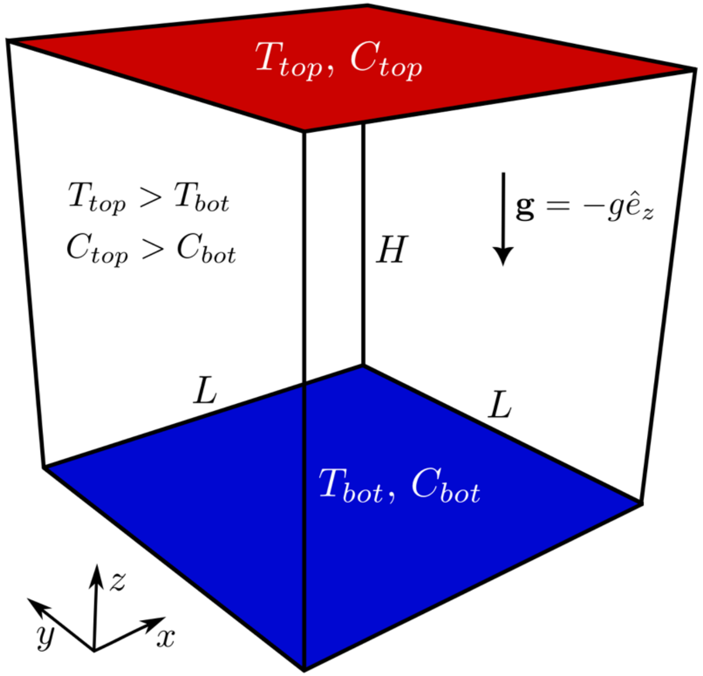
Fingering Convection: A Special Case of DDC



Stabilizing Thermal Buoyancy Force
(Stably Stratified)



Destabilizing Compositional Buoyancy Force



Competition b/w Temp. and Comp. buoyancy!

- Compositional Buoyancy Drive the convection
- Thermal Buoyancy stabilizes it

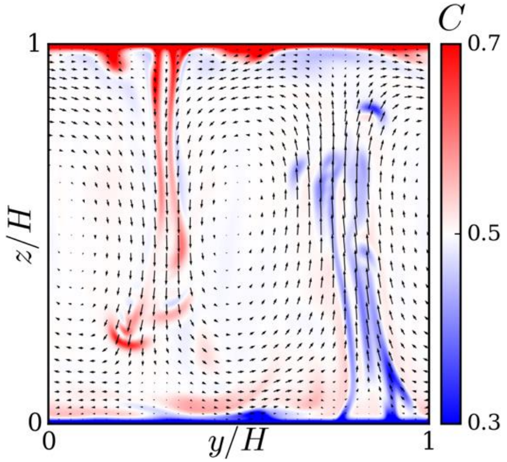
$$Le > 1 \quad i.e. \ \kappa_T > \kappa_C$$

- Fingering DDC plays an important role in ocean where salt fingers form
- Stable layer at the core-mantle-boundary (CMB), similar phenomenon is expected

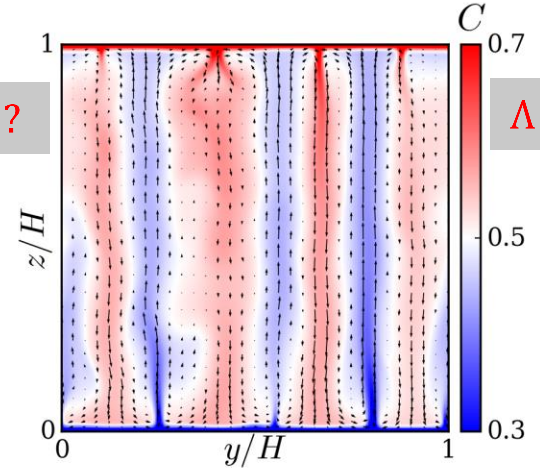
Objective: Regime Transitions?

Increasing Thermal Stabilization (Λ)

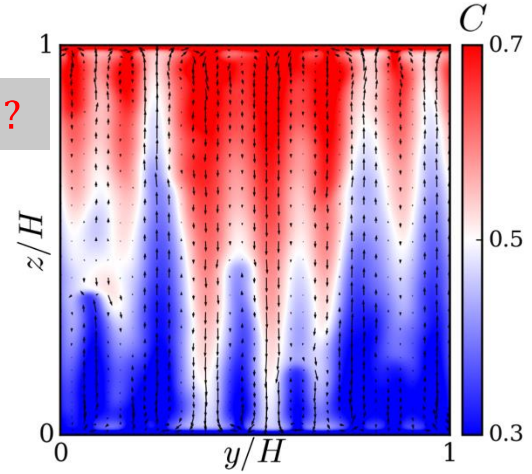
Stabilization-unaffected (Quasi-RB) Regime	$\Lambda = ?$	Stabilization-affected (Finger) Regime	$\Lambda = ?$	Stabilization-dominated (Damping) Regime	$\Lambda = Le$ ($Ra_T = Ra_C$)	No-convection (Diffusion) Regime



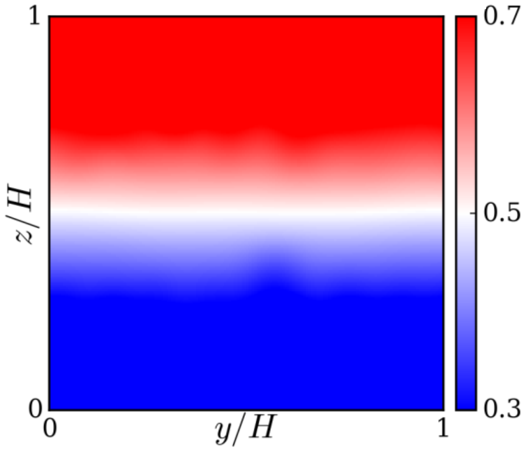
Weak Thermal Stabilization



Significant Thermal Stabilization



Strong Thermal Stabilization

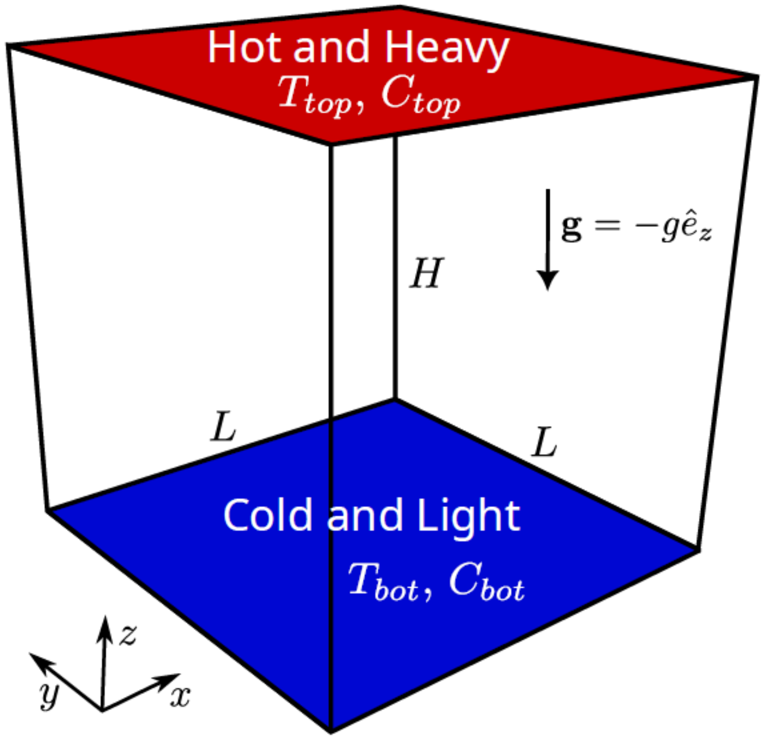


$\Lambda = Le$

$(Ra_T = Ra_C)$

Simulation Details

- Horizontally Periodic Box
- No-slip, constant temperature and composition at plates
- $Ra_C = 10^8$
- $Ra_T = 0$ to 10^8
- $Pr_T = 1$
- $Pr_C = 3, 10, 30, 100$
- $Le = \frac{Pr_C}{Pr_T} = 3, 10, 30, 100$
- Solver: GPU-accelerated Python Solver
- Computing Resources: swan cluster (swangpu)

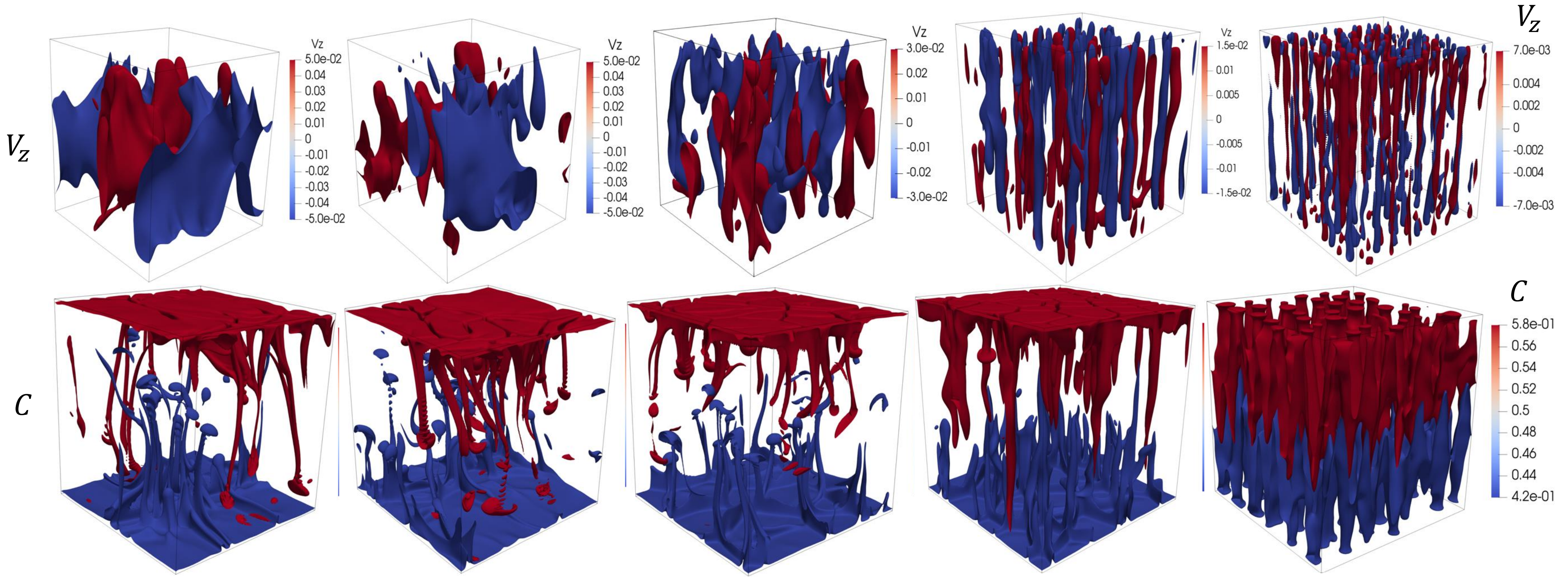


Flow Structures

Stabilization-unaffected (Quasi-RB) Regime

Stabilization-affected (finger) Regime

Stabilization-dominated (damping) Regime

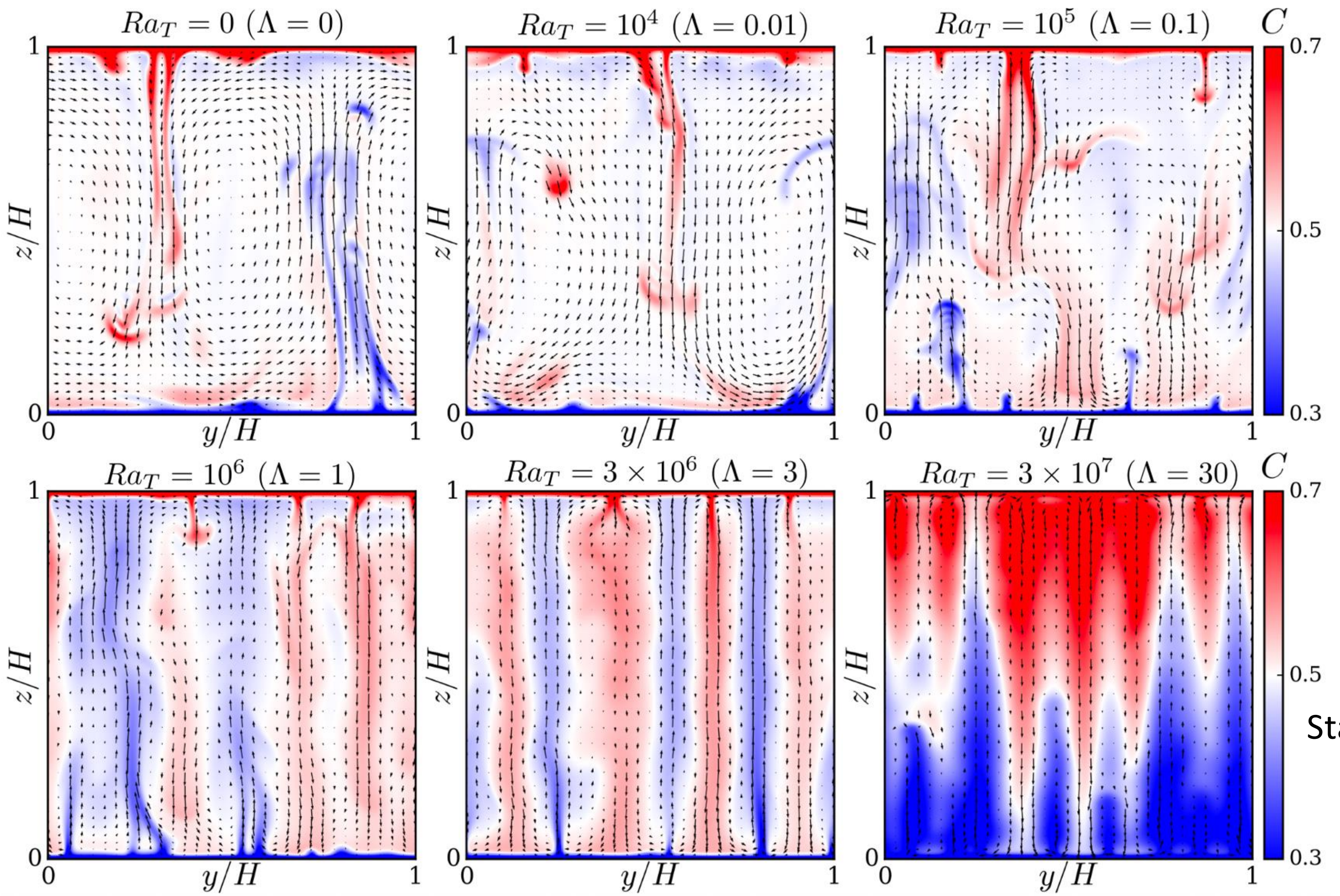


$Ra_T = 0\ (\Lambda = 0)$ $\quad$ $Ra_T = 10^4\ (\Lambda = 0.01)$ $\quad$ $Ra_T = 10^5 (\Lambda = 0.1)$ $\quad$ $Ra_T = 3 \times 10^6\ (\Lambda = 3)$ $\quad$ $Ra_T = 3 \times 10^7\ (\Lambda = 30)$

$Ra_C = 10^8, \qquad Le = 100$

Flow Structures

Stabilization-unaffected (Quasi-RB) Regime

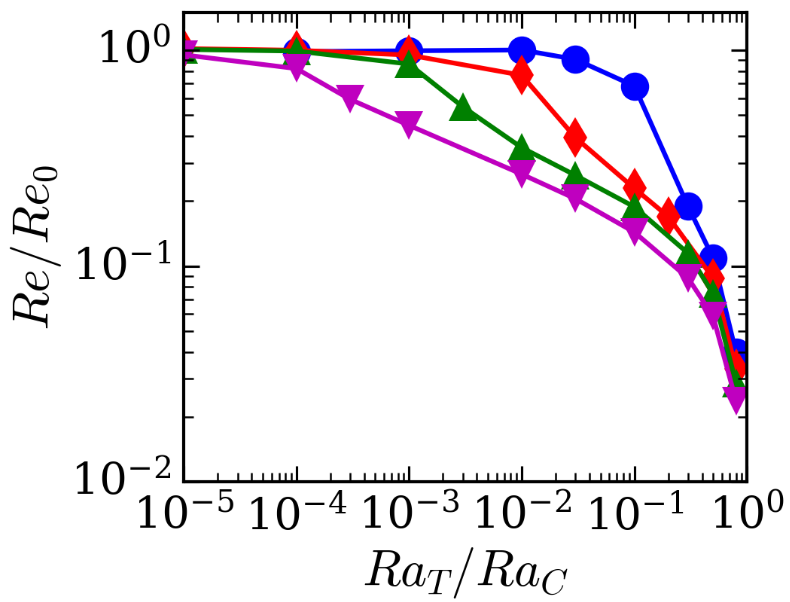


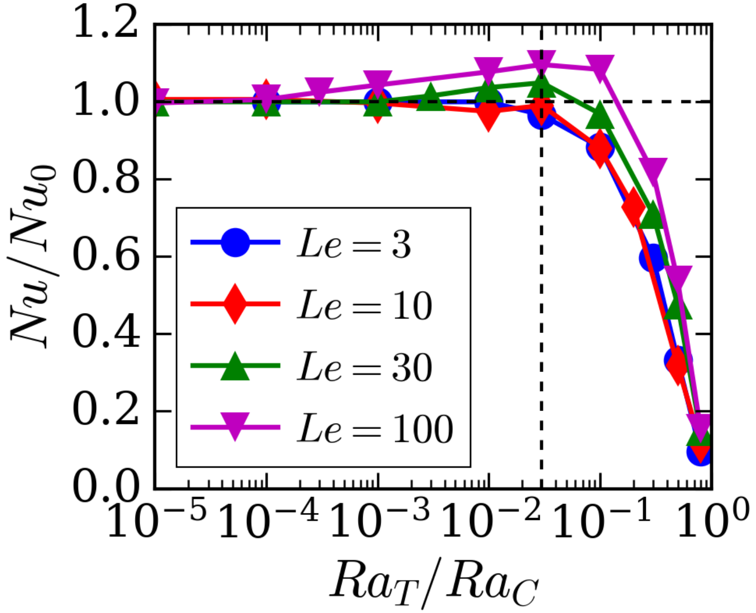
Stabilization-dominated (damping) Regime

Stabilization-affected (finger) Regime

Contours of composition with velocity vectors in yz-plane for various Ra_T (Λ) at $Ra_C = 10^8$ and $Le = 100$

Effect of Thermal Stabilization



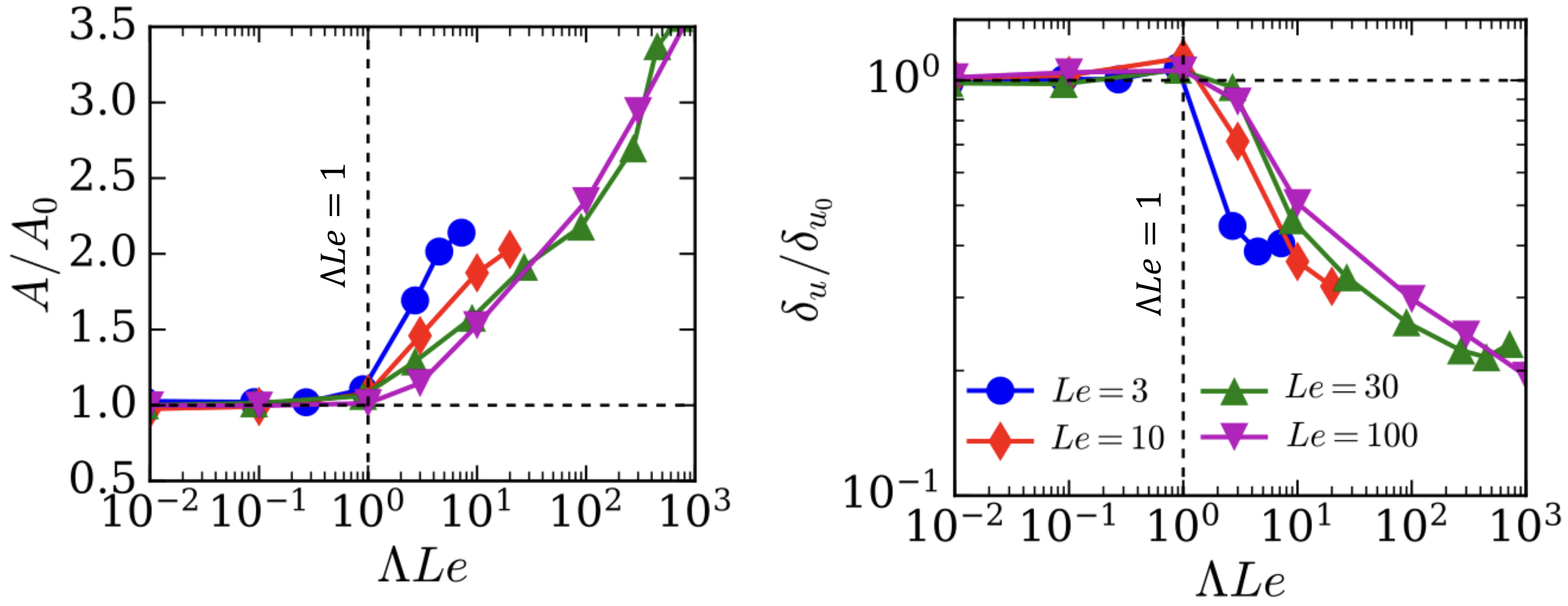


- $Re_0 = Re(Ra_T = 0)$
- $Nu_0 = Nu(Ra_T = 0)$

No-convection for $\frac{Ra_T}{Ra_C} \geq 1$

- Stabilization weakens flow strength
- Stabilization can enhance salinity transfer
- Transition to the damping regime: $Ra_T \sim 0.03 Ra_C$ (or $\Lambda \sim 0.03 Le$)

Transition to Finger Regime



Anisotropy $A = \sqrt{2}Re_V/Re_H$

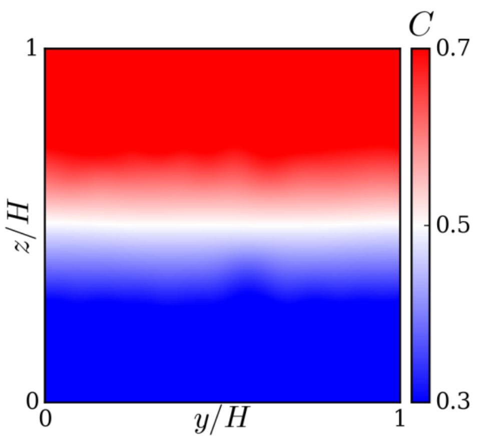
δ_u: velocity boundary layer thickness

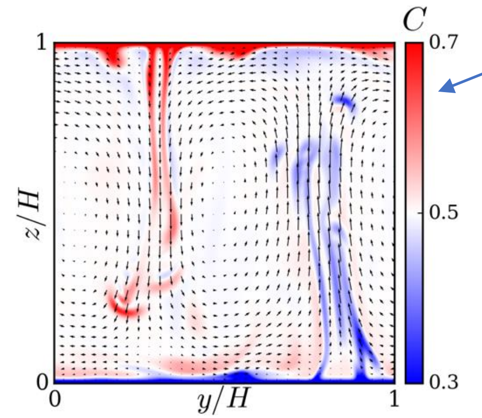
$A_0 = A(Ra_T = 0)$

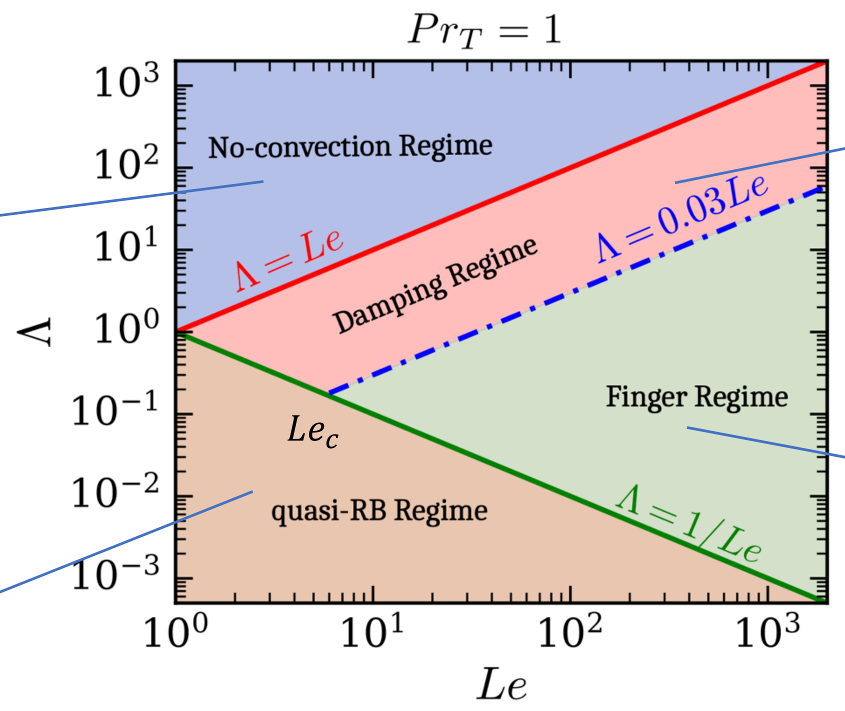
$\delta_{u0} = \delta_u(Ra_T = 0)$

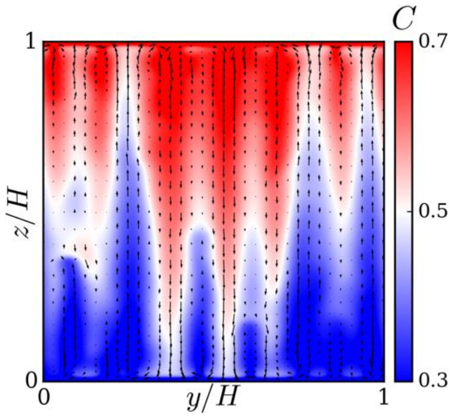
- Transition occurs at $\Lambda \sim 1/Le$
- Anisotropy increases and velocity boundary layer thickness decreases with stabilization

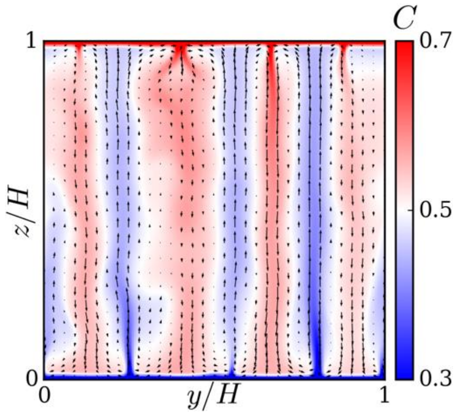
Regimes in Fingering Convection









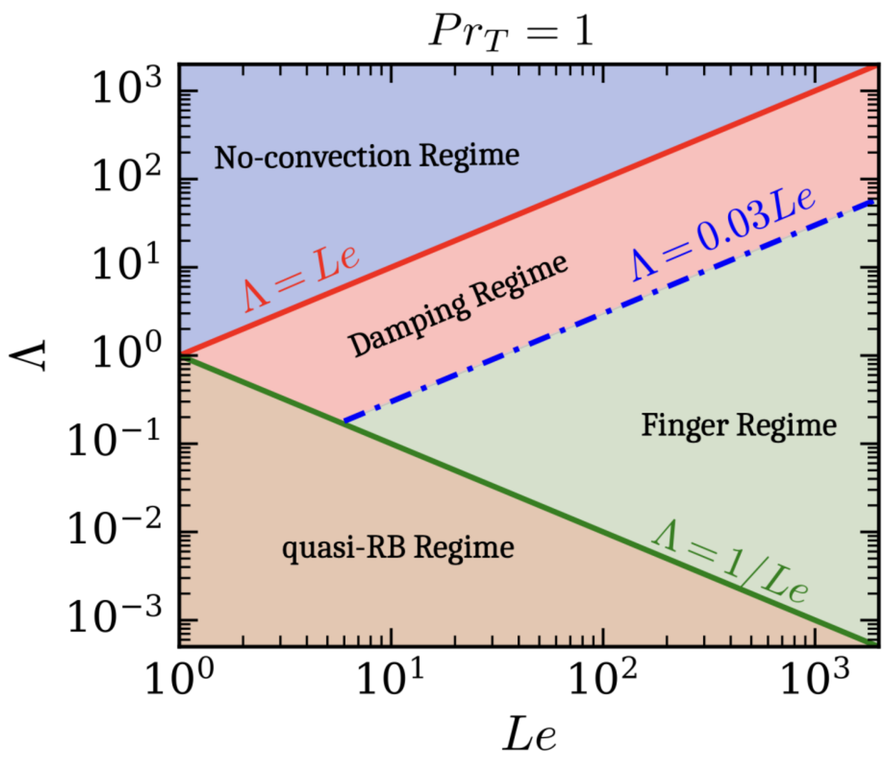


- $\boldsymbol{\Lambda \approx 1/Le}$ marks the transition from quasi-RB to finger regime
- $\boldsymbol{\Lambda \approx 0.03Le}$ marks the transition from finger to damping regime
- $\boldsymbol{\Lambda = Le}$ marks the transition from damping to diffusion regime

- **Critical Lewis Number ($Le_c \approx 6$)** below which flow directly transitions to damping regime, bypassing finger regime

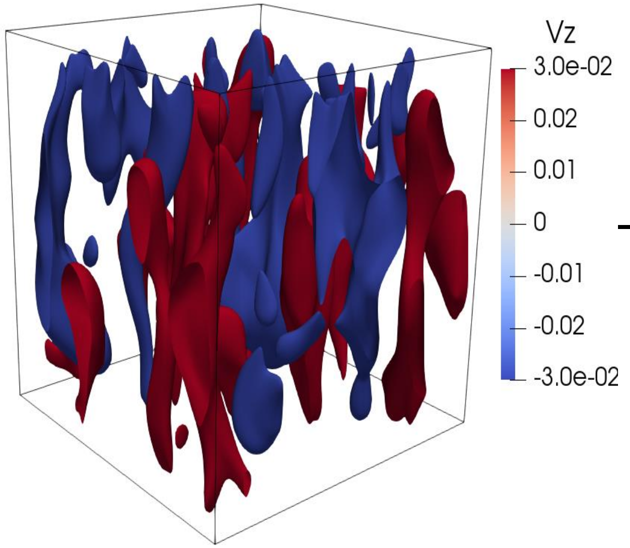
Summary

- Regime transitions in double-diffusive fingering convection
  - Quasi-RB regime: $\Lambda < 1/\mathrm{Le}$
  - Finger regime: $1/Le < \Lambda < 0.03\mathrm{Le}$
  - Damping regime: $0.03Le < \Lambda < \mathrm{Le}$
  - Diffusion regime: $\Lambda \geq Le$

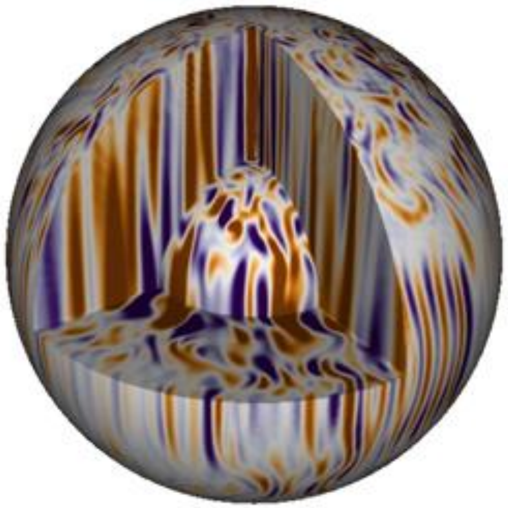


Upcoming Research Plan

- DDC with Rotation
- More Realistic boundary conditions for temperature and composition
- Dynamo with and without an inner core (in a Spherical Shell, using MagIC)
- Dynamo with a stable layer at CMB (in a Spherical Shell, using MagIC)



DDC simulations in Box



Dynamo simulations in Spherical Shell