Dynamo Solutions

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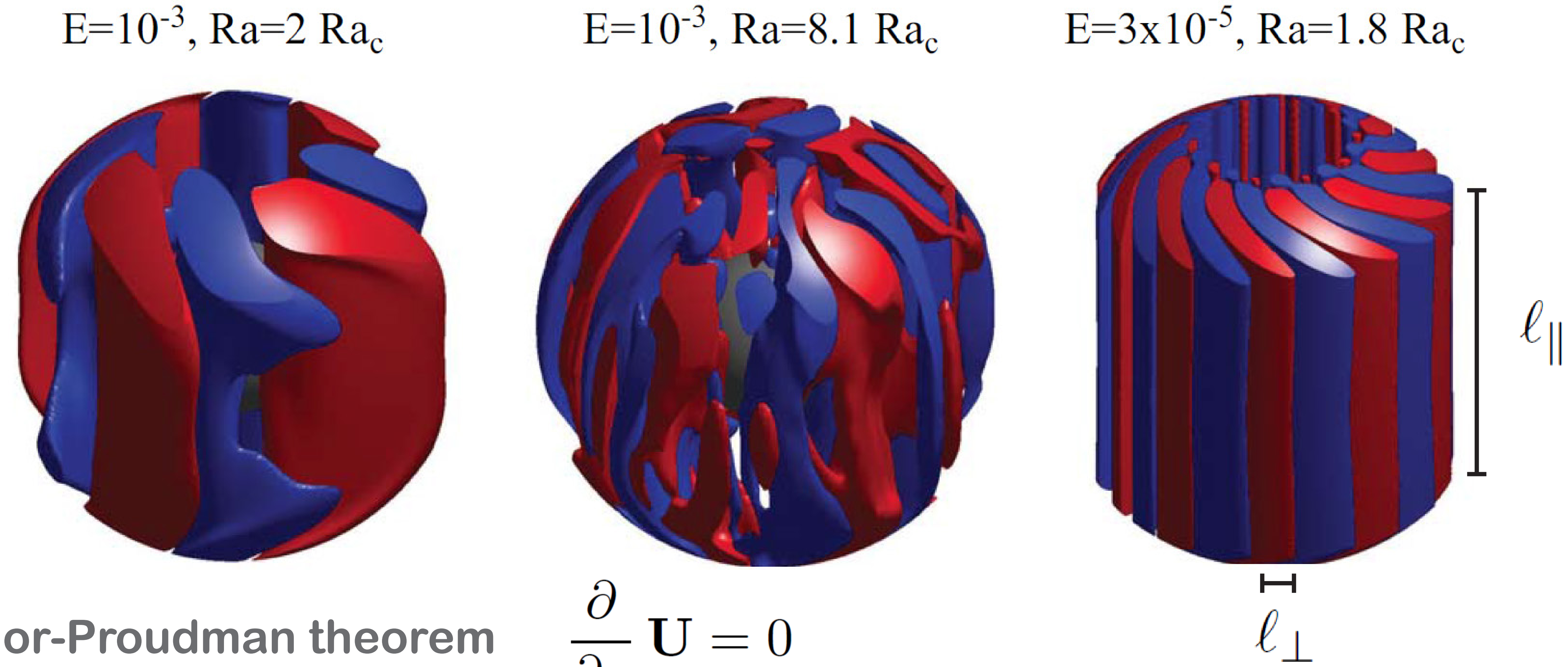
Overview

1.1 Dimensionless Parameter

symbol	definition	standard	high end	Earth
E	$\nu/(\Omega\ell^2)$	3×10^{-5}	10^{-7}	10^{-15}
Pm	ν/λ	1	0.1	10^{-6}
Pr	ν/κ	0.1 – 1	1	1??
$\mathcal{P}$	$P/(\ell^2\Omega^3)$	10^{-5}	10^{-7}	10^{-12}
Λ	$\mathrm{B}^2\sigma/(\rho\Omega)$	1 – 10	10	10
Le	$\mathrm{B}/(\rho^{1/2}\mu^{1/2}\Omega\ell)$	$5\times10^{-3} - 3\times10^{-2}$	3×10^{-3}	10^{-4}
Rm	$\mathrm{U}\ell/\lambda$	$10^2 - 10^3$	10^3	10^3
Re	$\mathrm{U}\ell/\nu$	$10^2 - 10^3$	10^4	10^9
Ro	$\mathrm{U}/(\ell\Omega)$	$3\times10^{-3} - 3\times10^{-2}$	10^{-3}	10^{-6}
M_A	$\mathrm{U}(\rho\mu)^{1/2}/\mathrm{B}$	0.2 – 2	10^{-1}	10^{-2}

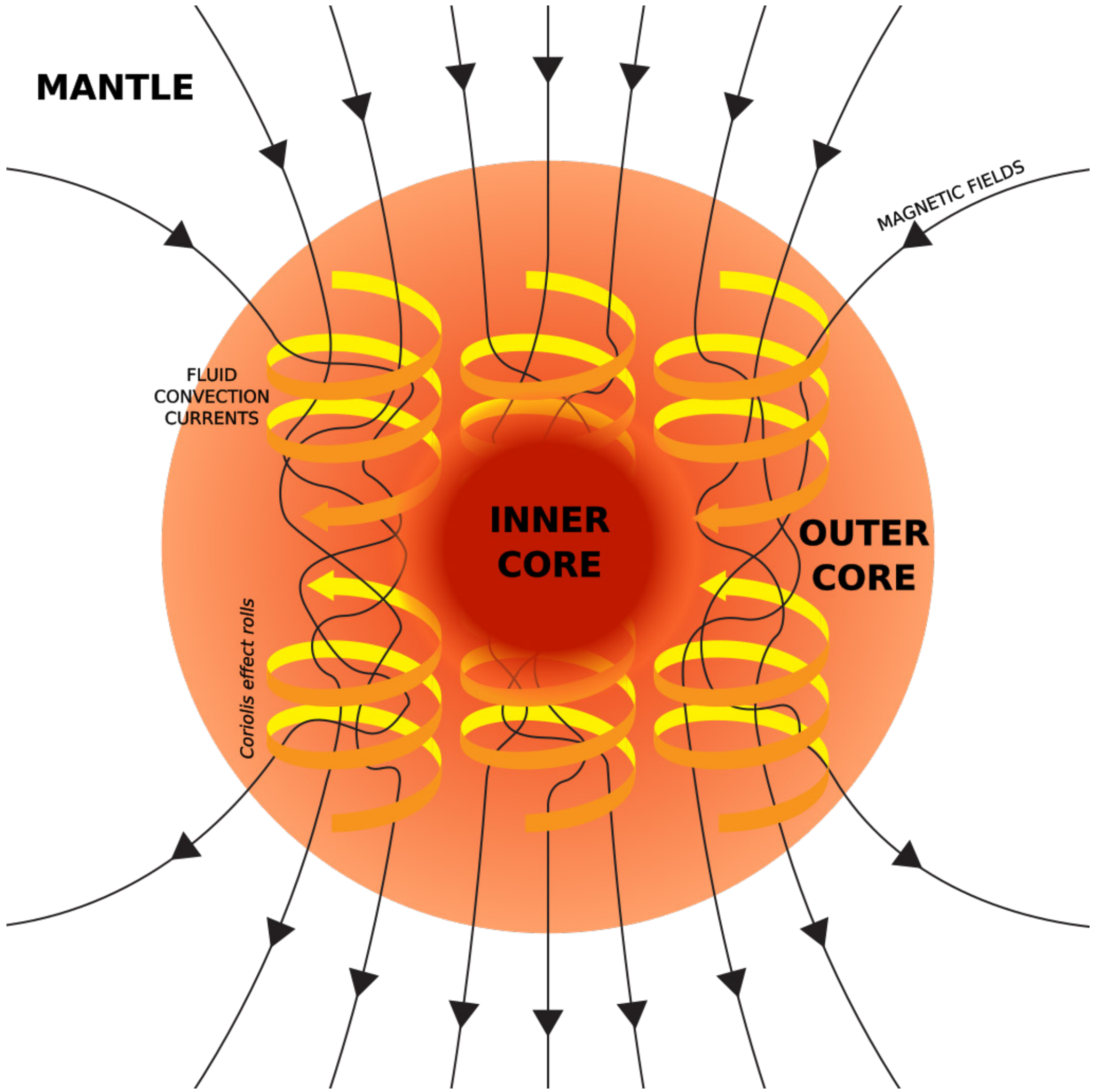
- All simulations are too viscous!
- Rayleigh number chosen to yield the desired regime!

1.2 Convection in Rotating Systems



- Taylor-Proudman theorem $\frac{\partial}{\partial z}\mathbf{U} = 0$
- Convective columns (isosurface of $\hat{\mathbf{z}} \cdot (\nabla \times \mathbf{U})$) aline with the rotation axis.
- Secondary flows up and down the column yield a helical motion.

1.3 Helicity

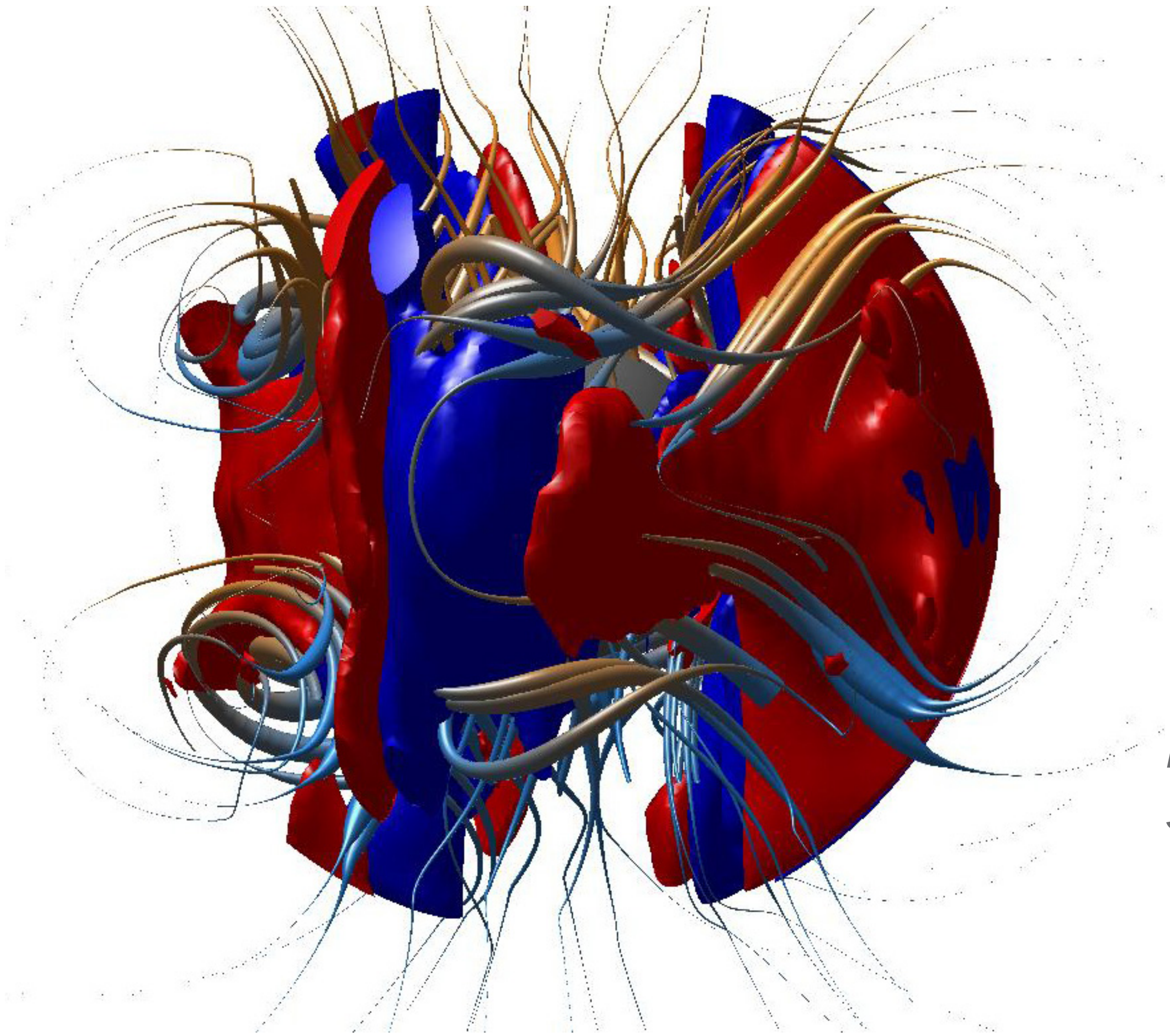


- Secondary flows up and down the column yield a helical motion.

$$H_z = U_z \cdot (\nabla \times \mathbf{U})$$

- Helicity has one sign in the north and the opposite in the south.
- This is ESSENTIAL for creating the large-scale dipole field.

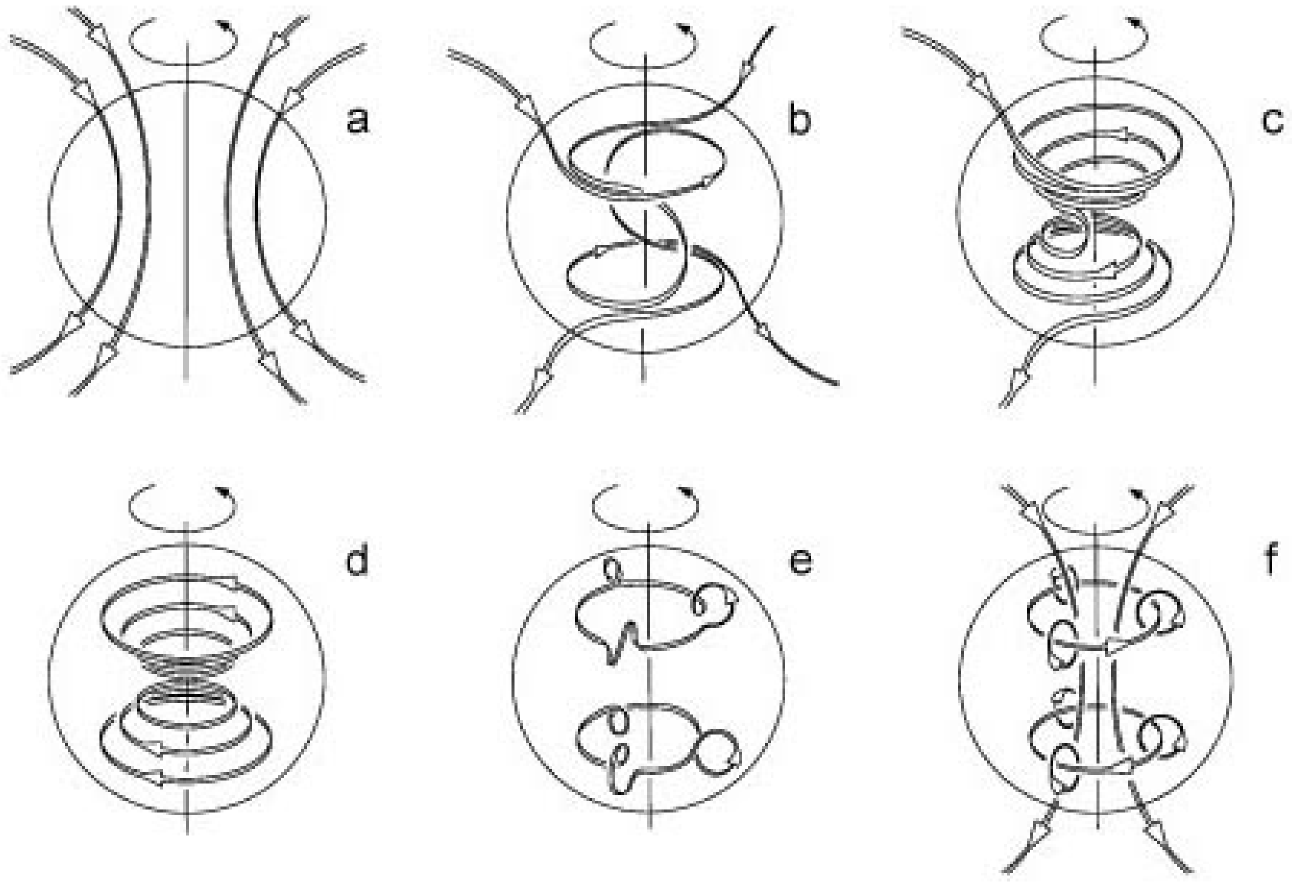
1.4 Magnetic Field Production



bechmark dynamo simulation

- Field line is twistes around (blue) anti-cyclones at lower latitudes.
- Advected pole-wards, sucked into the cyclone (red).
- Advected equator-wards in the cyclones, towards the anti-cyclones.
- This cycle maintains the field agains Ohmic decay.

1.5 Zonal Flows

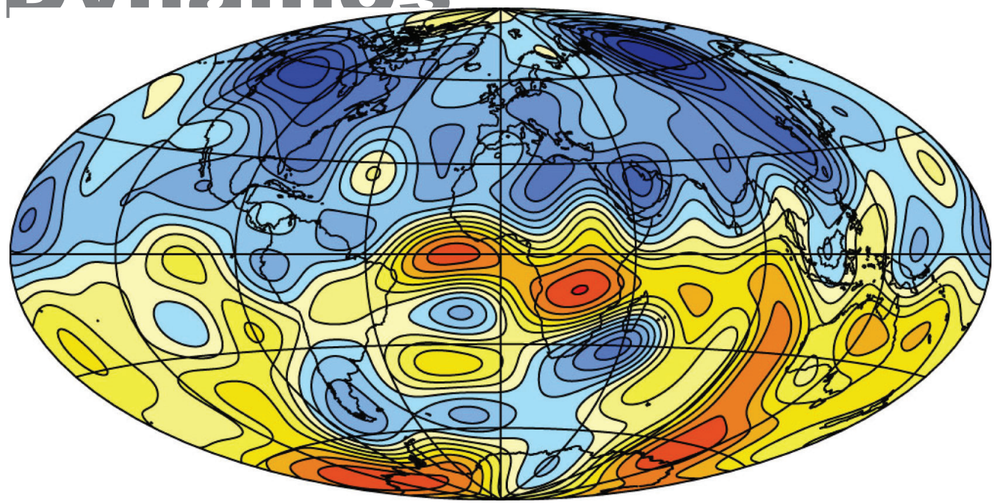


Love, J. J., 1999. Astronomy & Geophysics, 40, 6.14-6.19.

- Zonal flows wind up the magnetic field line (b-d).
- This is called an Ω–effect and can be an important dynamo ingredient.
- The remaining ingredient due to the helicity in called an α–effect.
- We distinguish $\alpha\Omega$–dynamos (above) and $\alpha\Omega$–dynamos (simulation).

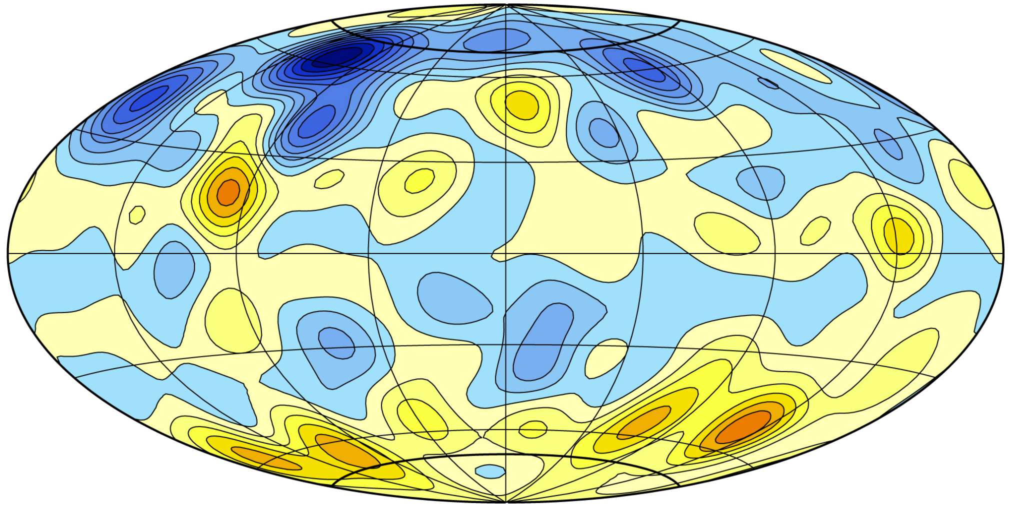
2.1 Simple and Complex Dynamos

gufm 1990: (l<15)



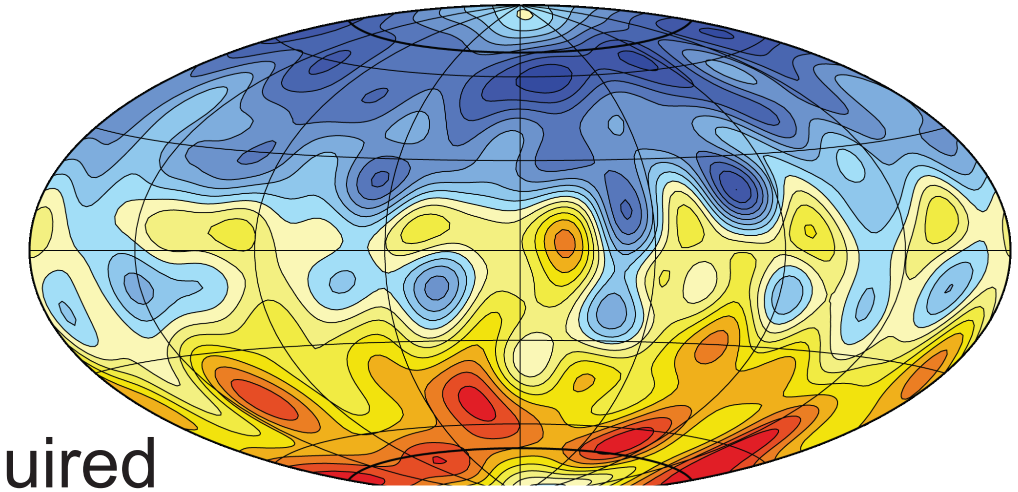
Easy-peasy model: (l<15)

$E=10^{-3}$
Pm=10
$Ra=8.1\ Ra_c$



Best model: (l<15)

$E=3x10^{-5}$
Pm=1
$Ra=36\ Ra_c$



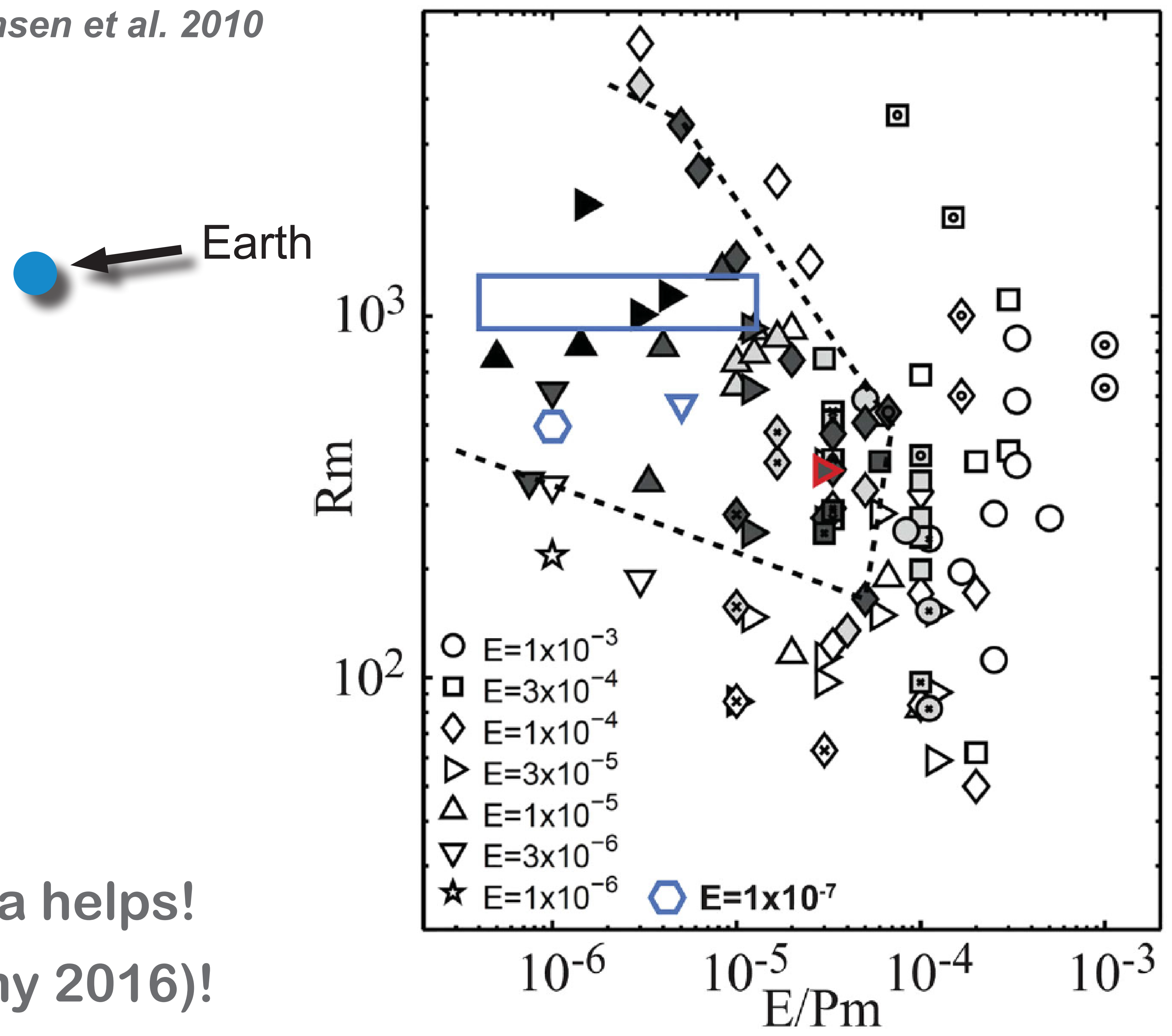
→ low E and large Ra required

2.2 Earth-like Dynamos

Christensen et al. 2010

checking
1) dipole dominance
2) axisymmetry
3) equatorial symmetry
4) kurtosis

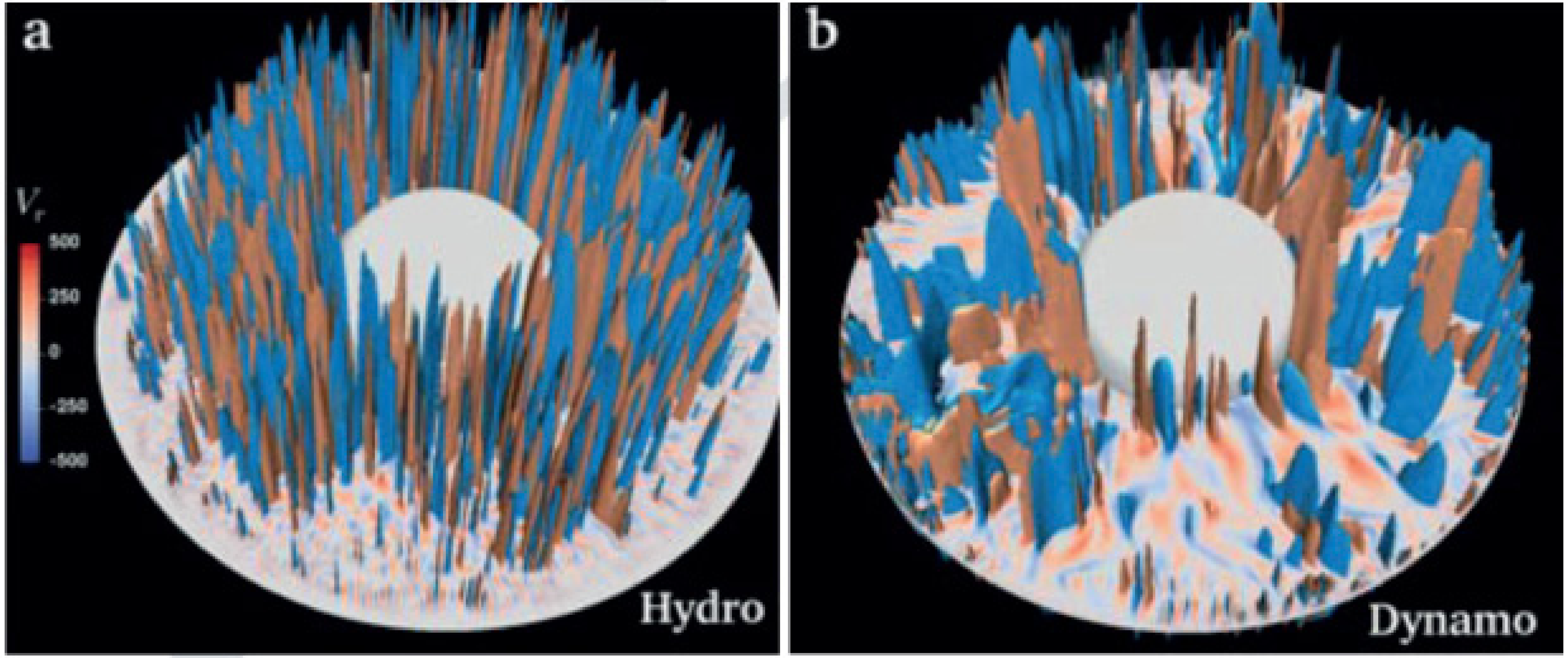
darker symbols more compliant with Earth



- **Decreasing E and increasing Ra helps!**
- **Larger Pm may also help (Dormy 2016)!**

2.3 Strong Lorentz

Yadav et al. 2016

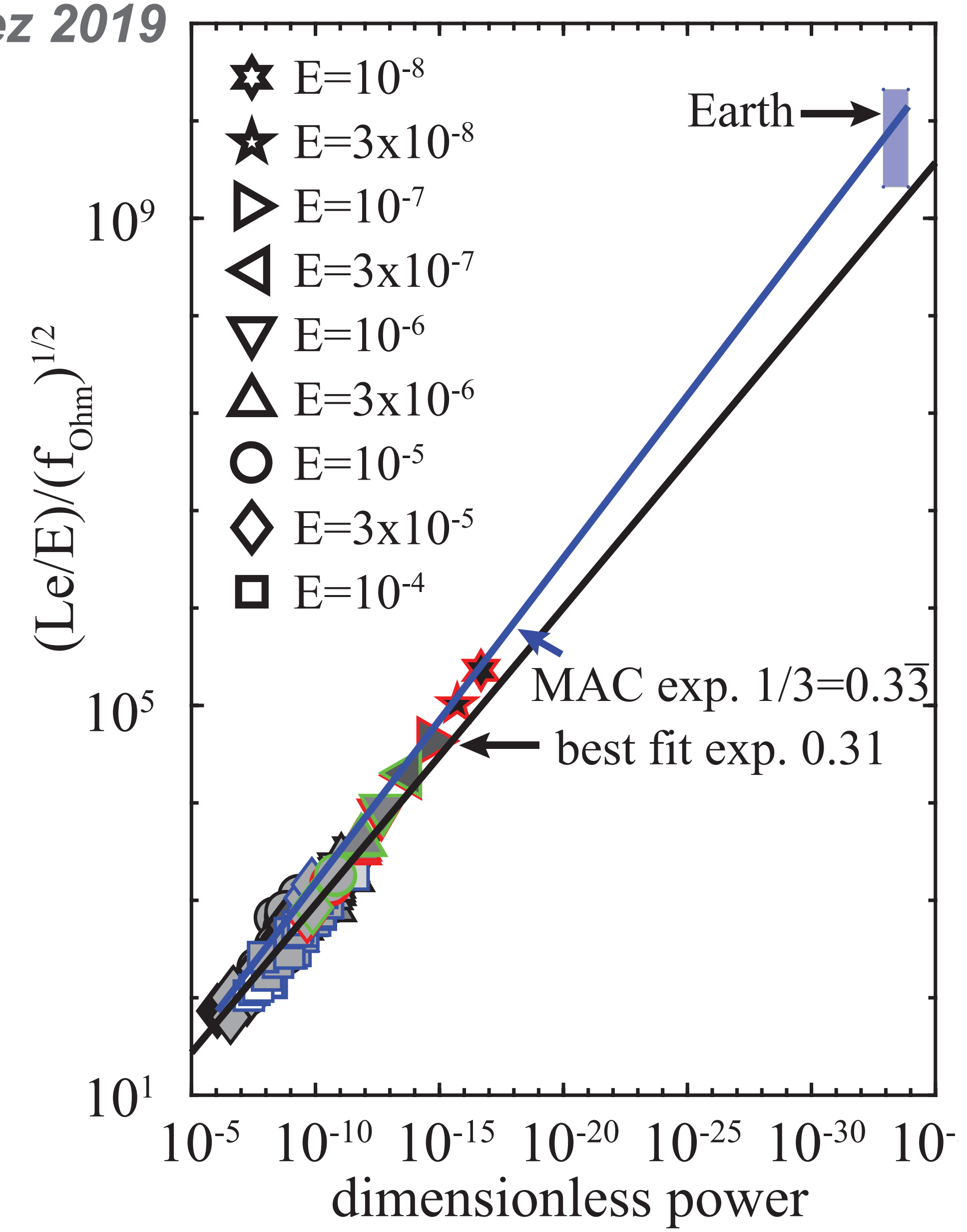


radial flow isosurfaces for simulations with $E=3x10^{-6}$ and Pm=0.5

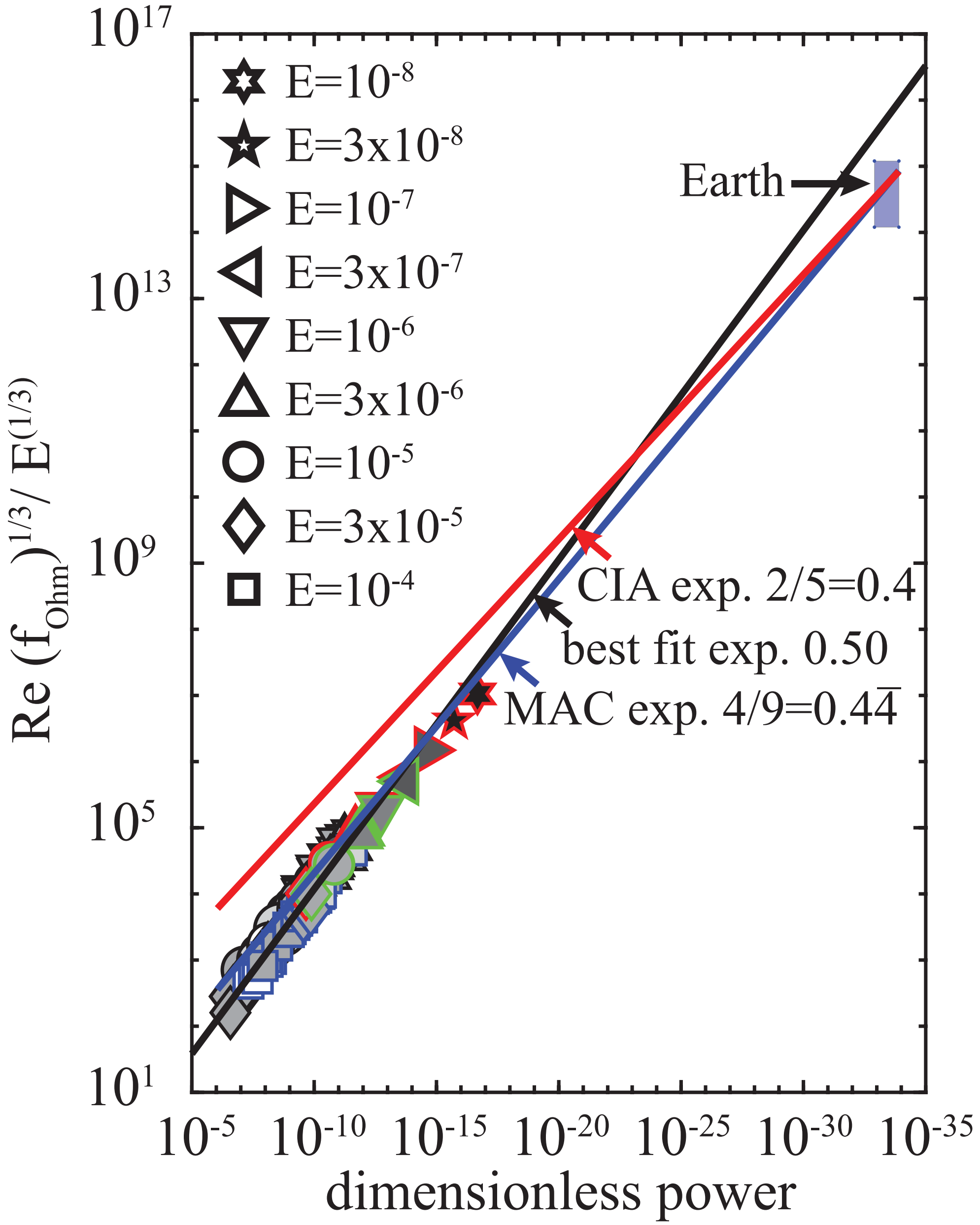
- Lorentz force should matter! (Enter leading order force ballance.)
- Active research field. (Teed & Formy 2023, Schaeffer et al. 2017, Sheyko et al. 2016, ...

2.4 Scaling Geodynamo Simulations

Wicht & Sanchez 2019



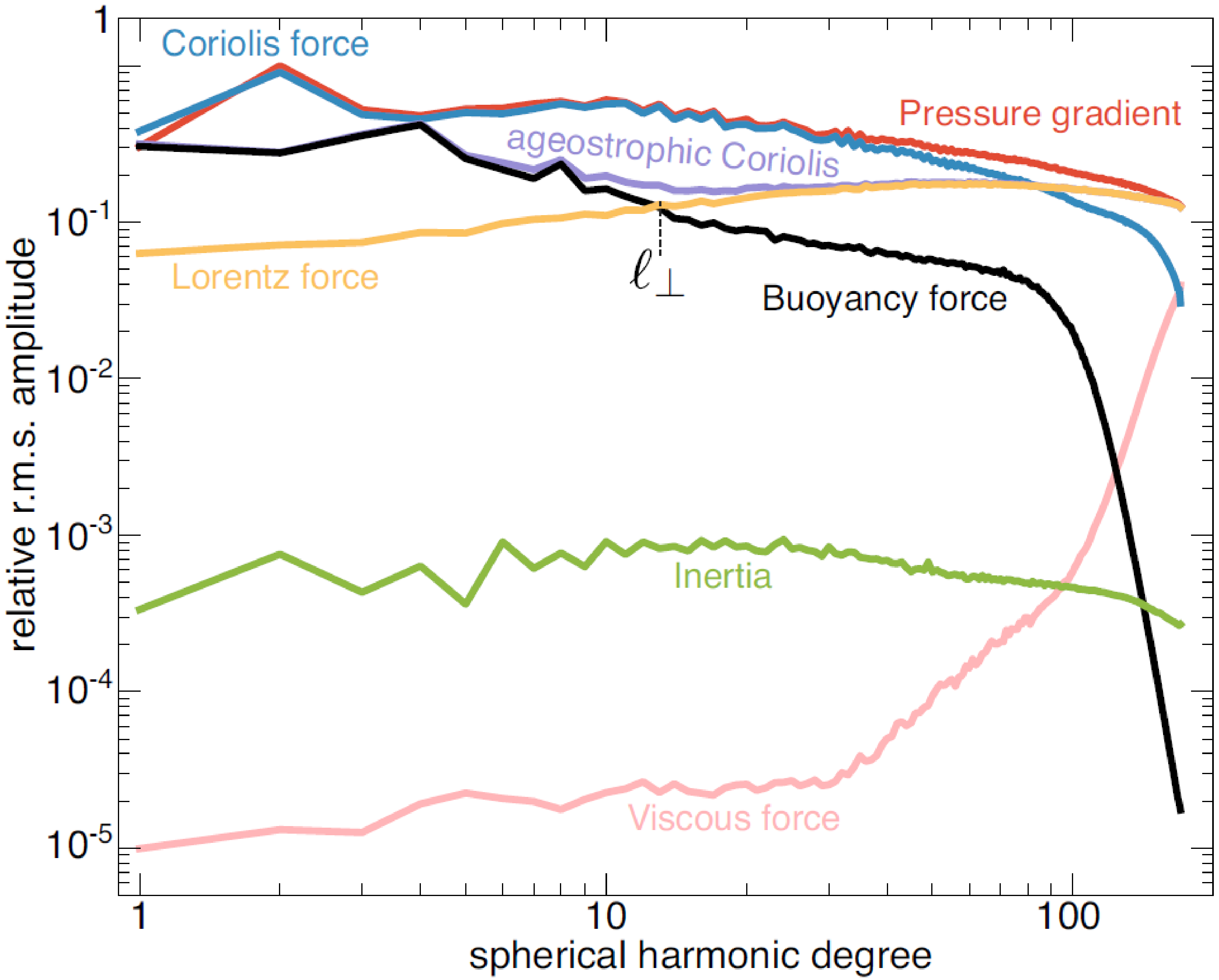
$$\mathrm{Le} = \mathrm{B}/\left(\tilde{\rho}^{1/2}\mu^{1/2}\Omega\ell_{\|}\right) \sim f_{\mathrm{Ohm}}^{1/2}\,\mathcal{P}^{1/3}$$



$$\mathrm{Ro} = \mathrm{U}/\left(\Omega\ell_{\|}\right) \sim f_{\mathrm{Ohm}}^{-1/3}\,\mathcal{P}^{4/9}$$

- Testing scaling laws with numerical simulations.

2.5 Force Balance with Hyperdiffusion



Aubert et al. 2022, Ek=3x10^{-10}, Pm=7.9x10^{-3}, using hyperdiffusion, 14 kyr

- Dominant flow length scale where Lorentz force starts to matter.
- Lorentz force yields much larger length scales!

2.6 Chosing Your Parameters: The PATH

- A path through the parameter regime that keeps the Earth-like properties.
- Keep Rm and Λ constant.
- Follow the Davidson scaling.
- One parameter with ϵ=1 for starting solution and ϵ=10^{-7} for Earth.

$$\mathrm{Ra}^\star = \epsilon\ \mathrm{Ra}_0^\star$$

$$\mathrm{E} = \epsilon\ \mathrm{E}_0$$

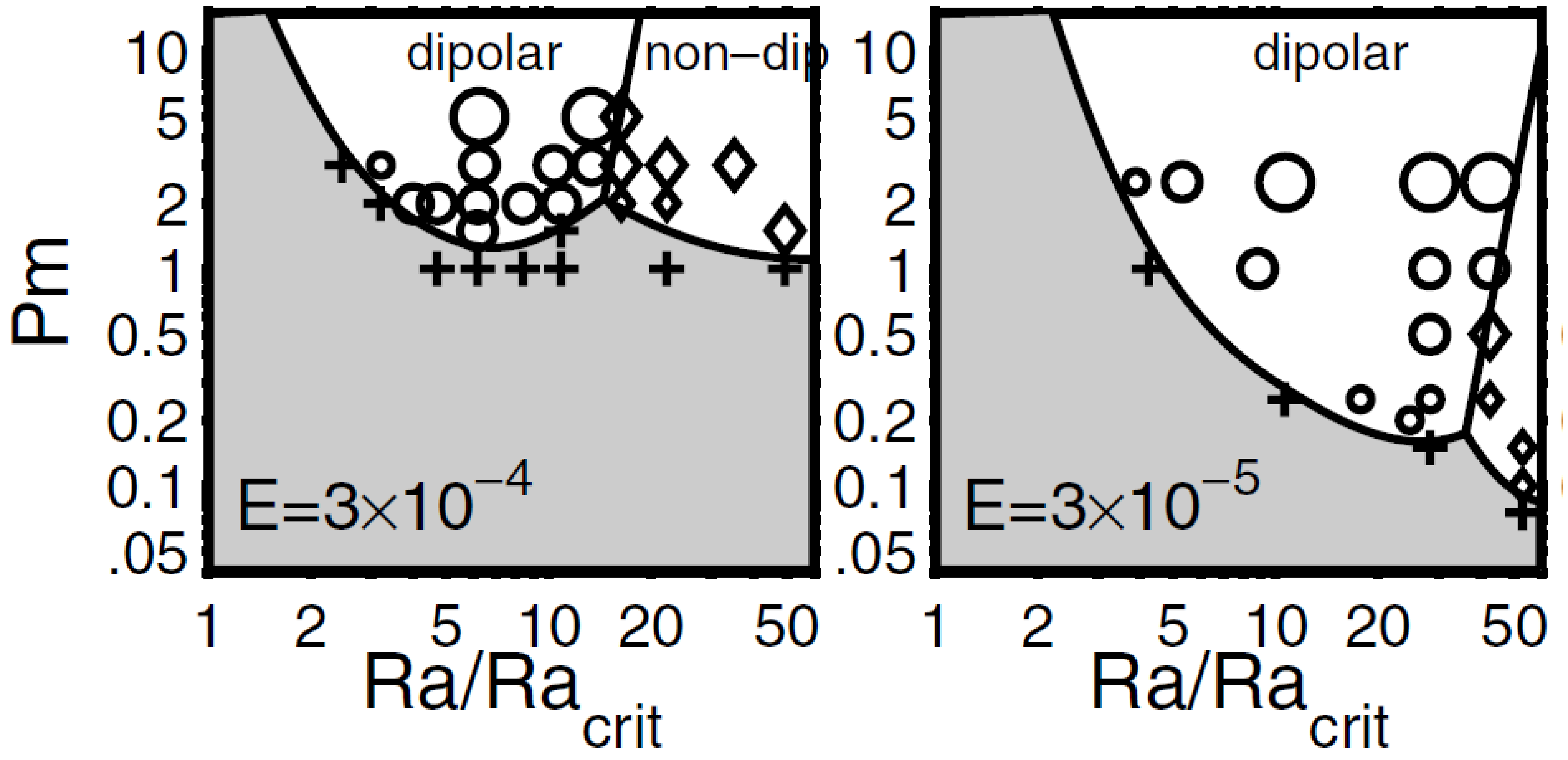
$$\mathrm{Pm} = \epsilon^{1/2}\ \mathrm{Pm}_0$$

$$\mathrm{Pr} = \mathrm{Pr}_0 = 1$$

- Start at E_0 = $3\mathrm{x}10^{-5}$, for E < $3\mathrm{x}10^{-6}$ hyperdiffusion used

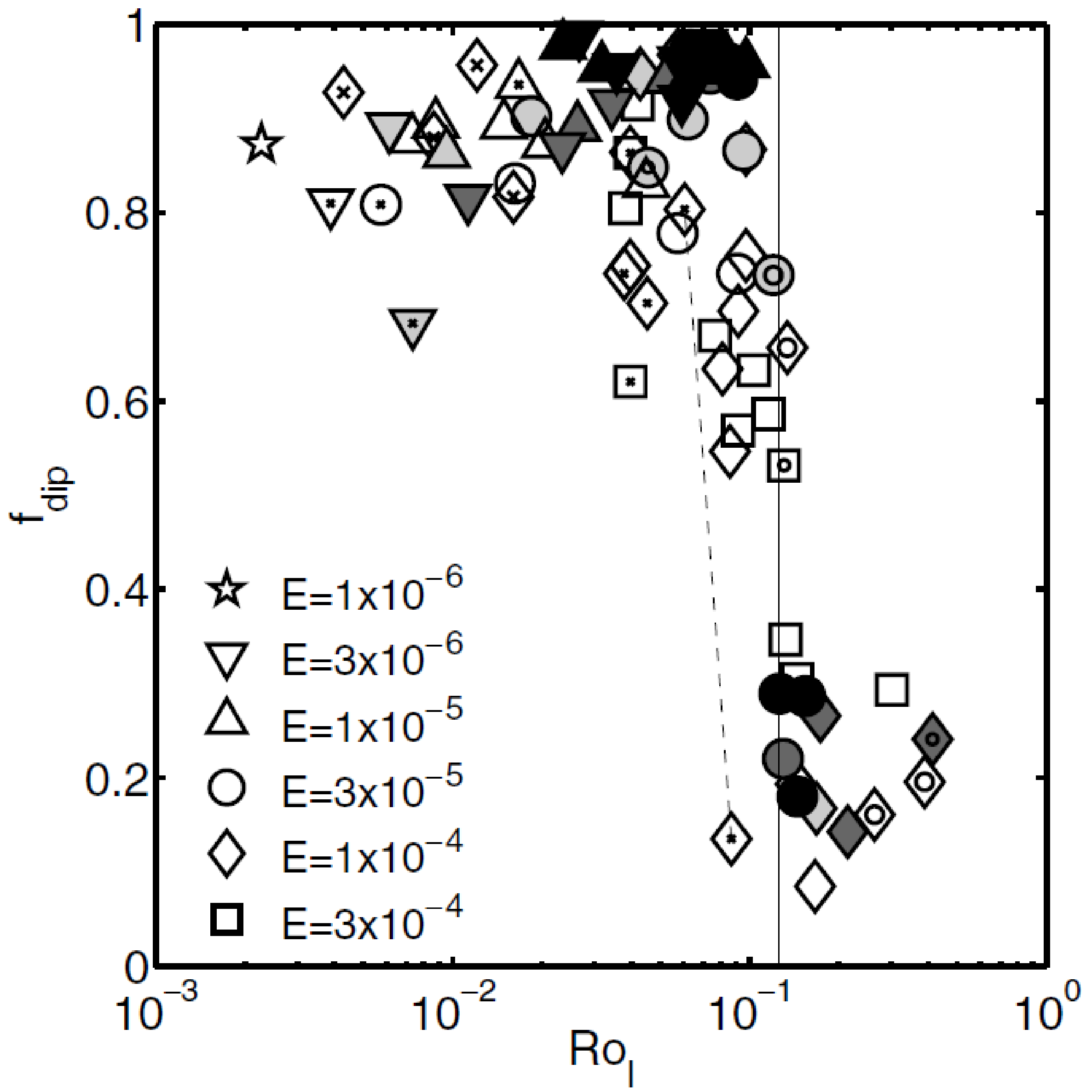
2.7 Regime Diagram

Christensen & Aubert 2006



- Regime Diagram for different Ekman numbers.

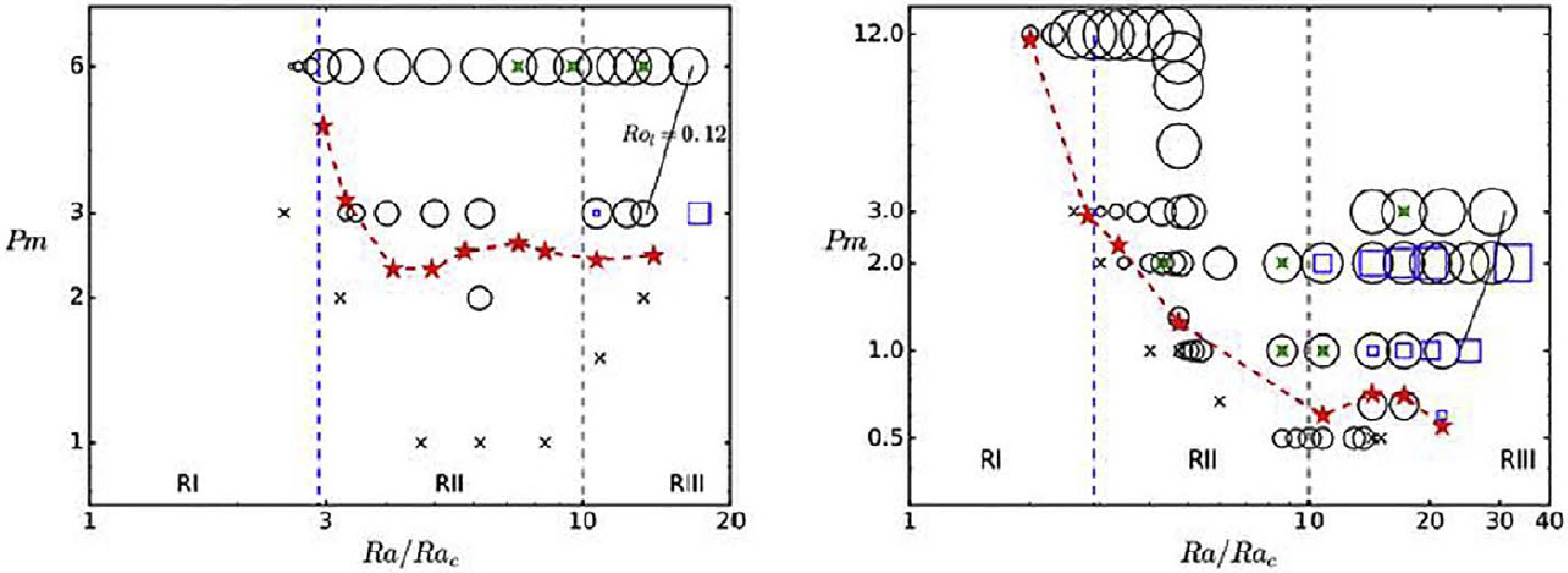
2.8 From Dipolar to Multipolar



Christensen & Aubert 2006

- Eventually all dynamos become multipolar (and reverse?).

2.9 Subcritical Dynamos and Bista-

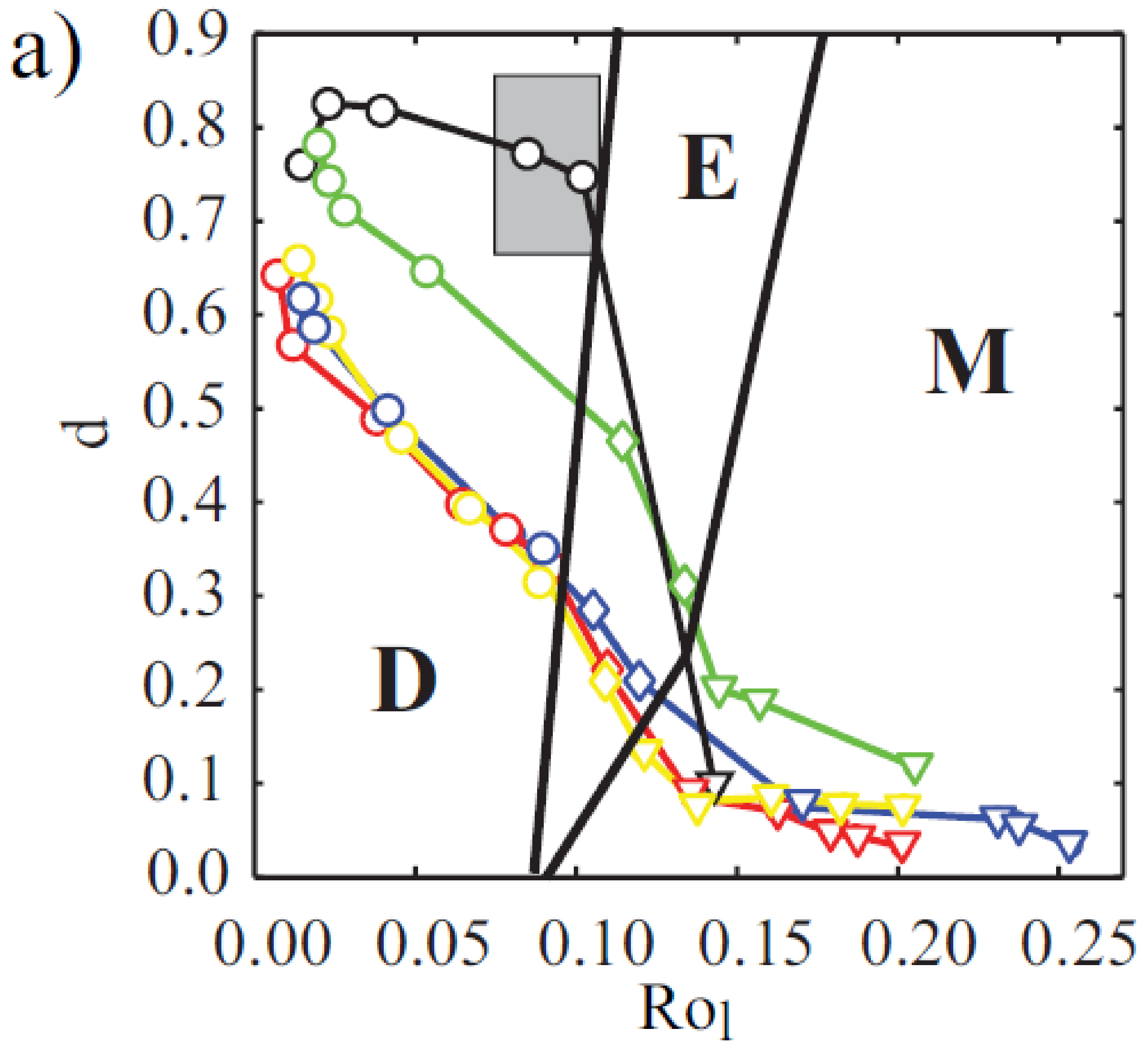


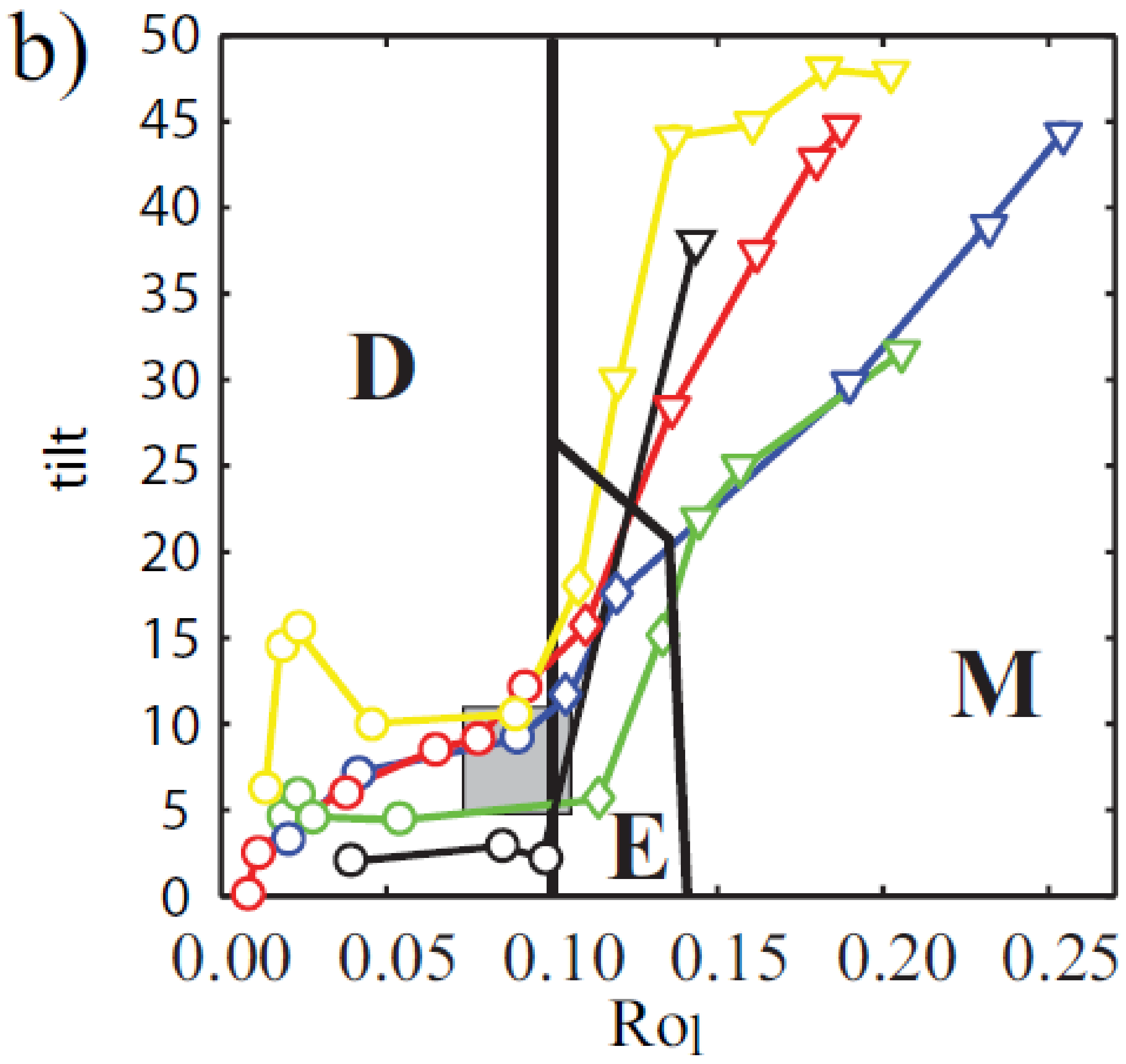
(a) $E = 3 \cdot 10^{-4}$ (b) $E = 10^{-4}$

Petitdemange 2018

- Many dipolar dynamos need a strong initial field (circles). These are called subcritical dynamos.
- At larger Ra you find dipolar dynamos with a strong initial field (circles) and multipolar dynamos (squares) with a weak initial field.

3.1 Earth-like Reversals

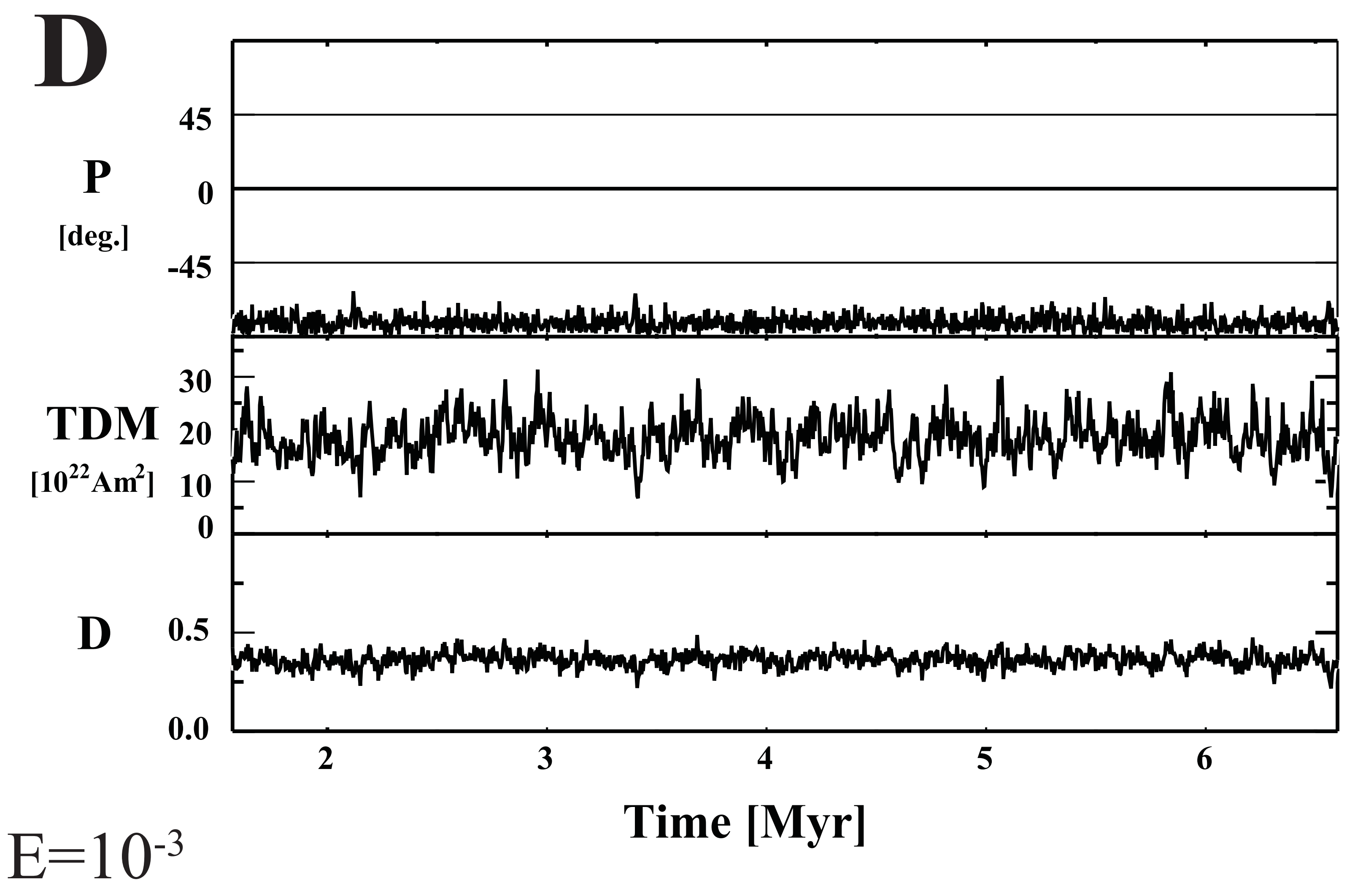




Wicht et al. 2011

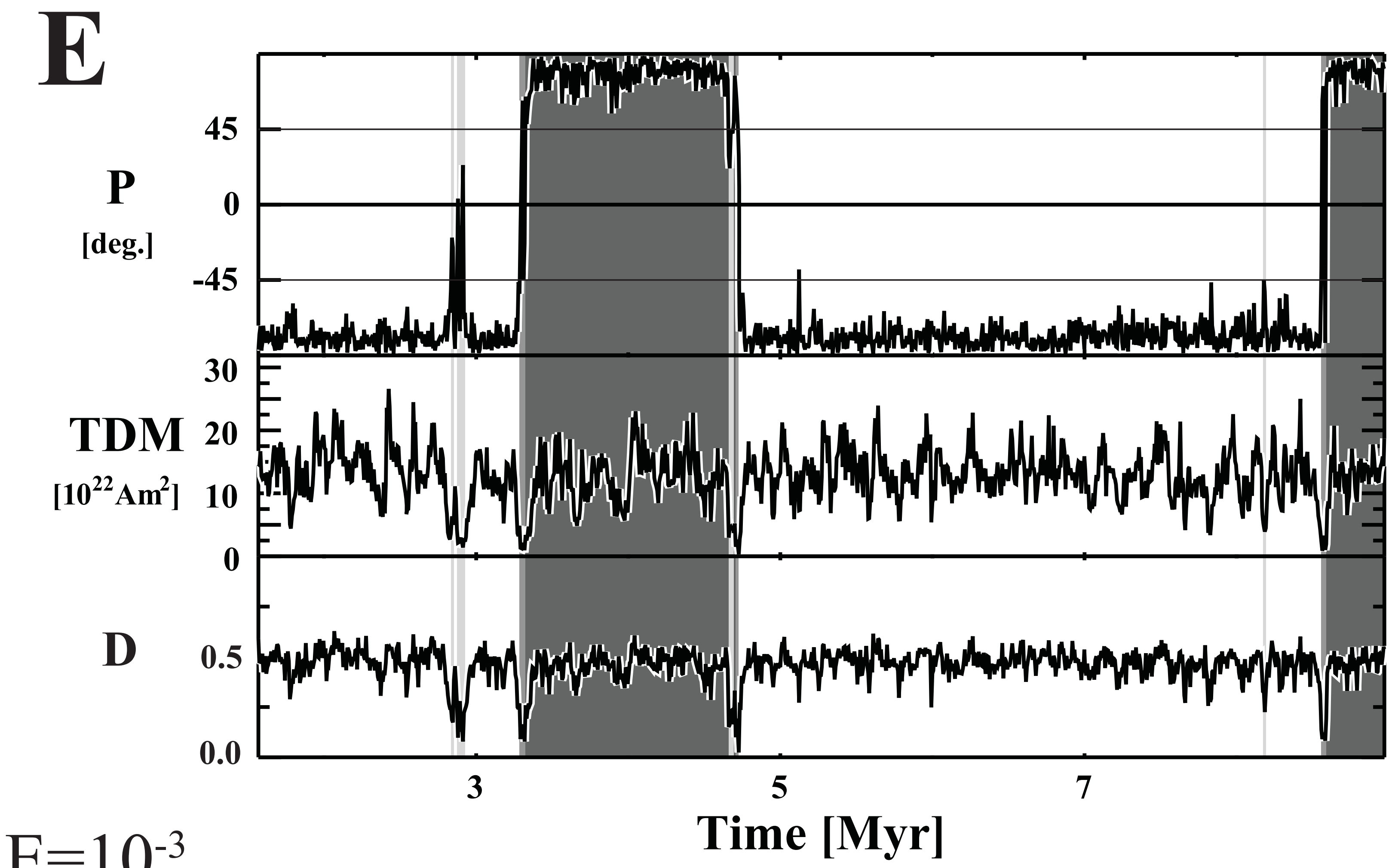
- Earth-like rare reversals are found at the transition from D to M.

3.2 Weakly Driven Dynamo



- Weakly driven dynamos never reverse and are also too boring otherwise.

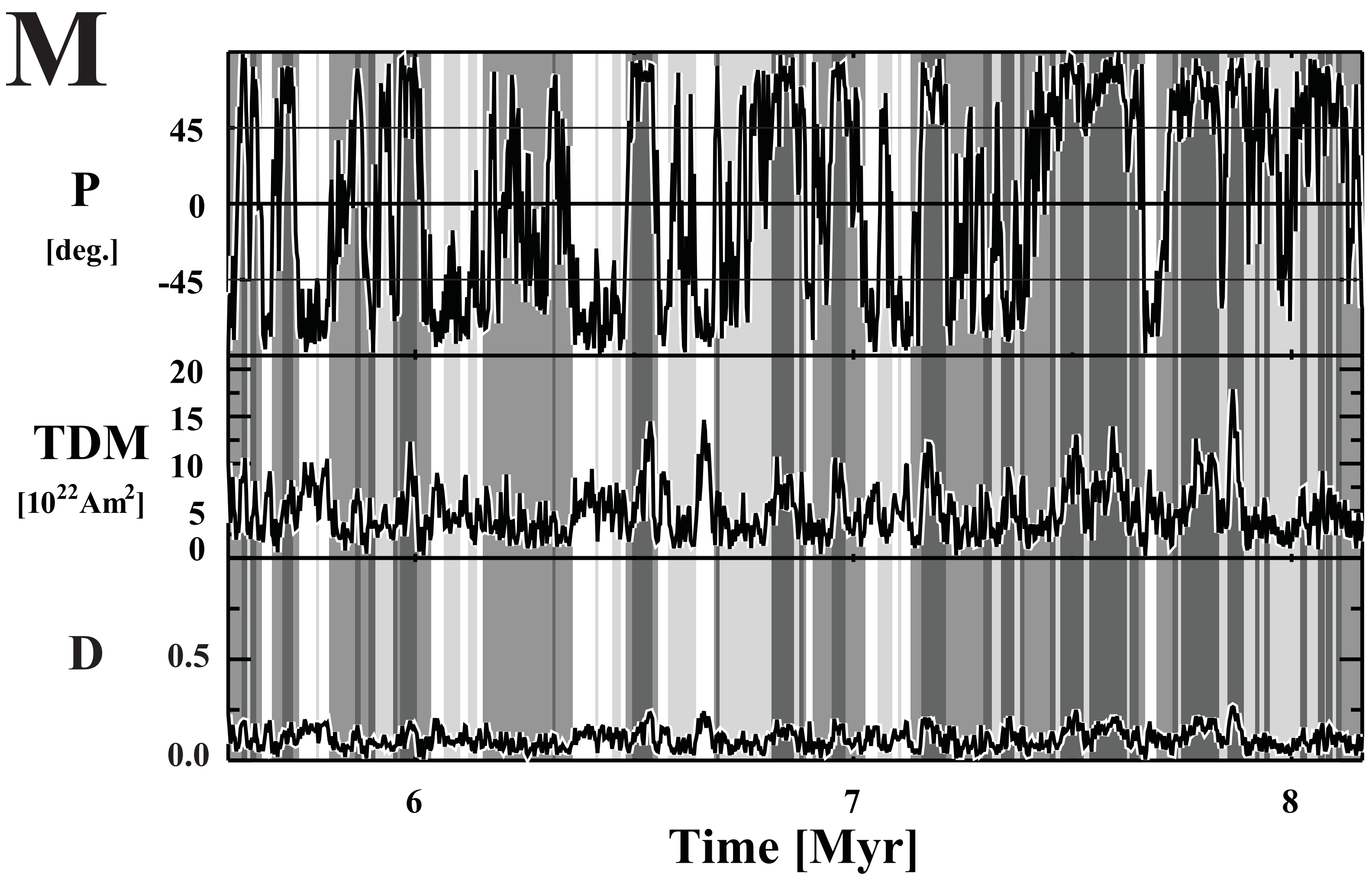
3.3 Stronger Driven Dynamo



$E=10^{-3}$

- When driven strongly enough the simulations show Earth-like rare reversals.
- But inertia is too strong!

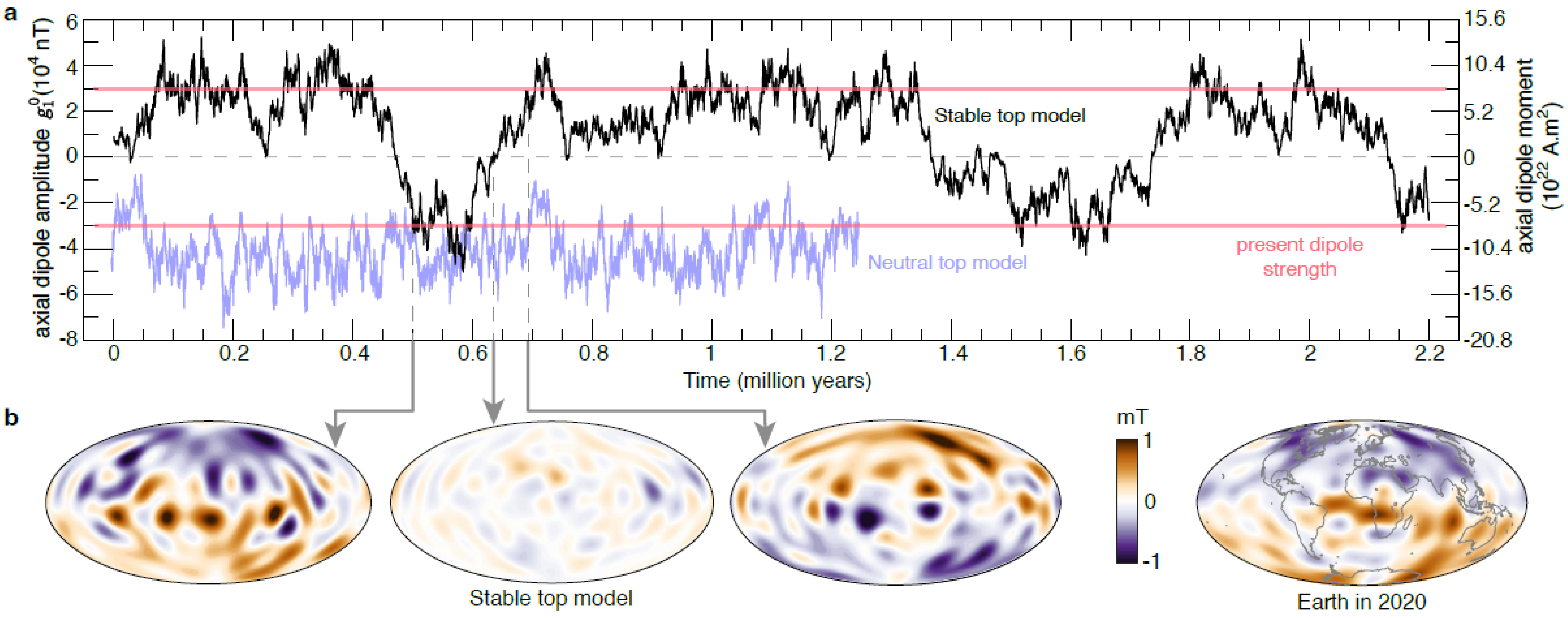
3.4 Too Strongly Driven Dynamo



$E=10^{-3}$

- When driven driven too strongly the simulations reverses all the time.

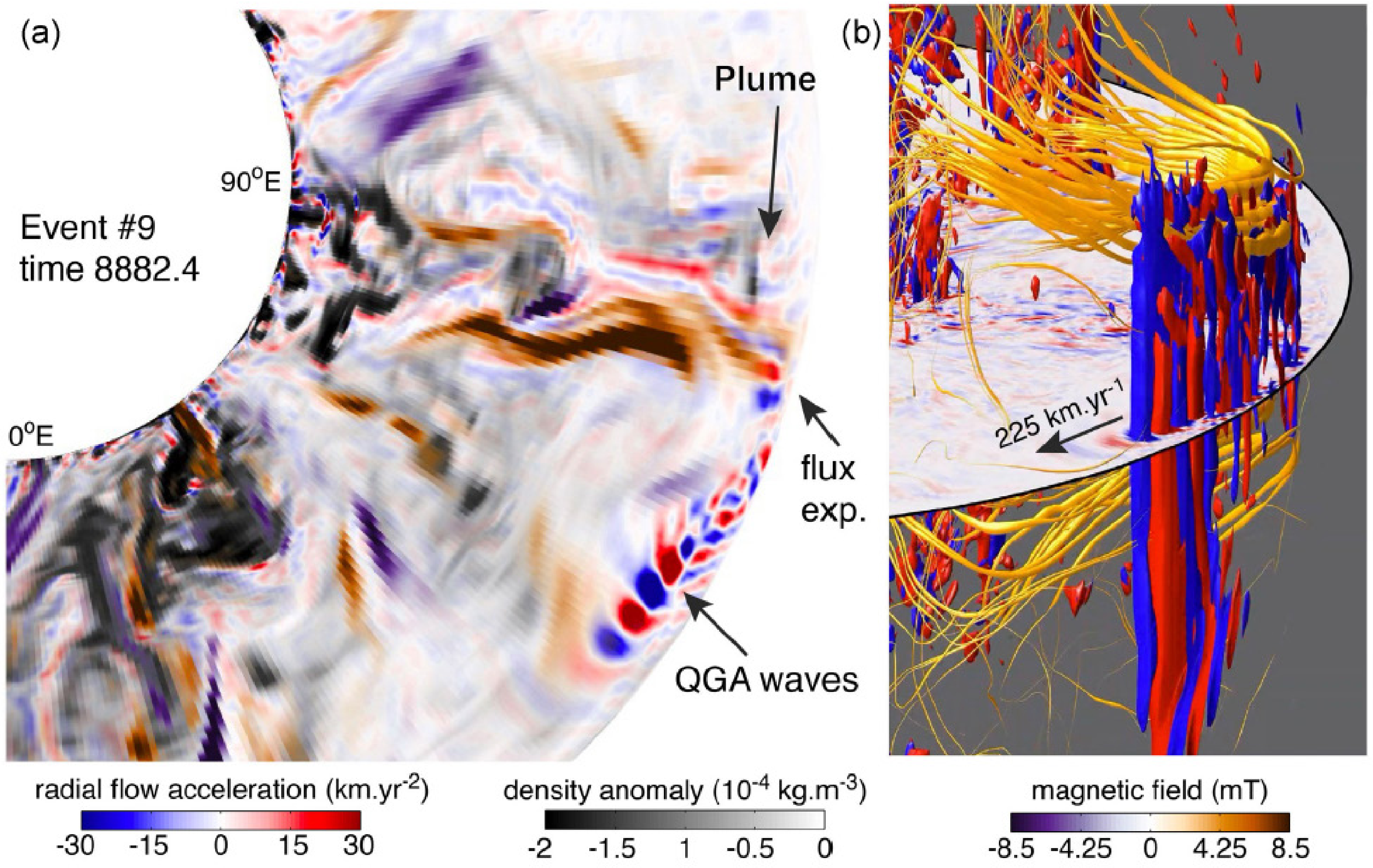
3.5 Alternative Reversals



Aubert et al. 2025

- A stable layer underneath the CMB triggers reversals.

3.6 Extreme Simulations Yield Interesting Waves



Aubert et al. 2022, E=3x10^{-10}, Pm=7.9x10^{-3}, using hyperdiffusion, 14 kyr

- Waves only seen at low Ekman number.
- Upwelling hits flux bundle and excites Alfven waves
- Low M_A and low CMB heat flux required

4.1 Chosing Your Parameters

- What do you want to simulate? Make up your mind!
- Reversal simulations can not be done at very low E (maybe Aubert can do that).
- Short simulations of variations of a background state can be done at low E

- Pick E according to the problem and the available computer resources.
- Chose Pr to represent the physics.
- Chose Ra and Pm to yield the desired regime.
- If your dynamo does not work, increase Ra and/or Pm.
- Lower E allow for higher Ra and thus stronger magnetic fields that are still dipolar.

4.2 Conclusion

- Dynamos rely on magnetic induction.
- They need fluid that is conducting enough and moves fast enough in a complex enough fashion.
- Rotation organizes the flow in such a was that dipole field is produced.
- Even simple dynamo simulations seem to explain many geomagnetic features.
- The Lorentz only clearly enters the leading order force ballance for more extreme parameters.
- More extreme simulations are also required to capture wave phenomena.
- When increasing Ra, the dynamo switches from dipole-dominated to multipolar.
- Most of the simulated reversals are found at the regime transition where inertial forces are likely too strong.
- A stably stratified layer underneath the CMB can trigger reversals where inertial forces are smaller.