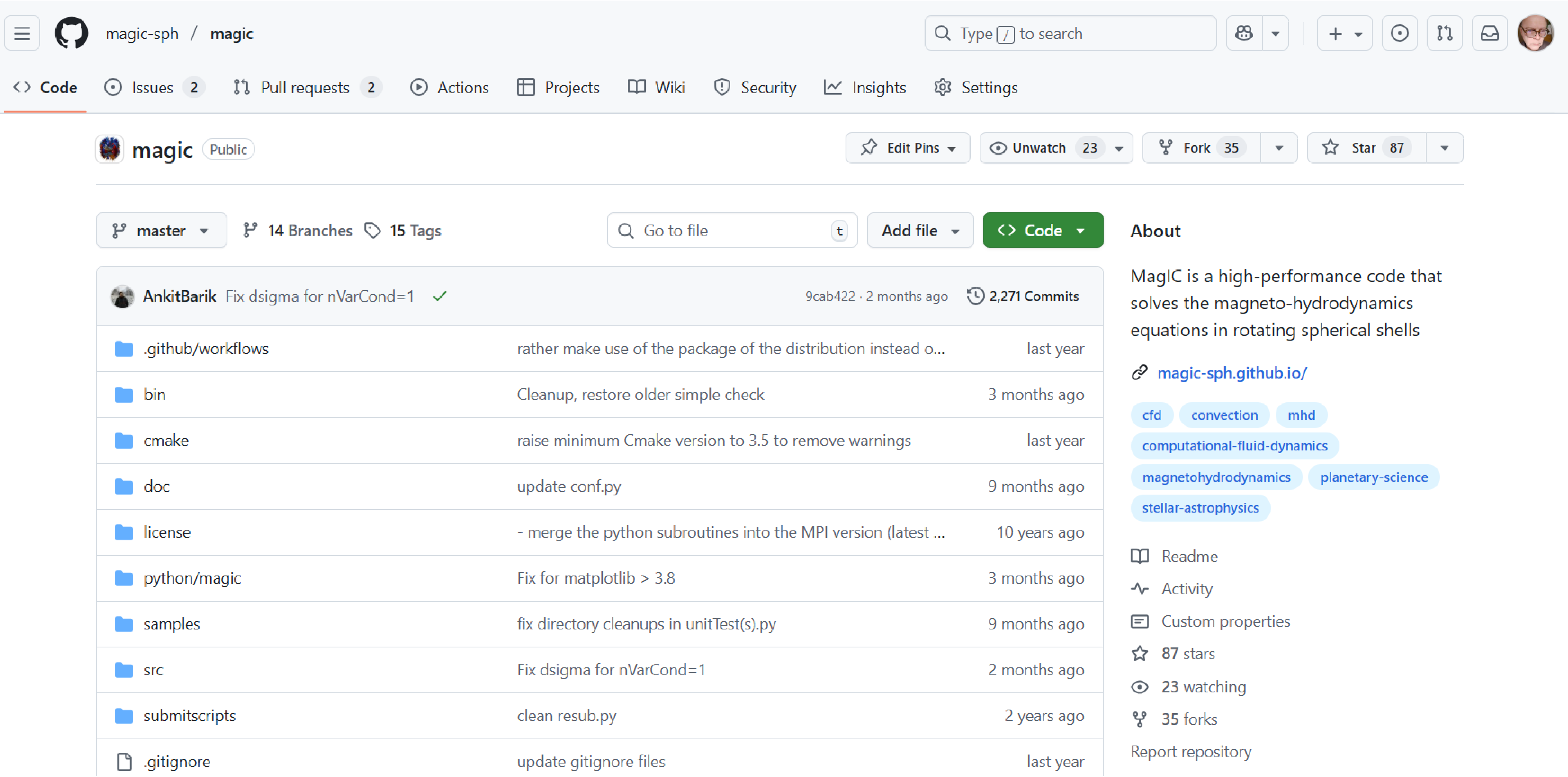
Introduction to MagIC

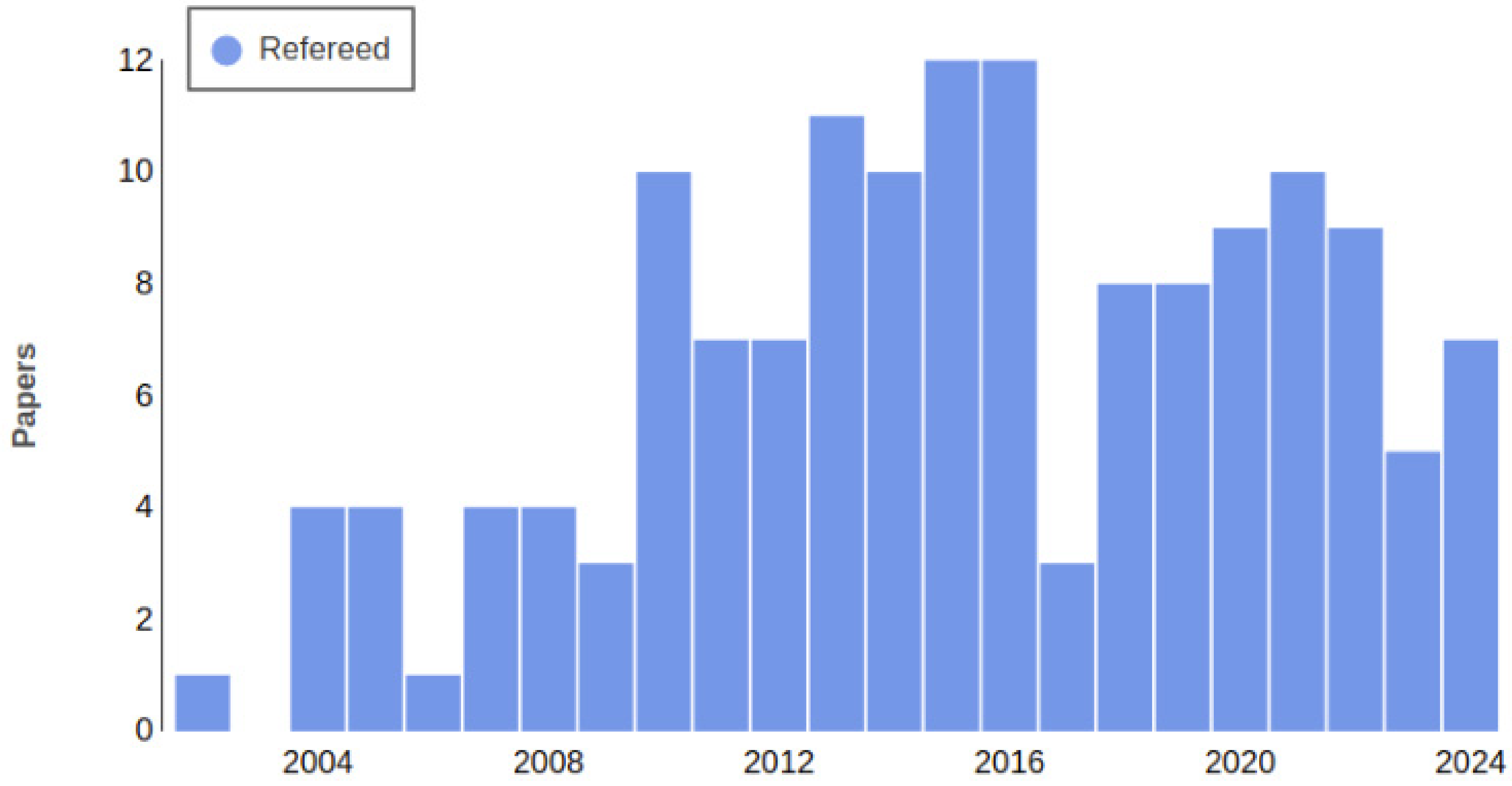
Johannes Wicht and Thomas Gastine (IPGP)

magic-sph on gitHub



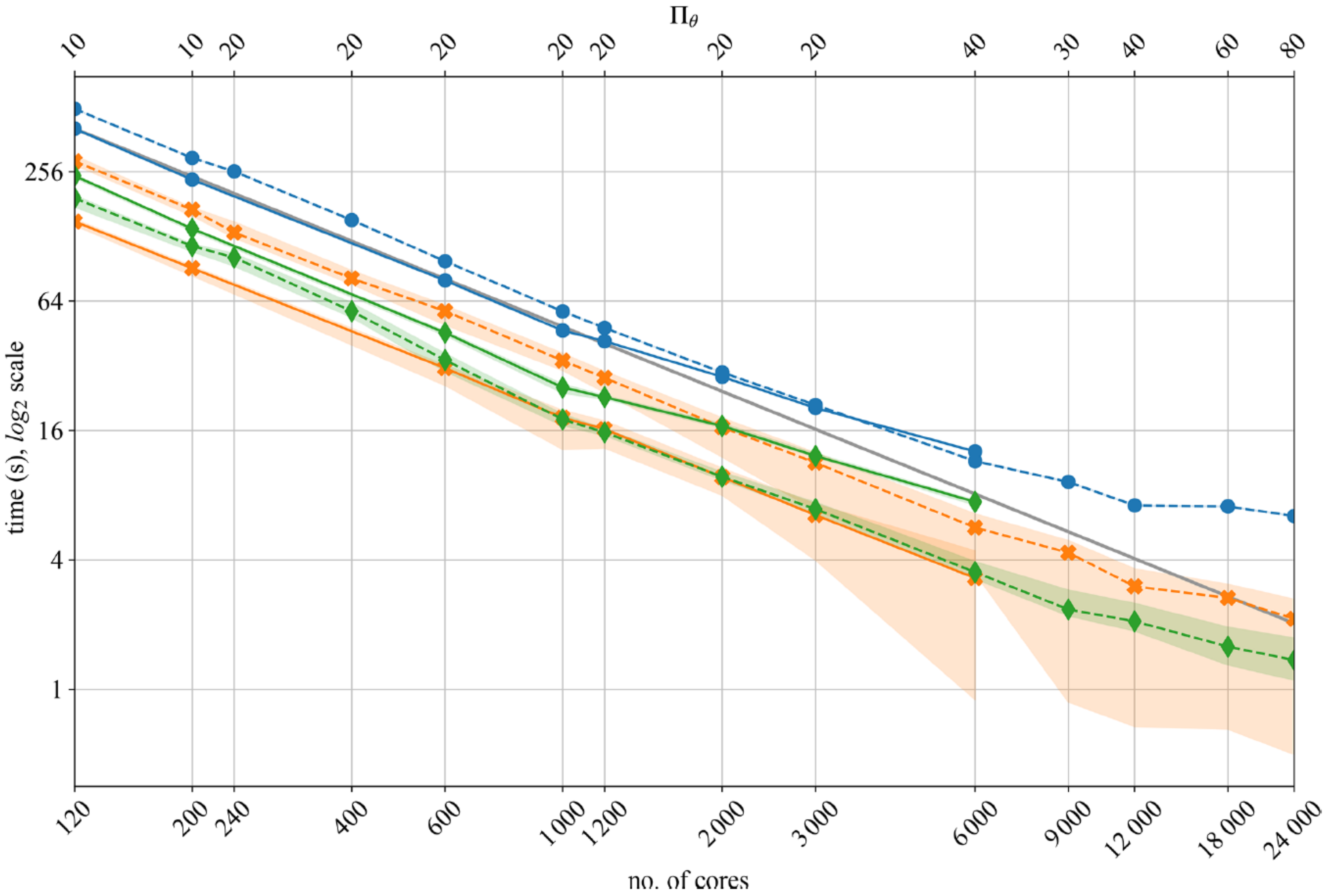
- Available on gitHub.

Publication Record



- More than 150 peer-reviewed publications!
- Used for many different problems.

Efficient Parallelization

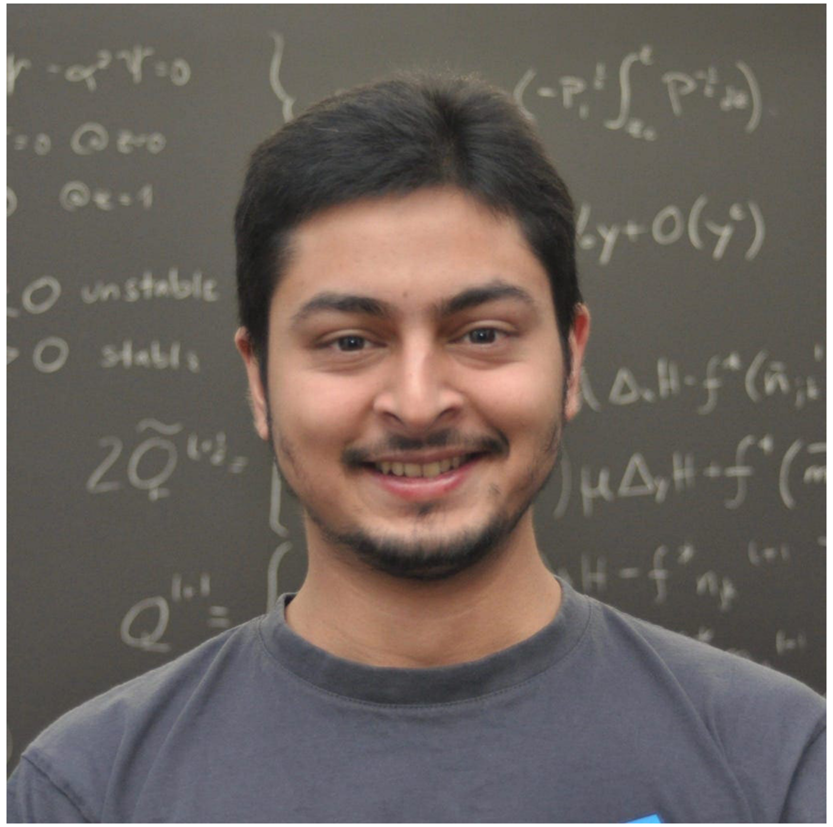


Lago et al 2022

- Fast, highly parallelized, runs on super computers but also notebooks.
- Efficient use of several thousand cores for large problems.

Well Maintained

Thomas Gastine



Ankit Barit

- Maintained by experienced scientists.

Documentation

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- Extensive online documentation.

Graphic Support

Ankit Barit

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Support for G_#.TAG files

class magic.**MagicGraph**(*ivar=None, datadir='.', quiet=True, ave=False, tag=None, precision=<class 'numpy.float32'>*) [source]

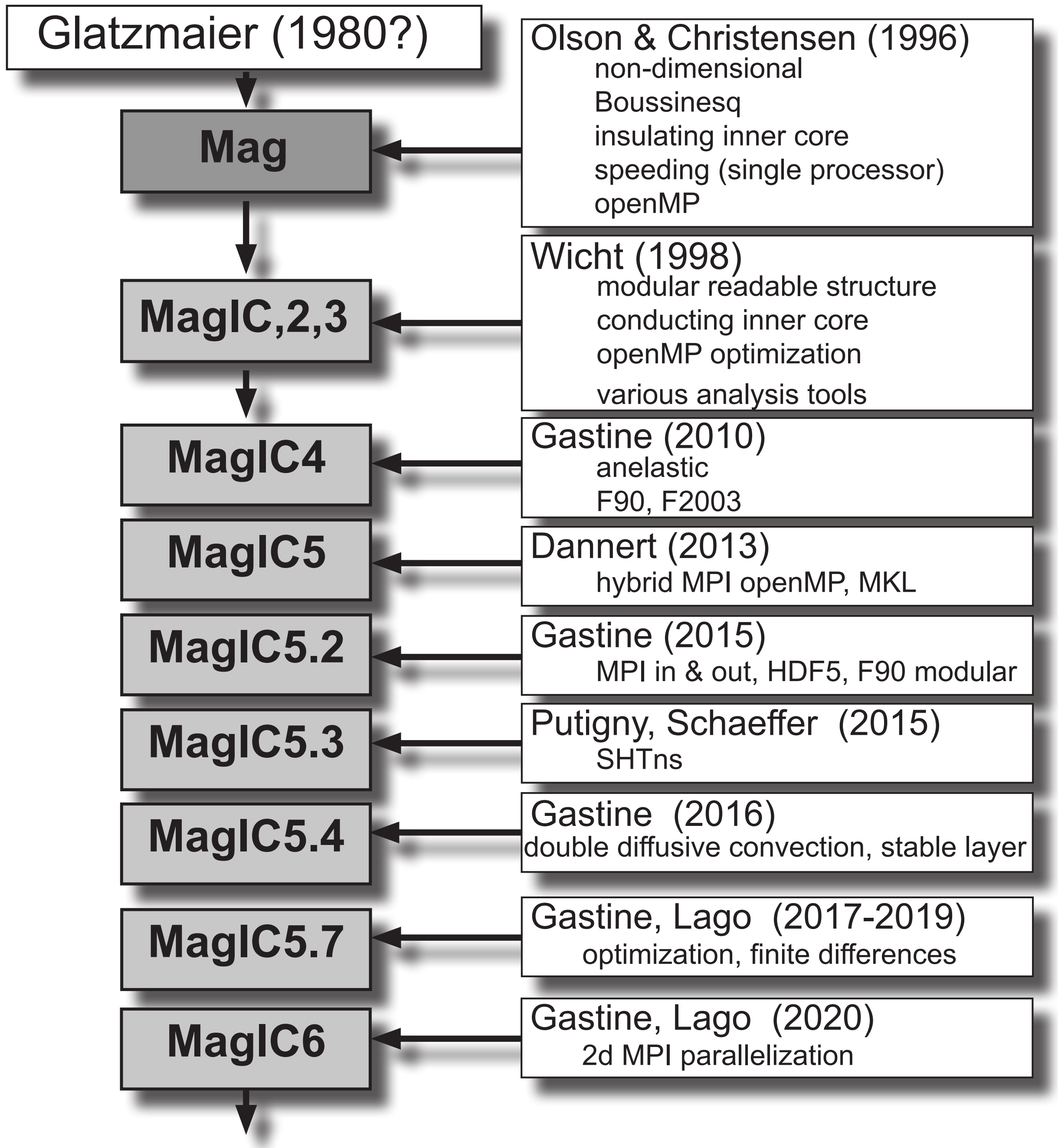
This class allows to read the 3-D graphic outputs of the MagIC code (G_#.TAG and G_ave.TAG) files. Those are binary unformatted outputs, there are therefore two ways to load them:

- If buildLib=True in magic.cfg and the fortran libraries were correctly built, then the reader uses a fortran program that is expected to be much faster than the pure python routine.
- If buildLib=False, then a pure python program is used to read the G files.

```
>>> # Regular G files
>>> gr = MagicGraph(ivar=1, tag='N0m2a')
>>> print(gr.vr.shape) # shape of vr
>>> print(gr.ek) # print ekman number
>>> print(gr.minc) # azimuthal symmetry
>>> # Averaged G file with double precision
>>> gr = MagicGraph(ave=True, tag='N0m2', precision=np.float64)
```

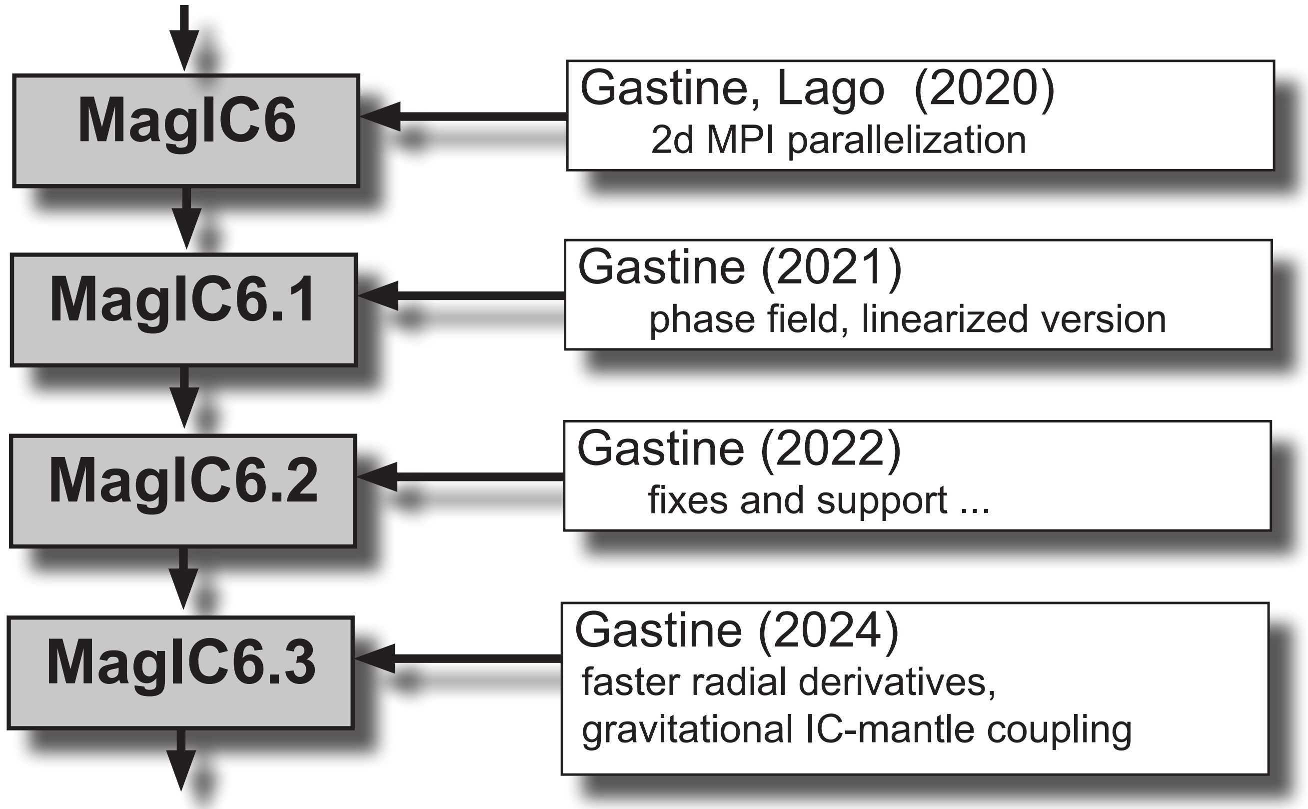
- Readily available visualization tools using Python.

MagIC Heritage



- Other codes: Ash, Rayleigh, Parody (JA), Xshells, UK-Mhd, Pencil

MagIC Heritage



- We all rely on Thomas Gastine.

MagIC in Words 1

- Open access
- Extensive manual
- Extensive post-processing including visualization
- High degree of parallelization
- Solves for convection and magnetic field generation
- Rotating frame of reference
- Spherical shell or full sphere
- Dimensionless formulation

Poloidal/Toroidal Decomposition

- Poloidal/toroidal representation for flow and magnetic field:

$$\mathbf{U} = \nabla \times \nabla \times \hat{\mathbf{r}}\, w \;+\; \nabla \times \hat{\mathbf{r}} z$$

$$\mathbf{B} = \nabla \times \nabla \times \hat{\mathbf{r}}\, b \;+\; \nabla \times \hat{\mathbf{r}} j$$

- Poloidal potentials w and b, toroidal potentials z and j
- Continuity equations are automatically fulfilled.

$$\nabla \cdot \mathbf{U} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

- This procedure allows to go from 9 equations with 8 unknowns to 6 equations for 6 unknowns.

NOTE:

$$\mathbf{U} = -\nabla^2 w \hat{\mathbf{r}} + \nabla_H \frac{\partial}{\partial r} w + \nabla \times \hat{\mathbf{r}} z$$

Radial component is purely poloidal.

Horizontal poloidal component depends on radial derivative.

Poloidal/Toroidal Equations

- Poloidal/toroidal equations:

$$\hat{\mathbf{r}} \cdot \frac{\partial}{\partial t}\mathbf{U} = -\nabla_H^2 \frac{\partial}{\partial t} w$$

$$\hat{\mathbf{r}} \cdot \nabla \times \frac{\partial}{\partial t}\mathbf{U} = -\nabla_H^2 \frac{\partial}{\partial t} z$$

with horizontal Laplacian

$$\nabla_H^2 = \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

- Spherical surface harmonics are Eigen function:

$$\nabla_H^2 \, Y_{\ell m} = -\frac{\ell(\ell+1)}{r^2} \, Y_{\ell m}$$

- Pressure equation:

$$\nabla_H \frac{\partial}{\partial t}\mathbf{U} = \nabla_H^2 \frac{\partial}{\partial t}\frac{\partial}{\partial r} w$$

7 Equations for 7 Unknowns

- Radial component of NS equation for poloidal flow potential w .
- Radial component of the curl of the NS equation for toroidal flow potential z .
- Horizontal divergence of the NS equation for pressure p.
- Heat equation for temperature T.
- Radial component of induction equation for poloidal field potential b .
- Radial component of the curl of the induction equation for toroidal field potential j .
- Continuity equations automatically fulfilled!

Boundary Conditions

- Flow:

  either rigid boundaries with vanishing flow

$$w = \frac{\partial}{\partial r} w = z = 0$$

  or stress-free

$$w = \left(\frac{\partial^2}{\partial r^2} - \frac{2}{r} \right) w = \left(\frac{\partial}{\partial r} - \frac{2}{r} \right) z = 0$$

- Magnetic field is matched to a potential field at the outer boundary and to a simplified solution for a rigid inner core in the inner boundary.

- Matching condition to a potential field at the CMB reads:

$$b_{\ell m} + \frac{r_o}{\ell} \frac{\partial}{\partial r} b_{\ell m} = 0$$

- Temperature boundary conditions are either fixed temperature or fixed heat flux (radial gradient of temperature).

Pseudospectral Approach

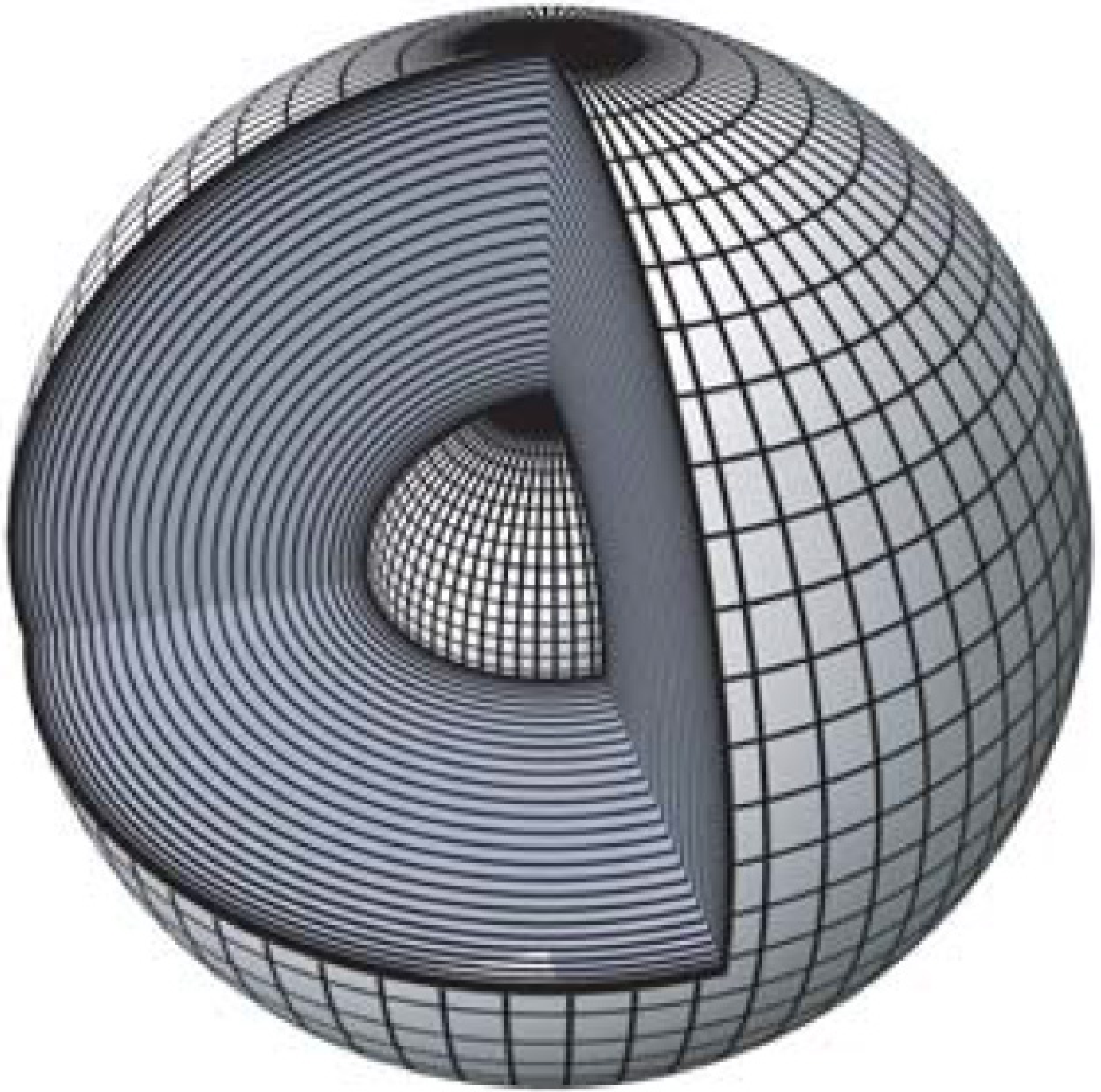
- Derivatives are best calculated in spectral space (recurrence relations).
- The magnetic boudary condition is easily formulated in spectral space (Ylm).
- The Ylm are Eigen functions of the Laplace operator (dissipation).

- Output wanted in physical grid.
- Non-linear terms are beste calculated on physical grid.

- We thus need a spectral and a grid representations
and transformations between the two.

Numerical Grid

- Equidistant points in longitude for FFT
- Gauss-Legendre grid points in latitude for Gauss-Legendre transform
- Special grid points in radius to use FFT for Chebychev polynomials C_n:



Horizontal Representation

- Spherical surface harmonics on longitude and colatitude:

$$Y_\ell^m(\theta,\phi) = P_\ell^m(\cos\theta)\, e^{im\phi}$$

$$\int_0^{2\pi} d\,\phi \int_0^{\pi} \sin\theta\, d\theta\; Y_\ell^m(\theta,\phi)\, Y_{\ell'}^{m'}(\theta,\phi) \;=\; \delta_{\ell\ell'}\delta^{mm'}$$

- Grid representation with degree and order up to ℓ_{max}

$$g(r,\theta,\phi) = \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} g_{\ell m}(r)\, Y_\ell^m(\theta,\phi)$$

- Spectral representation:

$$g_m(r,\theta) = \frac{1}{2\pi} \int_0^{2\pi} d\phi\; g(r,\theta,\phi)\; e^{-im\phi}.$$

$$g_{\ell m}(r) = \frac{1}{\pi} \int_0^{\pi} d\theta \sin\theta\; g_m(r,\theta)\; P_\ell^m(\cos\theta)$$

Horizontal Transforms

- FFT for ϕ–>m and m–>ϕ , with $N_\phi = 2 N_\theta$ grid points
- Gauss-Legendre integration for θ–>ℓ

$$g_{\ell m}(r) = \frac{1}{N_\theta} \sum_{j=1}^{N_\theta} w_j \, g_m(r, \theta_j) \; P_\ell^m(\cos\theta_j)$$

- Clever vector multiplication for ℓ–>θ
- Full dealiazing with $\ell_{max} = [\min(2N_\theta, N_\phi) - 1]/3$
- Both transforms faster with SHTns

Horizontal Derivatives

- Horizontal Laplacian:

$$\Delta_H Y_\ell^m = -\frac{\ell(\ell+1)}{r^2} Y_\ell^m$$

- θ derivative

$$\vartheta_2 = \sin\theta \frac{\partial}{\partial\theta}$$

$$\vartheta_2 P_\ell^m(\cos\theta) = \ell\, c_{\ell+1}^m\, P_{\ell+1}^m(\cos\theta) - (\ell+1)\, c_\ell^m\, P_{\ell-1}^m(\cos\theta)$$

with

$$c_\ell^m = \sqrt{\frac{(\ell+m)(\ell-m)}{(2\ell-1)(2\ell+1)}}$$

so that

$$\int_0^\pi d\theta\, \sin\theta\, P_\ell^m \vartheta_2 \sum_{\ell'} f_{\ell'}^m P_{\ell'}^m = (\ell-1)\, c_\ell^m\, f_{\ell-1}^m - (\ell+2)\, c_{\ell+1}^m\, f_{\ell+1}^m$$

Radial Representation

- Chebychev polynomials up to degree N

$$g_{\ell m}(r) = \sum_{n=0}^{N} g_{\ell m n}\, \mathcal{C}_n(r)$$

$$\mathcal{C}_n(x) = \cos\left[n\ \arccos(x)\right] \qquad -1 \leq x \leq 1$$

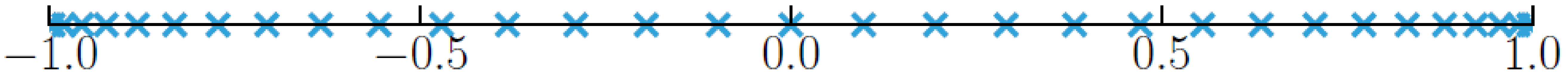
- Grid points are the N extrema of $\mathcal{C}_{N_r-1}$

$$x_k = \cos\left(\pi \frac{(k-1)}{N_r - 1}\right) \quad , \quad k = 1, 2, \ldots, N_r$$

so that

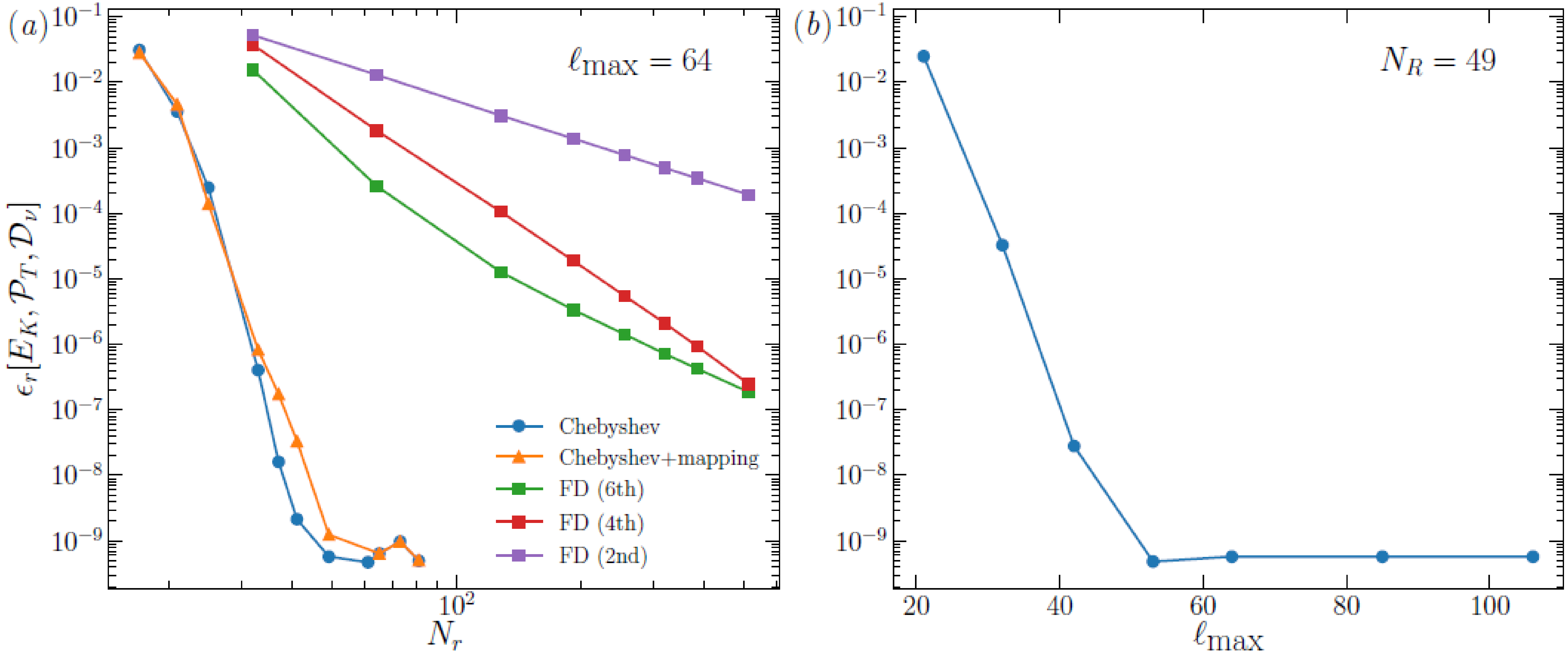
$$\mathcal{C}_{nk} = \mathcal{C}_n(x_k) = \cos\left(\pi \frac{n(k-1)}{N_r - 1}\right)$$

- This allows us to use FFT in radius.
- Derivatives are calculated with recurrence relations.
- Finite difference schemes are also available!



Collocation versus Finite Differences

A comparison for the benchmark BM0 (Christensen *et al.* 2001)



Calculating the Non-Linear Term

- Calculate horizontal components of EMF $\mathcal{F} = \boldsymbol{u} \times \boldsymbol{B}$ on grid

$$\mathcal{F}_\theta = u_\phi B_r - u_r B_\phi, \quad \mathcal{F}_\phi = u_r B_\theta - u_\theta B_r$$

- Then

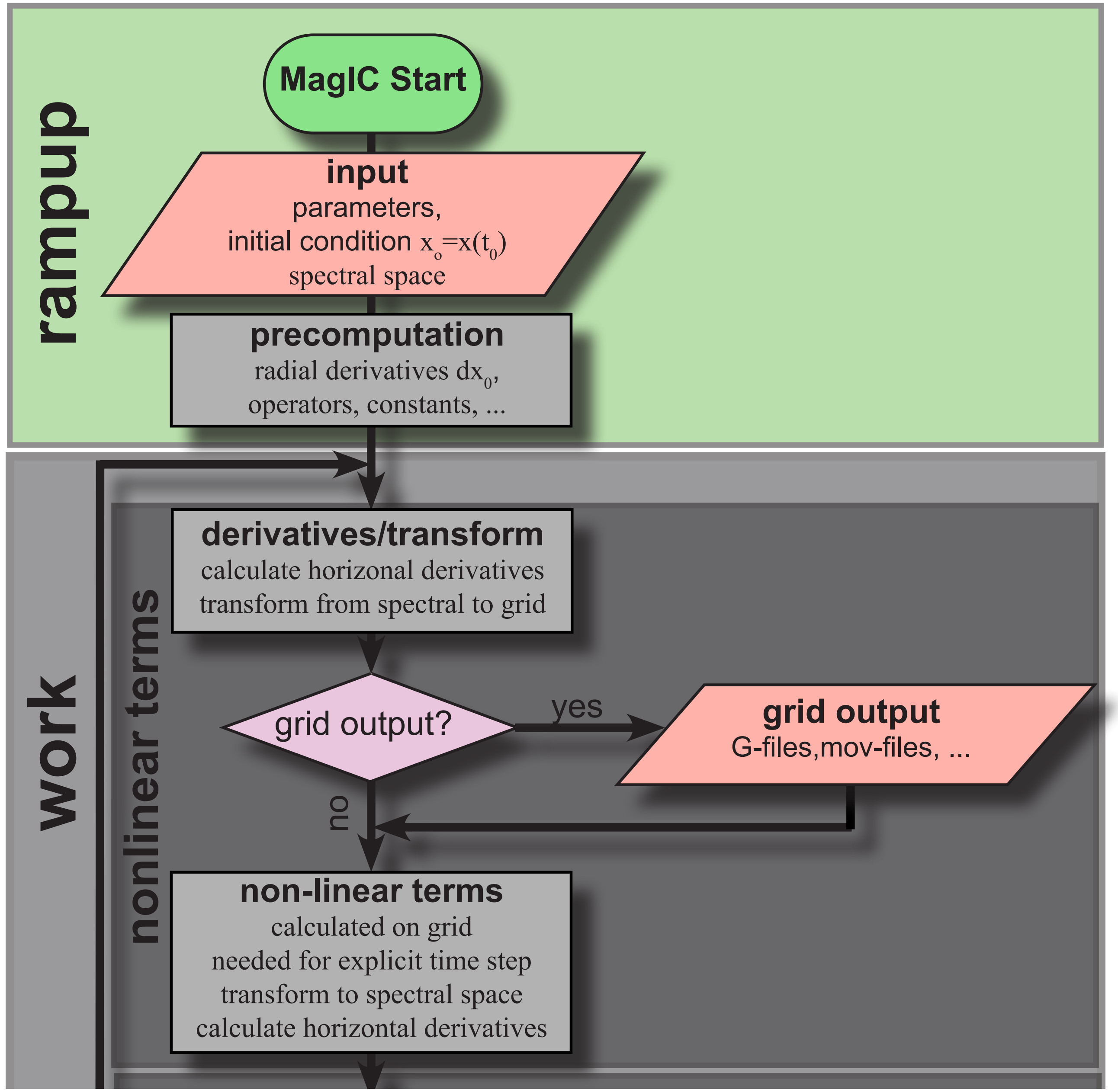
$$\mathcal{N}^g = \boldsymbol{e_r} \cdot [\boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B})] = \frac{1}{r\sin\theta}\left[\frac{\partial\left(\sin\theta\mathcal{F}_\phi\right)}{\partial\theta} - \frac{\partial\mathcal{F}_\theta}{\partial\phi}\right]$$

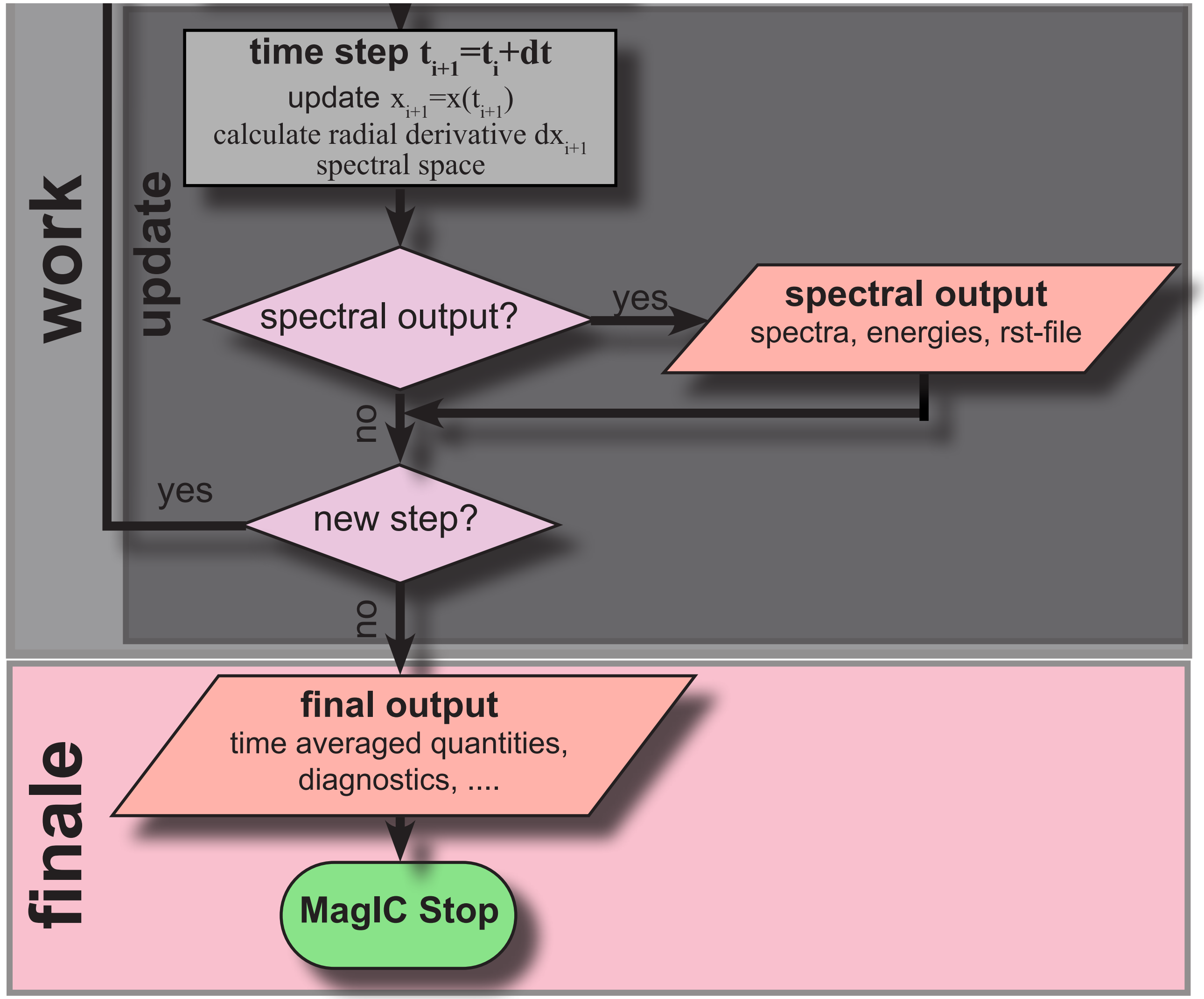
- Transform components to spectral space: ${\mathcal{F}_\theta}_\ell^m \quad {\mathcal{F}_\phi}_\ell^m$

- Then calculate derivative in spectral space:

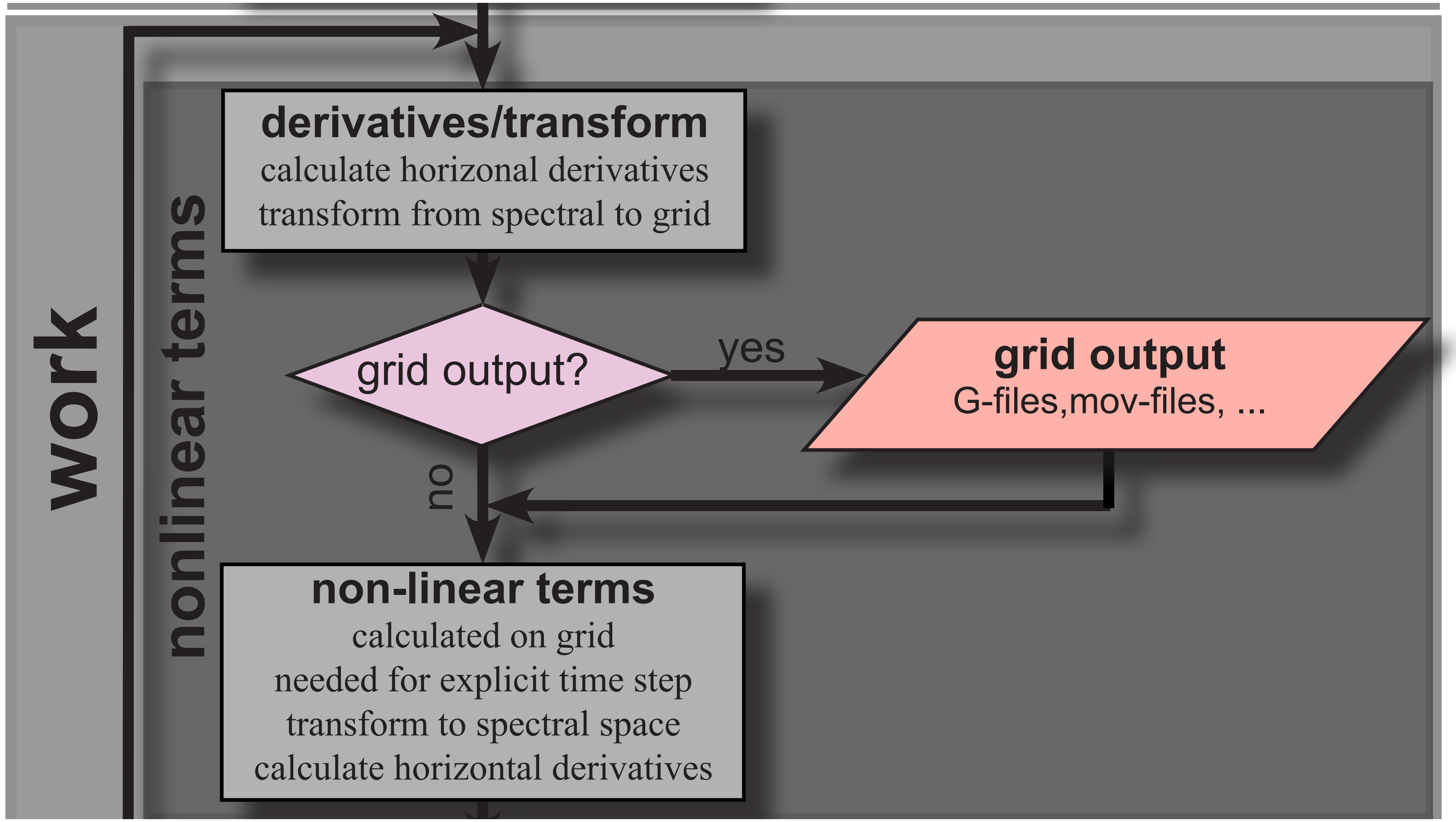
$$\mathcal{N}^g_{\ell m} = (\ell+1)\, c_\ell^m\, {\mathcal{F}_\phi}_{\ell-1}^m - \ell\, c_{\ell+1}^m\, {\mathcal{F}_\phi}_{\ell+1}^m - im\, {\mathcal{F}_\theta}_\ell^m$$

MagIC Structure





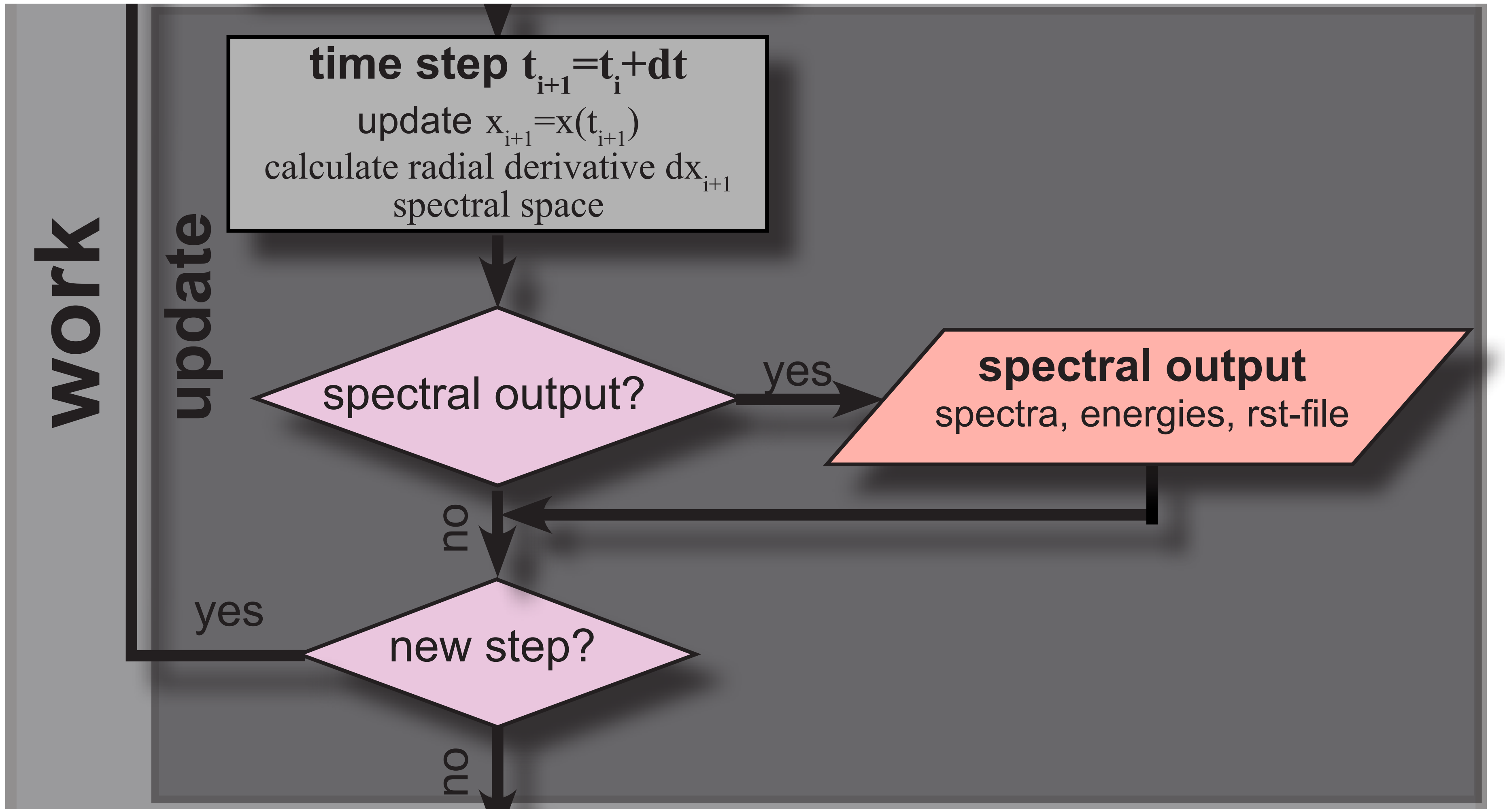
Radial Loop



$$b(r,\ell,m)$$
$$\downarrow$$
$$b(r,\theta,\phi)$$
$$\downarrow$$
$$b(r,\ell,m)$$

- Solving the non-linear terms takes most of the time. They are calculated in grid space and then transformed to spectral space.

LM Loop



$$b(t, r, \ell_i, m_i) \;\rightarrow\; b(t+\delta t, r, \ell_i, m_i)$$

- The time step is performed for each spherical harmonic mode, solving a linear system of equations, one equation per radial grid point. Boundary equations are replaced by the bounbary conditions.

Time Discretization

- Generic evolution equation with:

  terms $\mathcal{I}(x,t)$ to be treated implicitely and
  terms $\mathcal{E}(x,t)$ to be treated explicitely

$$\frac{\partial}{\partial t} x + \mathcal{I}(x,t) = \mathcal{E}(x,t)$$

- Implicit Crank-Nicolson time step:

$$\left(\frac{x(t+\delta t) - x(t)}{\delta t} \right)_I = -\alpha \, \mathcal{I}(x, t+\delta t) - (1-\alpha) \, \mathcal{I}(x,t)$$

- Explicit Adams-Bashforth time step:

$$\left(\frac{x(t+\delta t) - x(t)}{\delta t} \right)_E = \frac{3}{2} \, \mathcal{E}(x,t) - \frac{1}{2} \, \mathcal{E}(x, t-\delta t)$$

- Combined mixed time step yields algebraic equation

$$\frac{x(t+\delta t)}{\delta t} + \alpha \, \mathcal{I}(x, t+\delta t) = \frac{x(t)}{\delta t} - (1-\alpha) \, \mathcal{I}(x,t) + \frac{3}{2} \, \mathcal{E}(x,t) - \frac{1}{2} \, \mathcal{E}(x, t-\delta t)$$

- Time step is checked with a modified Courant-Friedrich-Levy criterion.
- Note: Several other higher order methods have been implemented.

Example Algebraic Equation for Poloidal Magnetic Field

$$(\mathcal{A}_{kn} + \alpha\,\mathcal{I}_{kn})\;g_{\ell mn}(t+\delta t) = (\mathcal{A}_{kn} - (1-\alpha)\,\mathcal{I}_{kn})\;g_{\ell mn}(t) + \frac{3}{2}\,\mathcal{E}_{k\ell m}(t) - \frac{1}{2}\,\mathcal{E}_{k\ell m}(t-\delta t)$$

with

$$\mathcal{A}_{kn} = \frac{\ell(\ell+1)}{r_k^2}\,\frac{1}{\delta t}\mathcal{C}_{nk}$$

$$\mathcal{I}_{kn} = \frac{\ell(\ell+1)}{r_k^2}\,\frac{1}{Pm}\left(\frac{\ell(\ell+1)}{r_k^2}\,\mathcal{C}_{nk} - \mathcal{C}''_{nk}\right)$$

$$\mathcal{E}_{k\ell m}(t) = \mathcal{N}^{g}_{k\ell m} = \int d\Omega\; Y_\ell^{m\star}\;\boldsymbol{e_r}\cdot\boldsymbol{D}(t, r_k, \theta, \phi)$$

- For each spectral mode we solve a system of algebraic linear equations, one equation for each radial grid point.
- Equations for outer and inner radial grid points are replace with boundary conditions.

MagIC in Words 2

- Open access
- Extensive manual
- Extensive post-processing including visualization
- High degree of parallelization
- Solves for convection and magnetic field generation
- Rotating frame of reference
- Spherical shell or full sphere
- Dimensionless formulation
- Poloidal/toroidal decomposition
- Pseudo-spectral approach
- Choice of pseudospectral or finite differences in radius
- Choice of different time stepping schemes
- Automatic check of adapted Courant-Friedrich-Levy criterium

Spectral Poloidal Dynamo Equation

- Radial component of induction equation

$$\boldsymbol{e_r} \cdot \left(\frac{\partial \boldsymbol{B}}{\partial t} \right) = \frac{\partial}{\partial t} (\boldsymbol{e_r} \cdot \boldsymbol{B}) = -\Delta_H \frac{\partial g}{\partial t}$$

- Plug in spectral representation of g, multiply with spherical harmonic, integrate over spherical surface, use othogonality relations.
  This yields the spectral form:

$$\frac{\ell(\ell+1)}{r^2} \left[\left(\frac{\partial}{\partial t} + \frac{1}{Pm} \lambda \frac{\ell(\ell+1)}{r^2} \right) \mathcal{C}_n - \frac{1}{Pm} \lambda \mathcal{C}_n'' \right] g_{\ell m n}$$

$$= \mathcal{N}^g_{\ell m} = \int d\Omega \, Y_\ell^{m\star} \, \mathcal{N}^g = \int d\Omega \, Y_\ell^{m\star} \, \boldsymbol{e_r} \cdot \boldsymbol{D}$$

- D is the radial induction term calculated in grid space

Final Spectral Equations

- This yields a purely spectral equation:

$$\frac{\ell(\ell+1)}{r^2}\left[\left(\frac{\partial}{\partial t}+\frac{1}{Pm}\lambda\,\frac{\ell(\ell+1)}{r^2}\right)\mathcal{C}_n-\frac{1}{Pm}\,\lambda\,\mathcal{C}_n''\qquad\right]\quad g_{\ell mn}\;=\mathcal{N}^g_{\ell m}$$

- Treating the other equations similarly means that all spatial derivatives are taken care off!
- What do we do with the time derivative?