CAS^LMU Research Group
"Algebraic Groups, Motives and Applications."

Motifs and Motives

29 June to 3 July 2026

Abstracts

Joseph Ayoub

On the classicality of the motivic Galois group

I will outline a new approach to prove that the motivic Galois group, which is a priori a derived affine group scheme, is actually classical, i.e., an affine group scheme in the classical sense.

Kęstutis Česnavičius

Generically trivial torsors under constant groups

For a smooth variety X over a field k and a smooth k-group scheme G, Grothendieck and Serre predicted that every generically trivial G-torsor over X trivializes Zariski locally on X. I will explain a resolution of the Grothendieck--Serre problem, the main new case being when k is imperfect, in which pseudo-reductive and quasi-reductive groups play a central role. The argument is built on new purity and extension theorems for torsors valid for pseudo-finite, pseudo-proper, and pseudo-complete groups, and it also rests on several other new results on algebraic groups in positive characteristic. The talk is based on joint work with Alexis Bouthier and Federico Scavia.

Jean Fasel

Chow-Witt groups of quadrics (joint work with A. Ananyevskiy)

I will survey a few recent results of the computation of the Chow-Witt groups of smooth quadrics.

Terry Gannon

Descent considerations for Vertex Algebras
Part 2. Representations

Vertex algebras exist primarily for their representations. In my talk I'll survey their representation theory, with an eye to descent. This is joint work with Robin Mader and Arturo Pianzola.

Stefan Gille

Rost nilpotence

This is an overview talk on Rost nilpotence and generalizations of it.

Olivier Haution

Algebraic cobordism and fixed locus dimension of abelian groups

I will investigate the restrictions on the actions of an abelian groupon a smooth projective variety that come from the Chern numbers of the ambient variety. Given an abelian group G and a cobordism class, what are all the possible fixed locus dimensions for the G-actions on a variety representing that class?

Determining lower bounds on the dimension in the case of p-group actions turns out to be the main challenge. We proceed by constructing explicit examples of actions with low-dimensional fixed locus, and by showing, through an analysis of the equivariant cobordism ring, that these exhaust all possibilities.

Detlev Hoffmann

Equivalence relations for quadratic forms and their quadrics

We consider equivalence relations for quadratic forms (in characteristic not 2) that can be interpreted in terms of algebraic properties of quadratic forms as well as in terms of geometric properties of the associated quadrics: similarity, birational and stably birational equivalence, motivic equivalence.

We address the Quadratic Zariski Problem that asks if two stably birationally equivalent quadrics of the same dimension are already birationally equivalent. This is an open problem and no counterexamples are known. We give an overview over positive results.

We also address a question originally investigated by Izhboldin:

Are two quadratic forms that are motivically equivalent already similar?

He showed that the answer is positive in odd dimensions or in dimensions less than 8, but that there are counterexamples in all even dimensions at least 8 except possibly 12. Izhboldin's arguments don't yield counterexamples in terms of explicitly given forms (e.g., in terms of a diagonalization over a specific field). Using the interplay between the above mentioned equivalence relations, we present an explicit construction of counterexamples in all dimensions at least 8 that are divisible by 4, thus also settling the last remaining open case of dimension 12 (joint work with Nils Poprawski).

Nikita Karpenko

Fields of any u-invariant but a 2-power minus 1 and 3

The u-invariant of a field is the highest dimension of a non-degenerate anisotropic quadratic form over this field. Let u be a natural number not of the form a 2-power minus 1 or a 2-power minus 3. Given any field, we find its overfield of the u-invariant u.

Bruno Klingler

Constructing algebraic cycles from variations of Hodge structures

Given a smooth algebraic variety S over the complex numbers, I will explain how to construct finitely generated subrings of its Chow ring using Hodge theory.

Marc Levine

Toward a Riemann-Roch theorem for 0-cycles on a singular variety

Chuck Weibel and I defined a group of 0-cycles modulo a suitable notion of rational equivalence, $CH_0(X, X_{sing})$, on a quasi-projective k-scheme over an infinite perfect field k, together with a cycle class map $cl_0:CH_0(X, X_{sing})\to F_0K_0(X)$, where the latter group is the subgroup of $K_0(X)$ generated by classes $[k(x)]$ for x a closed point in $X\text{-}X_{sing}$. By construction, cl_0 is surjective.

Various authors (Biswas-Srinivas, Gupta-Krishna, …) have studied the kernel of cl_0 in various settings, for example, Biswas-Srinivas show cl_0 is injective for X of dimension ≤ 2 and Gupta-Krishna prove injectivity for X affine over an algebraically closed field; there are other results in other similar settings.

In this lecture, we describe a work-in-progress that uses some of the methods developed by Biswas-Srinivas, together with a «representable» presentation of $K_0(X)$ via Grassmannians and flag varieties, to construct a Chern class map

$c_d:K_0(X)\to CH_0(X, X_{sing})$, $d=\dim_k X$,

as a map of pointed sets, and to prove a Riemann-Roch theorem relating the restriction of c_d to $F_0K_0(X)$ with cl_0. This will show that for X of dimension d over an infinite perfect field k, the kernel of cl_0 is (d-1)!-torsion.

Fabien Morel

How to use $\mathbb{A}^1$-homotopy theory to (try to) classify rational smooth projective varieties over a field?

In this talk I will try to explain how $\mathbb{A}^1$-homotopy theory provides new types of invariant which can be used to distinguish rational smooth projective varieties over a field.

I will give a brief recollection and basic computations in $\mathbb{A}^1$-homotopy and try to explain these invariants, two important of them being the $\mathbb{A}^1$-fundamental group sheaf and the cellular $\mathbb{A}^1$-chain complex, the latter having been recently found in collaboration with A. Sawant.

I will give explicit examples of computations and results, mostly concerning rational smooth projective surfaces over a perfect field, which is itself already non trivial.

This work is based on my two main collaborations in the past 15 years, with A. Asok and more recently with A. Sawant.

Arturo Pianzola

Descent considerations for Vertex Algebras

Part 1. Structure theory

Anne Quéguiner-Mathieu

Critical varieties

Let G be an absolutely almost simple algebraic group and X a G-projective homogeneous variety. We call X critical if its Chow motive (with finite coefficients) determines the isomorphism class of the Chow motive of every G-projective homogeneous variety. We discuss the existence of such a variety, notably for groups of classical type over arbitrary fields. This is based on joint works with several authors, in particular Barry, De Clercq and Zhykhovich.

Alexander Samokhin

Highest weight category structures on rep($\mathbb{B}$) and full exceptional collections on generalized flag varieties over $\mathbb{Z}$

This talk is based on a joint work with Wilberd van der Kallen available at https://arxiv.org/abs/2407.13653.

For a split simply connected and connected algebraic group scheme $\mathbb{G}$ over $\mathbb{Z}$ and a split parabolic subgroup scheme $\mathbb{P}\subset\mathbb{G}$, we construct semi-orthogonal decompositions of the bounded derived category $D^b(\mathrm{rep}(\mathbb{P}))$ of noetherian representations of $\mathbb{P}$ with each semi-orthogonal component being equivalent to the bounded derived category $D^b(\mathrm{rep}(\mathbb{G}))$ of noetherian representations of $\mathbb{G}$. The semi-orthogonal components of those decompositions are stable under the monoidal action of $D^b(\mathrm{rep}(\mathbb{G}))$ on $D^b(\mathrm{rep}(\mathbb{P}))$. The decompositions depend on an arbitrarily chosen total order on the Weyl group that refines the Bruhat order. The semi-orthogonal decompositions are also compatible with the Bruhat order on cosets of the Weyl group of $\mathbb{P}$ in the Weyl group of $\mathbb{G}$. Their construction builds upon the foundational results on $\mathbb{B}$-modules from the works of Mathieu, Polo, and van der Kallen, and upon properties of the Steinberg basis of the $\mathbb{T}$-equivariant K-theory of $\mathbb{G}/\mathbb{B}$. As a corollary, we obtain full exceptional collections in the bounded derived category of coherent sheaves on generalized flag schemes $\mathbb{G}/\mathbb{P}$ over $\mathbb{Z}$.

Time permitting, we will mention potential applications to geometric representation theory.

Anand Sawant

Weil restriction, cellular structures and Milnor-Witt K-theory

The talk is based on joint work with Fabien Morel.

The category of strictly $\mathbb{A}^1$-invariant sheaves is central to the development of $\mathbb{A}^1$-algebraic topology over a field. It is a nontrivial fact that this is an abelian, symmetric monoidal category. I will outline the computation of the tensor product of the unramified Milnor-Witt K-theory with the so-called free strictly $\mathbb{A}^1$-invariant sheaf on a point, defined over a field extension of the base field. The computation is made possible by cellular $\mathbb{A}^1$-homology introduced in our earlier joint work.

Federico Scavia

Splitting Brauer classes by genus one curves

Let F be a field. A basic way to study a Brauer class over F is through its splitting fields, that is, field extensions of F over which the class becomes trivial. I will discuss a question of Clark and Saltman: is every Brauer class over F split by a smooth projective genus one curve over F? Equivalently, does every Severi-Brauer variety X/F admit a morphism C → X, with C a smooth projective curve of genus one? I will present joint work with Zinovy Reichstein showing that the answer is no in general. I will then discuss joint work with Ben Antieau and Asher Auel showing that, in contrast, the answer is yes over number fields.

Stephen Scully

On symmetric bilinear forms of height 2 in characteristic 2

The height of an anisotropic symmetric bilinear form b over a field F is the maximal length of an irredundant splitting tower for b. The forms of height 1 are the general Pfister forms and their codimension-1 subforms. The problem of classifying the forms of height 2, however, is open. When the characteristic of F is not 2, a conjectural classification was proposed by Kahn, but only partial results are known (Kahn, Rost, Izhboldin, Vishik), with the open cases being closely tied to difficult descent problems for quadratic forms and cohomology classes over function fields of quadrics. In this talk, I'll discuss some recent results on the case where F has characteristic 2, which can be approached by elementary means. Here, additional examples appear, revealing a broader connection to another significant open problem about which little is known - the classification of the «low-dimensional» forms in the powers of the fundamental ideal in the Witt ring.

Pavel Sechin

Tate-stable equivalence of integral Chow motives

We exhibit integral Chow motives that are non-isomorphic, yet become isomorphic after adding suitable Tate motives. We call such motives Tate-stably equivalent. Examples include motives of Severi-Brauer varieties, genus 1 curves, and K3 surfaces. This is joint work with H.-Y. Lin and E. Shinder.

Tamás Szamuely

A motivic height pairing over function fields of complex varieties

In his celebrated paper on height pairings, Beilinson has defined a height pairing for homologically trivial algebraic cycles on varieties defined over the function field of a curve. A few years ago, Damian Roessler and I have extended Beilinson's construction to varieties defined over a function field of a base variety B of arbitrary dimension. The values of our pairing are not numbers any more but elements in the second cohomology group of B. We have conjectured that this pairing is of geometric origin, i.e. comes from a pairing with values in the Picard group of B (tensored with $\mathbb{Q}$). I shall report on recent work with P. Brosnan and G. Pearlstein in which we verify this conjecture in the case when the base field is $\mathbb{C}$ by means of a motivic construction.

Matthias Wendt

Equivariant real cycle class map and Witt-sheaf cohomology of classifying spaces

In the talk I will discuss an equivariant version of the real cycle class map relating Witt-sheaf cohomology and singular cohomology of real points. I will explain some examples where this topological comparison can provide new information, such as the Witt-sheaf cohomology of classifying spaces of orthogonal groups or the Witt-sheaf cohomological invariants of spin groups over the real numbers. This is joint work with Lorenzo Mantovani and Akos Matszangosz.

Kirill Zainoulline

A hyperplane decomposition formula for equivariant oriented motives

We extend and generalize Chow-motivic decomposition techniques used to compute motives of hyperplane sections of twisted Milnor hypersurfaces to an arbitrary oriented cohomology in the sense of Levine-Morel. As a byproduct, we prove Rost nilpotence for all those examples of motivic decompositions. This is a recent joint work with Evan Marth and Rui Xiong.